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SOME RESULTS ON CONNECTED ACCURATE EDGE DOMINATION IN FUZZY GRAPHS USING STRONG ARCS

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ABSTRACT

In this paper, connected accurate edge dominating set of a fuzzy graph is discussed. An accurate edge dominating set D is said to be a connected accurate edge dominating set, if an induced fuzzy subgraph <D> of G is connected. The connected accurate edge domination number, $\gamma_{fcae}(G)$ is the minimum fuzzy cardinality taken over all connected accurate edge dominating sets of G. We also determine, upper bounds of connected accurate edge domination number for some standard fuzzy graphs. Relations between connected accurate edge domination number and some other edge domination parameters are discussed.

Keywords: *Connected edge dominating sets, Connected accurate edge domination numbers, Accurate edge dominating set & domination numbers, Strong arc and Strong neighbourhood.*

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INTRODUCTION

Ore [11] and Berge [1] introduced the concept of dominating sets in graphs. Then, the domination number and independent domination number was introduced by Cockayne and Hedetniemi [2]. An accurate domination and connected accurate domination was introduced by Kulli and Kattimani [3, 4]. And also, accurate edge domination number in graph was introduced by Kulli and Kattimani [5]. Kulli and Srgarkanti [6] introduced the concept of connected edge domination number of a graph. Then accurate connected edge domination number in graphs was studied by Venkanagouda M. Goudar *et al.* [14].

The concept of fuzzy relation was introduced by Zadeh[15] in his classical paper in 1965. The notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness was introduces by Rosenfeld [12]. Somasundaram and Somasundaram [13] discussed domination in fuzzy graphs using effective edges. Nagoorgani and Chandrasekaran [7] discussed domination in fuzzy graphs using strong arcs. Nagoorgani and Vadivel [9, 10] discussed domination, independent domination and irredundance in fuzzy graphs using strong arcs. Nagoorgani and Prasanna Devi [8] discussed the concepts of Edge domination and independence in fuzzy graph. We discuss about accurate connected edge domination in fuzzy graphs using strong arcs.

PRELIMINARIES

A *fuzzy graph* $G = \langle\sigma, \mu\rangle$ is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$, where for all $x, y \in V$ we have $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$.

A fuzzy graph $H = \langle\tau, \rho\rangle$ is called a *fuzzy subgraph* of G if $\tau(v_i) \leq \sigma(v_i)$ for all $v_i \in V$ and $\rho(v_i, v_j) \leq \mu(v_i, v_j)$ for all $v_i, v_j \in V$.

An arc (u, v) in a fuzzy graph G is said to be strong, if $\mu^{\infty}(u, v) = \mu(u, v)$. The strong neighbourhood of an edge e_i in a fuzzy graph G is denoted by $N_S(e_i) = \{e_j \in S / S \subseteq E(G), e_j \text{ is adjacent to } e_i\}$, where S is a set of all strong arcs in G.

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An arc $e_i \in E(G)$ in fuzzy graph G is called an *isolated edge* if it is not adjacent to any strong arcs $e_j \in E(G)$ of fuzzy graph G.

The *underlying crisp graph* of a fuzzy graph $G = \langle \sigma, \mu \rangle$ is denoted by $G^* = \langle \sigma^*, \mu^* \rangle$, where $\sigma^* = \{v_i \in V / \sigma(v_i) > 0\}$ and $\mu^* = \{(v_i, v_j) \in V \times V / \mu(v_i, v_j)\ is\ a\ strong\ arc\}$.

The strong neighbourhood edge degree of an edge e_i is, $dN_S(e_i) = \sum_{e_j \in N_S(e_j)} \mu(e_j)$. The minimum strong neighbourhood edge degree of a fuzzy graph G is defined as, $\delta_{EN_S}(G) = min\{dN_S(e_i) / e_i \in S\}$. The maximum strong neighbourhood edge degree of a fuzzy graph G is defined as, $\Delta_{EN_S}(G) = max\{dN_S(e_i) \ / \ e_i \in S\}$.

A fuzzy graph $G = \langle \sigma, \mu \rangle$ is said to be a *complete fuzzy graph*, if $\mu(v_i, v_j) = \sigma(v_i) \wedge \sigma(v_j)$ for all $v_i, v_j \in \sigma^*$.

A fuzzy graph $G = \langle \sigma, \mu \rangle$ is said to be a *connected fuzzy graph*, if there exists a strong path between every pair of nodes.

A fuzzy graph G is said to be a *complete bipartite* fuzzy graph, if the vertex set $V(G)$ can be partitioned into two non-empty sets, $V_1\ and\ V_2$, such that $\mu(v_1, v_2) = 0$, if $v_1, v_2 \in V_1$ or $v_1, v_2 \in V_2$ and $\mu(v_1, v_2) = \sigma(v_1) \wedge \sigma(v_2)$ for all $v_1 \in V_1\ and\ v_2 \in V_2$.

A subset D' of S is said to be an edge dominating set of G, where $S \subseteq E(G)$, set of all strong arcs in $E(G)$, if for every $e_j \in E(G) - D'$ there exists $e_i \in D'$ such that e_i dominates e_j. The minimum cardinality taken overall edge dominating sets of fuzzy graph G is called an edge domination number and it is denoted by $\gamma_{fe}(G)$.

Let a subset D' of S be an edge dominating set of fuzzy graph G and it is said to be an accurate edge dominating set of G, if $S - D'$ has no edge dominating set with same fuzzy cardinality $|D'|$. The accurate edge domination number, which is denoted as $\gamma_{fae}(G)$, of a fuzzy graph G is the minimum fuzzy cardinality taken over all accurate edge dominating sets of G.

3. CONNECTED ACCURATE EDGE DOMINATING SET IN FUZZY GRAPH

Definition 3.1: Let G be the connected fuzzy graph and $D' \subseteq S$, where $S \subseteq E(G)$, set of all strong arcs in $E(G)$.Then the accurate edge dominating set D' is said to be a connected accurate edge dominating set of fuzzy graph G, if the induced fuzzy graph $\langle D' \rangle$ is a connected fuzzy graph.

The connected accurate edge domination number of fuzzy graph G, is the minimum fuzzy cardinality taken over all connected accurate edge dominating sets of G, and it is denoted by $\gamma_{fcae}(G)$.

Example 3.2:

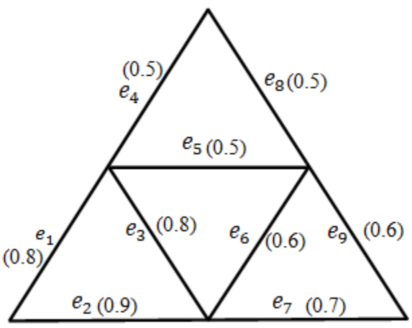

Figure-3.1: Connected Fuzzy Graph $G = (\sigma, \mu)$

From the above fig. 3.1, some of the edge dominating sets are $D_1' = \{e_1, e_9\}$, $D_2' = \{e_3, e_6\}$, $D_3' = \{e_3, e_9\}$, $D_4' = \{e_1, e_6\}$, $D_5' = \{e_4, e_6\}$, $D_6' = \{e_3, e_8\}$, $D_7' = \{e_4, e_7\}$, $D_8' = \{e_4, e_6, e_8\}$, $D_9' = \{e_3, e_7, e_9\}$ and $D_{10}' = \{e_3, e_4, e_7\}$.

Here, D_1', D_2', D_3' and D_4' are edge dominating sets but not accurate edge dominating sets of figure 3.1.
And D_5', D_6', D_7', D_8', D_9', D_{10}' are accurate edge dominating sets of figure 3.1, but D_5', D_6', D_7' are not connected accurate edge dominating sets of figure 3.1.

Then, D'_8, D'_9 and D'_{10} are the connected accurate edge dominating sets of figure 3.1, whose fuzzy cardinality are 1.6, 2.1 and 2.0 respectively. Therefore, the connected accurate edge domination number, $\gamma_{fcae}(G) = min\{1.6, 2.1, 2.0\} = 1.6$.

Remark:

(i) The connected accurate edge dominating set of a fuzzy graph G may or may not be a minimal edge dominating set.

(ii) Every minimum connected accurate edge dominating set of a fuzzy graph G is an accurate edge dominating set and also edge dominating set of G.

Theorem 3.3: For any connected fuzzy graph G, $\gamma_{fe}(G) \leq \gamma_{fae}(G) \leq \gamma_{fcae}(G)$.

Proof: Let G be a connected fuzzy graph and $S \subseteq E(G)$, where S be the set of all strong arcs in G.

Case (i): $\gamma_{fe}(G) \leq \gamma_{fae}(G)$

First we prove that $\gamma_{fe}(G) = \gamma_{fae}(G)$.

Let a subset D' of S, be the minimum edge dominating set of a fuzzy graph G, then $\gamma_{fe}(G) = |D'|$. Suppose that, $\langle S - D' \rangle$ has no edge dominating set with same fuzzy cardinality $|D'|$, then D' itself forms an accurate edge dominating set of G. Therefore, $\gamma_{fe}(G) = \gamma_{fae}(G)$.

Then, from the result, every minimum accurate edge dominating set, $D'_1 \subseteq S$, of a fuzzy graph G is also the edge dominating set of G. Then, the accurate edge domination number is $\gamma_{fae}(G) = |D'_1|$.
That is, $D' \subseteq D'_1$
Then, $|D'| \leq |D'_1|$
Therefore, $\gamma_{fe}(G) \leq \gamma_{fae}(G)$ (3.1)

Case (ii): $\gamma_{fae}(G) \leq \gamma_{fcae}(G)$

Similarly, by case (i),

Let D' be the minimum accurate edge dominating set of fuzzy graph G. If $\langle D' \rangle$ is connected, then D' be the connected accurate edge dominating set of G and the connected accurate edge domination number of fuzzy graph G, is $\gamma_{fcae}(G) = |D'|$.

Also, every minimum connected accurate edge dominating set of a fuzzy graph G is an accurate edge dominating set of G.
That is, $\gamma_{fae}(G) \leq \gamma_{fcae}(G)$ (3.2)

From (3.1) and (3.2), we get,
Since, $\gamma_{fe}(G) \leq \gamma_{fae}(G)$
Then, $\gamma_{fae}(G) \leq \gamma_{fcae}(G)$
Therefore, $\gamma_{fe}(G) \leq \gamma_{fae}(G) \leq \gamma_{fcae}(G)$.

Theorem 3.4: For any connected fuzzy graph G, $\gamma_{fce}(G) \leq \gamma_{fcae}(G)$.

Proof: Let G be a connected fuzzy graph. Let a subset D' of S, $S \subseteq E(G)$ be the set of all strong arcs, be the edge dominating set of G. If $\langle D' \rangle$ is connected, then D' be the connected edge dominating set of G and the connected edge domination number of fuzzy graph G, is $\gamma_{fce}(G) = |D'|$ and $|D'|$ be the minimum fuzzy cardinality taken over all edge dominating sets of G.

Similarly, If $\langle S - D' \rangle$ does not contain any edge dominating set of same fuzzy cardinality $|D'|$ then, D' itself forms an accurate edge dominating set of G. If, the accurate edge dominating set, D', of the fuzzy graph G is with minimum fuzzy cardinality among all accurate edge dominating set of G and the fuzzy subgraph induced by $\langle D' \rangle$ is connected, then, the accurate edge dominating set D' itself forms the connected accurate edge dominating set of G.

Clearly, every minimum connected accurate edge dominating set of a connected fuzzy graph G is also the connected edge dominating set of G. Hence the result.

Theorem 3.5: Let G be a connected fuzzy graph, then $\left\lfloor \frac{|S|}{\Delta_{EN_S}(G)+1} \right\rfloor \leq \gamma_{fcae}(G)$.

Proof: Let G be a connected fuzzy graph. Let $S \subseteq E(G)$ be the set of all strong arcs of G and an edge set, $D' \subseteq S$, be the connected accurate edge dominating set of G, where its fuzzy cardinality is minimum taken over all connected accurate edge dominating sets of G, therefore $\gamma_{fcae}(G) = |D'|$. If there exists a strong arc $e_i \in S$, where the strong edge neighbourhood degree is maximum among for all $e_j \in S$, that is, $\Delta_{EN_S}(G) = |N_S(e_i)|$.
Then,

$$|S| - |D'| \leq |D'||N_S(e_i)|$$
$$|S| \leq |D'||N_S(e_i)| + |D'|$$
$$|S| \leq |D'|(|N_S(e_i)| + 1)$$
$$\frac{|S|}{|N_S(e_i)| + 1} \leq |D'|$$
$$\frac{|S|}{\Delta_{EN_S}(G) + 1} \leq |D'|$$
$$\therefore \left\lfloor \frac{|S|}{\Delta_{EN_S}(G) + 1} \right\rfloor \leq \gamma_{fcae}(G)$$

Theorem 3.6: For any connected fuzzy graph,

$$\gamma_{fcae}(G) \leq \left\lceil \frac{|S| - \delta_{EN_S}(G) + 1}{2} \right\rceil$$

Proof: By above theorem 3.5, Let G be a connected fuzzy graph. Let $S \subseteq E(G)$ be the set of all strong arcs of G and an edge set, $D' \subseteq S$, be the connected accurate edge dominating set of G, where its fuzzy cardinality is minimum taken over all connected accurate edge dominating sets of G, therefore $\gamma_{fcae}(G) = |D'|$. If there exists a strong arc, $e_i \in S$, where the strong edge neighbourhood degree is minimum among for all $e_j \in S$, that is, $\delta_{EN_S}(G) = |N_S(e_i)|$.

$$|D'| + |N_S(e_i)| \leq |S| - |D'| + 1$$
$$|D'| + |D'| \leq |S| - |N_S(e_i)| + 1$$
$$2|D'| \leq |S| - |N_S(e_i)| + 1$$
$$\therefore \gamma_{fcae}(G) \leq \left\lceil \frac{|S| - \delta_{EN_S}(G) + 1}{2} \right\rceil.$$

Theorem 3.7: Let G be any connected fuzzy graph, then $\gamma_{fcae}(G) \leq 2|S| - \gamma_{fae}(G)$.

Proof: Let G be a connected fuzzy graph. A subset S of $E(G)$ be the set of all strong arcs in G. Let $D_1', D_2' \subseteq S$, be the minimum connected accurate edge dominating set and the minimum accurate edge dominating set of G respectively.
Since, G is a connected fuzzy graph, $D_1' \cup D_2' \subseteq S$.
By theorem 3.3, $\gamma_{fae}(G) \leq \gamma_{fcae}(G)$, that is, $|D_2'| \leq |D_1'|$.
And, $|D_1'| \geq |S| - |D_2'|$

$$|D_1'| \leq |S| + |D_1'|$$
$$|D_1'| \leq |S| + |S| - |D_2'|$$
$$|D_1'| \leq 2|S| - |D_2'|$$

Therefore, $\gamma_{fcae}(G) \leq 2|S| - \gamma_{fae}(G)$

Corollary: For any connected fuzzy graph, $\gamma_{fcae}(G) \leq 2|S| - \gamma_{fce}(G)$.

4. Upper bounds of connected accurate edge domination number for some standard fuzzy graphs:

Observation 4.1: If the connected fuzzy graph G is a path P_n, with n nodes, then

$$\gamma_{fcae}(P_n) \leq n - 3$$

Observation 4.2: If the connected fuzzy graph G is a cycle C_n, with n nodes, then

$$\gamma_{fcae}(C_n) \leq \begin{cases} n - 1 \; if \; n \leq 4 \\ n - 2 \; if \; n \geq 5 \end{cases}$$

Observation 4.3: If the connected fuzzy graph G is a complete fuzzy graph K_n, with n nodes, then

$$\gamma_{fcae}(K_n) \leq \left\lfloor \frac{n(n-1)}{4} \right\rfloor + 1$$

Observation 4.4: If the connected fuzzy graph G is a complete bipartite fuzzy graph $K_{m,n}$, $m \leq n$, with $(m+n)$ nodes, then

$$\gamma_{fcae}\left(K_{m,n}\right) \leq \left\lfloor \frac{mn}{2} \right\rfloor + 1$$

Observation 4.5: If the connected fuzzy graph G is a star (S_n), with n nodes, then

$$\gamma_{fcae}(S_n) \leq \left\lfloor \frac{n}{2} \right\rfloor$$

Observation 4.6: If the connected fuzzy graph G is a wheel W_n, with n nodes, then

$$\gamma_{fcae}(W_n) \leq n$$

CONCLUSION

Some results on connected accurate edge dominating sets and the relations between connected accurate edge domination number with other parameters of edge domination numbers are determined. Also, upper bounds of connected accurate edge domination number for some standard fuzzy graphs are found.

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