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INTERVAL VALUED INTUITIONISTIC Q-FUZZY IDEALS OF NEAR-RINGS

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ABSTRACT

In this paper, we introduce the notion of interval valued Q-fuzzy ideal of near-rings and investigate some related properties.

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1. INTRODUCTION

The theory of fuzzy sets was introduced by Zadeh [12]. Rosenfeld [10] first introduce the fuzzification of the algebraic structures and defined fuzzy subgroups. The intuitionistic fuzzy sets (IFSs) are substantial extensions of the ordinary fuzzy sets. IFSs are objects having degrees of membership and non-membership such that their sum is less than or equal to 1. The most important property of IFSs not shared by the fuzzy sets is that modal-like operators can be defined over IFSs. The IFSs have essentially higher describing possibilities than fuzzy sets.

The notion of intuitionistic fuzzy sets was introduced by Atanasov [2] as a generalization of the notion of fuzzy sets. Biswas [4] applied the concept of intuitionistic fuzzy sets to the theory of groups and studied intuitionistc fuzzy subgroups of a group. The notion of an intuitionistic fuzzy ideal of a near ring is given by Jun,Kim and Yon [7]. Also Cho and Jun [5] introduced the notion of intuitionistic fuzzy R-subgroups in a near-ring and related propertied are investigated. Kazanci,Yamak and Yilmaz [8] introduced intuitionistic Q -fuzzy R -subgroups of near-rings. Kim [9] introduced intuitionistic Q -fuzzy semiprime ideals of semigroups. The notion of interval valued intuitionistic fuzzy sets was introduced by Atanasov [3] as a generalization of the notion of intuitionistic fuzzy sets. In this paper we introduce the notion of interval valued intuitionistic Q -fuzzy ideals of near-rings and investigate some related properties.

2. PRELIMINARIES

We recall some definitions for the sake of completeness. Throughout this paper Q denotes any non-empty set and N is a near-ring.

By a near-ring we mean a non-empty set N with two binary operations "$+$" and "$.$" satisfying the following axioms:

(i) ($N,+$) is a group;

(ii) ($N,.$) is semi-group;

(iii) $x.(y+z) = x.y + x.z$ for all $x, y, z \in N$.

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Precisely speaking, it is a left near-ring because it satisfies the left distributive law. We will use the word "near-ring" instead of "left near-ring". We denote xy instead of $x.y$. Note that $x0=0$ and $x(-y)=-xy$, but $0x\neq 0$ for $x,y\in N$. A function $\mu : N\times Q\to[0,1]$ is called a Q-fuzzy set. Let μ be a Q-fuzzy set in N and $t\in[0,1]$ then an upper t-level cut of μ is defined by $U(\mu;t)=\{x\in N \mid \mu(x,q)\geq t, q\in Q\}$. Let μ be a Q-fuzzy set in N and $t\in[0,1]$, then an lower t-level cut of μ is defined by $L(\mu;t)=\{x\in N \mid \mu(x,q)\leq t, q\in Q\}$.

An intuitionistic Q-fuzzy set A is an object having the form $A=\{((x,q),\mu_A(x,q),\lambda_A(x,q)) \mid x\in N, q\in Q\}$, where the functions $\mu_A : N\times Q\to[0,1]$ and $\lambda_A : N\times Q\to[0,1]$ denote the degree of membership and the degree of non membership of each element $(x,q)\in N\times Q$ to the set A, respectively, such that
$0\leq\mu_A(x,q)+\lambda_A(x,q)\leq 1$ for $x\in N, q\in Q$.
A Q-fuzzy set μ is called a fuzzy N-subnear-ring of N over Q if

(i) $\mu(x-y,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)$,

(ii) $\mu(xy,q)\geq\mu_A(x,q)\wedge(y,q)$

for all $x,y\in N$ and $q\in Q$.

A Q-fuzzy set μ is called a fuzzy N-subgroup of N over Q if

(i) $\mu(x-y,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)$

(ii) $\mu(rx,q)\geq\mu_A(x,q)$,

(iii) $\mu(xr,q)\geq\mu_A(x,q)$

for all $x,y,r\in N$ and $q\in Q$.

A Q-fuzzy set μ is called a fuzzy ideal of N over Q if

(i) $\mu(x-y,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)$

(ii) $\mu(rx,q)\geq\mu_A(x,q)$,

(iii) $\mu((x+i)y-xy)\geq\mu_A(i,q)$

for all $x,y,i\in N$ and $q\in Q$.

Definition 2.11: An interval number $\bar{a}$ on [0,1] is a closed subinterval of [0,1], that is, $\bar{a}=[a^-,a^+]$ such that $0\leq a^-\leq a^+\leq 1$ where a^- and a^+ are the lower and upper end limits of $\bar{a}$ respectively. The set of all closed subintervals of $[0,1]$ is denoted by $D[0,1]$. We also identify the interval $[a,a]$ by the number $a\in[0,1]$. For any interval numbers $\bar{a}_i=[a_i^-,a_i^+], \bar{b}_i=[b_i^-,b_i^+]\in D[0,1], i\in I$, we define

$$max^i\{\bar{a}_i,\bar{b}_i\}=[max^i\{a_i^-,b_i^-\},max^i\{a_i^+,b_i^+\}],$$

$$min^i\{\bar{a}_i,\bar{b}_i\}=[min^i\{a_i^-,b_i^-\},min^i\{a_i^+,b_i^+\}],$$

$$inf^i\,\bar{a}_i=[\bigcap_{i\in I}a_i^-,\bigcap_{i\in I}a_i^+], sup^i\,\bar{a}_i=[\bigcup_{i\in I}a_i^-,\bigcup_{i\in I}a_i^+]$$

In this notation $\bar{0}=[0,0]$ and $\bar{1}=[1,1]$. For any interval numbers $\bar{a}=[a^-,a^+]$ and $\bar{b}=[b^-,b^+]$ on $[0,1]$, define

(1) $\bar{a}\leq\bar{b}$ if and only if $a^-\leq b^-$ and $a^+\leq b^+$.

(2) $\bar{a}=\bar{b}$ if and only if $a^-=b^-$ and $a^+=b^+$.

(3) $\bar{a}<\bar{b}$ if and only if $\bar{a}\leq\bar{b}$ and $\bar{a}\neq\bar{b}$

(4) $k\bar{a}=[ka^-,ka^+]$, whenever $0\leq k\leq 1$.

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Definition 2.2:2 Let X be any set. A mapping $\overline{A}: X \to D[0,1]$ is called an interval-valued fuzzy subset (briefly, i-v fuzzy subset) of X where $D[0,1]$ denotes the family of all closed subintervals of $[0,1]$ and $\overline{A}(x) = [A^-(x), A^+(x)]$ for all $x \in X$, where A^- and A^+ are fuzzy subsets of X such that $A^-(x) \le A^+(x)$ for all $x \in X$.
Note that $\overline{A}(x)$ is an interval (a closed subset of [0,1]) and not a number from the interval [0,1] as in the case of fuzzy subset.

Definition 2.3:3 $[10]$ A mapping $min^i : D[0,1] \times D[0,1] \to D[0,1]$ defined by
$min^i(\overline{a}, \overline{b}) = [min\{a^-, b^-\}, min\{a^+, b^+\}]$ for all $\overline{a}, \overline{b} \in D[0,1]$ is called an interval min-norm.

Definition 2.4:4 A mapping $max^i : D[0,1] \times D[0,1] \to D[0,1]$ defined by
$max^i(\overline{a}, \overline{b}) = [max\{a^-, b^-\}, max\{a^+, b^+\}]$ for all $\overline{a}, \overline{b} \in D[0,1]$ is called an interval max-norm.
Let min^i and max^i be the interval min-norm and max-norm on $D[0,1]$ respectively. Then the following are true.

1. $min^i\{\overline{a}, \overline{a}\} = \overline{a}$ and $max^i\{\overline{a}, \overline{a}\} = \overline{a}$ for all $\overline{a} \in D[0,1]$.
2. $min^i\{\overline{a}, \overline{b}\} = min^i\{\overline{b}, \overline{a}\}$ and $max^i\{\overline{a}, \overline{b}\} = max^i\{\overline{b}, \overline{a}\}$ for all $\overline{a}, \overline{b} \in D[0,1]$.
3. If $\overline{a} \ge \overline{b} \in D[0,1]$, then $min^i\{\overline{a}, \overline{c}\} \ge min^i\{\overline{b}, \overline{c}\}$ and $max^i\{\overline{a}, \overline{c}\} \ge max^i\{\overline{b}, \overline{c}\}$ for all $\overline{c} \in D[0,1]$.

Definition 2.5:5 An interval valued intuitionistic fuzzy set (i-v IFS,shortly) $\overline{A}''$ over X is an object having the form $\overline{A} = \{(x, \overline{\mu}_A, \overline{\lambda}_A) : x \in X\}$, where $\overline{\mu}_A(x) : X \to D[0,1]$ and $\overline{\lambda}_A(x) : X \to D[0,1]$, the intervals $\overline{\mu}(x)$ and $\overline{\lambda}_A(x)$ denotes the intervals of the degree of membership and the degree of the non-membership of the element x to the set $\overline{A}$, where $\overline{\mu}_A(x) = [\overline{\mu}_A^-, \overline{\mu}_A^+(x)]$ and $\overline{\lambda}_A(x) = [\overline{\lambda}_A^-, \overline{\lambda}_A^+(x)]$ for all $x \in X$ with the condition $[0,0] \le [\overline{\mu}_A(x), \overline{\lambda}_A(x)] \le [1,1]$ for all $x \in X$. For the sake of simplicity, we use the symbol $\overline{A} = (\overline{\mu}_A, \overline{\lambda}_A)$, where $D[0,1]$ is the set of all closed sub interval of $[0,1]$.

3. INTERVAL VALUED INTUITIONISTIC Q-FUZZY IDEALS OF NEAR-RINGS

Now we introduce the interval valued intuitionistic Q-fuzzy ideals of near-rings as follows.

Definition 3.1:6 An IVIQFS $A = (\overline{\mu}_A, \overline{\lambda}_A)$ of a near-ring N is called an interval valued intuitionistic Q-fuzzy subnear-ring of N if

(i) $\overline{\mu}_A(x-y,q) \ge min^i\{\overline{\mu}_A(x,q), \overline{\mu}_A(y,q)\}$ and $\overline{\lambda}_A(x-y,q) \le max^i\{\overline{\lambda}_A(x,q), \overline{\lambda}_A(y,q)\}$,

(ii) $\overline{\mu}_A(xy,q) \ge min^i\{\overline{\mu}_A(x,q), \overline{\mu}_A(y,q)\}$ and $\overline{\lambda}_A(xy,q) \le max^i\{\overline{\lambda}_A(x,q), \overline{\lambda}_A(y,q)\}$,
for all $x, y \in N$ and $q \in Q$.

Definition 3.2:7 An $IVIQFS\ \overline{A} = (\overline{\mu}_A, \overline{\lambda}_A)$ in a near-ring N is called an i-v intuitionistic Q- fuzzy ideal of N if

(i) $\overline{\mu}_A(x-y,q) \ge \overline{\mu}_A(x,q) \wedge \overline{\mu}_A(y,q)$ and $\overline{\lambda}_A(x-y,q) \le \overline{\lambda}_A(x,q) \vee \overline{\lambda}_A(y,q)$,

(ii) $\overline{\mu}_A(y+x-y,q) = \overline{\mu}_A(x,q)$ and $\overline{\lambda}_A(y+x-y,q) = \overline{\lambda}_A(x,q)$.

(iii) $\overline{\mu}_A(rx,q) \ge \overline{\mu}_A(x,q)$ and $\overline{\lambda}(rx,q) \le \overline{\lambda}(x,q)$,

(iv) $\overline{\mu}((x+i)y - xy, q) \ge \overline{\mu}_A(i,q)$ and $\overline{\lambda}((x+i)y - xy, q) \le \overline{\lambda}_A(i,q)$ for all $x, y, i \in N$ and $q \in Q$.

If $\overline{A} = (\overline{\mu}_A, \overline{\lambda}_A)$ satisfies $(i), (ii)$ and (iii) then $\overline{A}$ is called an intuitionistic Q-fuzzy left ideal of N and if $\overline{A} = (\overline{\mu}_A, \overline{\lambda}_A)$ satisfies $(i), (ii)$ and (iv) then $\overline{A}$ is called an intuitionistic Q-fuzzy right ideal of N.

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Example 3.3:8 Let $N = \{a,b,c,d\}$ be a non-empty set with two binary operations "$+$" and "$.$" defined as follows:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

·	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

Then $(N,+,.)$ is a near-ring. Define an i-v intuitionistic Q-fuzzy set, $\overline{A} = (\overline{\mu}_A, \overline{\lambda}_A)$ in N as follows.
$\overline{\mu}_A(a,q) = [0.9,1], \overline{\mu}_A(b,q) = [0.3,0.4], \overline{\mu}_A(c,q) = [0,0], \overline{\mu}_A(d,q) = [0,0], \overline{\lambda}_A(a,q) = [0,0]$,
$\overline{\lambda}_A(b,q) = [0.3,0.4], \overline{\lambda}_A(c,q) = \overline{\lambda}_A(d,q) = [1,1]$
for all $q \in Q$. Then clearly $\overline{A} = \overline{\mu}_A, \overline{\lambda}_A$ is an i-v intuitionistic Q-fuzzy ideal of a near-ring N.

Basic properties of an i-v intuitionistic Q-fuzzy ideal of N are proved in the following theorem.

Theorem 3.4:9 If $\overline{A} = (\overline{\mu}_A, \overline{\lambda}_A)$ is an i-v intuitionistic Q- fuzzy ideal of N then

(i) $\overline{\mu}_A(0\ q) \geq \overline{\mu}_A(x,q)$ and $\overline{\lambda}_A(0\ q) \leq \overline{\lambda}_A(x,q)$,

(ii) $\overline{\mu}_A(-x,q) = \overline{\mu}_A(x,q)$ and $\overline{\lambda}_A(-x,q) = \overline{\lambda}(x,q)$,

(iii) $\overline{\mu}_A(x-y,q) \geq \overline{\mu}_A(0\ q) \Rightarrow \overline{\mu}_A(x,q) = \overline{\mu}_A(y,q)$,

(iv) $\overline{\lambda}_A(x-y,q) \leq \overline{\lambda}_A(0\ q) \Rightarrow \overline{\lambda}_A(x,q) = \overline{\lambda}_A(y,q)$

for all $x, y \in N$ and $q \in Q$.

Proof: Let $\overline{A} = (\overline{\mu}_A, \overline{\lambda}_A)$ be an i-v intuitionistic Q-fuzzy ideal of N.

(i) $\overline{\mu}_A(0\ q) = \overline{\mu}_A(x-x,q) \geq \overline{\mu}_A(x,q) \wedge \overline{\mu}_A(x,q)$.

Therefore $\overline{\mu}_A(0\ q) \geq \overline{\mu}_A(x,q)$ for all $x \in N$ and $q \in Q$.

Similarly, $\overline{\lambda}_A(0\ q) \leq \overline{\lambda}_A(x,q)$ for all $x \in N$ and $q \in Q$.

(ii) $\overline{\mu}_A(-x,q) = \overline{\mu}_A(0-x,q) \geq \overline{\mu}_A(0\ q) \wedge \overline{\mu}_A(x,q) \geq \overline{\mu}_A(x,q) \wedge \overline{\mu}_A(x,q) = \overline{\mu}_A(x,q)$.

Now $\overline{\mu}_A(x,q) = \overline{\mu}_A(-(-x),q) \geq \overline{\mu}_A(-x,q) \geq \overline{\mu}_A(x,q)$. Thus we get $\overline{\mu}_A(-x,q) = \overline{\mu}_A(x,q)$ for all $x \in N, q \in Q$.

Similarly, $\overline{\lambda}_A(-x,q) = \overline{\lambda}_A(x,q)$ for all $x \in N, q \in Q$.

(iii) We have $\overline{\mu}_A(x-y,q) \geq \overline{\mu}_A(0\ q)$. But $\overline{\mu}_A(0\ q) \geq \overline{\mu}_A(x-y,q)$. Thus $\overline{\mu}_A(x-y,q) = \overline{\mu}(0\ q)$.
Now consider

$$
\begin{aligned}
\overline{\mu}_A(x,q) &= \overline{\mu}_A(x-y+y,q) \\
&= \overline{\mu}_A((x-y)+y,q) \\
&\geq \overline{\mu}_A(x-y,q) \wedge \overline{\mu}_A(y,q) \\
&\geq \overline{\mu}_A(0\ q) \wedge \overline{\mu}_A(y,q) \\
&= \overline{\mu}_A(y,q)
\end{aligned}
$$

Similarly, we can prove that $\overline{\mu}_A(y,q) \geq \overline{\mu}_A(x,q)$. Hence $\overline{\mu}_A(x,q) = \overline{\mu}_A(y,q)$ for all $x, y \in N, q \in Q$.

A necessary and sufficient condition for $\chi = (\chi_I, \chi_I^c)$ to be an intuitionistic Q- fuzzy ideal of N is given in the following theorem.

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Theorem 3.5: 10Let N be a near-ring and $\overline{\chi}_I$ be a characteristic function of a subset I of N. Then $\overline{\chi} = (\overline{\chi}_I, \overline{\chi}_I^c)$ is an i-v intuitionistic Q-fuzzy ideal of N if and only if I is an ideal of N.

Proof: Let I be right ideal of N.

(1) Let $q \in Q$ and $x, y \in N$. Then we have the following three cases.

Case-1: Let $x, y \in I$. Then $\overline{\chi}_I(x,q) = \bar{1}, \overline{\chi}_I(y,q) = \bar{1}, \overline{\chi}_I(x-y,q) = \bar{1}$ and $\overline{\chi}_I^c(x,q) = \bar{0}, \overline{\chi}_I^c(x-y,q) = \bar{0}$. Therefore $\overline{\chi}_I(x-y,q) \geq \overline{\chi}_I(x,q) \wedge \overline{\chi}_I(y,q)$ and $\overline{\chi}_I^c(x-y,q) \leq \overline{\chi}_I^c(x,q) \vee \overline{\chi}_I^c(y,q)$ for all $x, y \in N$ and $q \in Q$.

Similarly, we can easily obtain the following two cases.

Case 2: Let $x \in I$ and $y \notin I$

Case-3: Let $y \in N$ and $x \in I$

Thus in any case we have $\overline{\chi}_I(x-y,q) \geq \overline{\chi}_I(x,q) \wedge \overline{\chi}_I(y,q)$ and $\overline{\chi}_I^c(x-y,q) \leq \overline{\chi}_I^c(x,q) \vee \overline{\chi}_I^c(y,q)$ for all $x, y \in N$ and $q \in Q.$

(2) **Case 1:** Let $y \in N$ and $x \in I.$ Since $y + x - y \in I$ for all $y \in N$ and $x \in I.$ Therefore $\overline{\chi}_I(y+x-y) = \bar{1} = \overline{\chi}_I(x)$, i.e. $\overline{\chi}_I(y+x-y,q) = \bar{1} = \overline{\chi}_I(x,q)$ and $\overline{\chi}_I^c(y+x-y,q) = \bar{0} = \overline{\chi}_I^c(x,q)$.

Case-2: Let $y \in N$ and $x \notin I$. Then $\overline{\chi}_I(y+x-y,q) = \bar{0} = \overline{\chi}_I(x,q)$ and $\overline{\chi}_I^c(y+x-y,q) = \bar{1} = \overline{\chi}_I^c(x,q)$.

Hence in any case we get $\overline{\chi}_I(y+x-y,q) = \overline{\chi}_I(x,q)$ and $\overline{\chi}_I^c(y+x-y,q) = \overline{\chi}_I^c(x,q)$ for all $x, y \in N$ and $q \in Q$.

(3) Let $x, y \in N$.

Case-1: Let $i \in I$. Since I is a right ideal of $N, (x+i)y - xy \in I$ for all $x, y \in N$ and $i \in I$. Therefore

$\overline{\chi}_I((x+i)y - xy, q) = \bar{1}, \overline{\chi}_I(i,q) = \bar{1}$ and $\overline{\chi}_I^c((x+i)y - xy, q) = \bar{0}, \overline{\chi}_I^c(i,q) = \bar{0}.$ Thus

$\overline{\chi}_I((x+i)y - xy, q) \geq \overline{\chi}_I(i,q)$ and $\overline{\chi}_I^c((x+i)y - xy, q) \leq \overline{\chi}_I^c(i,q)$ for all $x, y, i \in N, q \in Q$.

Case 2: Let $i \notin I$. Then $\overline{\chi}_I((x+i)y - xy, q) = \overline{\chi}_I(i,q) = \bar{0}$ and $\overline{\chi}_I^c((x+i)y - xy, q) = \overline{\chi}_I^c(i,q) = \bar{1}$.

Hence $\overline{\chi}_I((x+i)y - xy, q) \geq \overline{\chi}_{(i,q)}$ and $\overline{\chi}_I^c((x+i)y - xy, q) \leq \overline{\chi}_I^c(i,q)$ for all $x, y, i \in N, q \in Q$. Thus $\overline{\chi} = (\overline{\chi}_I, \overline{\chi}_I^c)$ is an i-v intuitionistic Q-fuzzy right ideal of N.

Similarly if I is a left ideal of N then we can easily prove that $\overline{\chi} = (\overline{\chi}_I, \overline{\chi}_I^c)$ is an i-v intuitionistic Q-fuzzy left ideal of N.

Conversely, let $\overline{\chi} = (\overline{\chi}_I, \overline{\chi}_I^c)$ be an i-v intuitionistic Q-fuzzy ideal in N. Then we shall prove that I is an ideal of N.

(i) Since $\overline{\chi} = (\overline{\chi}_I, \overline{\chi}_I^c)$ is an i-v intuitionistic Q-fuzzy ideal of N. Therefore,

$\overline{\chi}_I(x-y,q) \geq \overline{\chi}_I(x,q) \wedge \overline{\chi}_I(y,q)$ and $\overline{\chi}_I^c(x-y,q) \leq \overline{\chi}_I^c(x,q) \wedge \overline{\chi}_I^c(y,q)$ for all $x, y \in N$ and $q \in Q$.

If $x, y \in I$ then $\overline{\chi}_I(x,q) \wedge \overline{\chi}_I(y,q) = \bar{1}$ implies that $\overline{\chi}_I(x-y,q) = \bar{1}$. Therefore $x, y \in I \Rightarrow x - y \in I$. Hence $(I,+)$ is a subgroup of $(N,+)$.

CONFERENCE PAPER

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(ii) Since $\overline{\chi}_I(y+x-y,q)=\overline{\chi}_I(x,q)$ and $\overline{\chi}_I^c(y+x-y,q)=\overline{\chi}_I^c(x,q)$, therefore clearly if $x\in I$ and $y\in N$ then $\overline{\chi}_I(y+x-y,q)=\overline{\chi}_I(x,q)=\bar{1}$ and $\overline{\chi}_I^c(y+x-y,q)=\overline{\chi}_I^c(x,q)=\bar{0}$ implies that $y+x-y\in I$ for all $x\in I$ and $y\in N$.

Hence $(I,+)$ is a normal subgroup of $(N,+)$.

(iii) Since $\overline{\chi}(rx,q)\geq\overline{\chi}_I(x,q)$ and $\overline{\chi}_I^c(rx,q)\leq\overline{\chi}_I^c(x,q)$, therefore if $x\in I$ then $\overline{\chi}_I(rx,q)\geq\overline{\chi}_I(x,q)=\bar{1}\Rightarrow\overline{\chi}_I(rx,q)=\bar{1}\Rightarrow rx\in I$ for all $r\in N, x\in I$.

Similarly if $x\notin I$ then $rx\notin I$.

(iv) Since $\overline{\chi}_I((x+i)y-xy,q)\geq\overline{\chi}_I(i,q)$ and $\overline{\chi}_I^c((x+i)y-xy,q)\leq\overline{\chi}_I^c(i,q)$, therefore if $x\in I$ then

$\overline{\chi}_I((x+i)y-xy,q)\geq\overline{\chi}_I(i,q)$

$\overline{\chi}_I((x+i)y-xy,q)=\bar{1}$

$(x+i)y-xy\in I$ for all $x,y\in N$ and $i\in I$. Similarly we can prove that if $i\notin I$ then $(x+i)y-xy\notin I$ for all $x,y\in I$.

Hence I is an ideal of N.

For intersection of any number of i-v intuitionistic Q- fuzzy ideal of N, we have

Theorem 3.6:11 If $\{\overline{A}_i \mid i\in I\}$ is a family of i-v intuitionistic Q-fuzzy ideals of N then $\cap\{\overline{A}_i\mid i\in I\}$ is an i-v intuitionistic Q-fuzzy ideal of N where I is an index set.

Proof: Let $x,y,a,r\in N$ and $q\in Q$. Then

(1)
$$\begin{aligned}(\cap\overline{\mu}_{A_i})(x-y,q)&=\wedge\{\overline{\mu}_{A_i}(x-y,q)\mid i\in I\}\\&\geq(\wedge\{\overline{\mu}_{A_i}(x,q)\mid i\in I\})\wedge(\wedge\{\overline{\mu}_{A_i}(y,q)\mid i\in I\})\\&=(\cap\overline{\mu}_{A_i})(x,q)\wedge(\cap\overline{\mu}_{A_i})(y,q)\end{aligned}$$

$$\begin{aligned}(\cup\overline{\lambda}_{A_i})(x-y,q)&=\vee\{\overline{\lambda}_{A_i}(x-y,q)\mid i\in I\}\\&\leq(\wedge\{\overline{\lambda}_{A_i}(x,q)\mid i\in I\})\wedge(\wedge\{\overline{\lambda}_{A_i}(y,q)\mid i\in I\})\\&=(\cup\overline{\lambda}_{A_i})(x,q)\vee(\cup\overline{\lambda}_{A_i})(y,q),\end{aligned}$$

(2)
$$\begin{aligned}(\cap\overline{\mu}_{A_i})(y+x-y,q)&=\wedge\{\overline{\mu}_{A_i}(y+x-y,q)\mid i\in I\}\\&=\wedge\{\overline{\mu}_{A_i}(x,q)\mid i\in I\}\\&=(\cap\overline{\mu}_{A_i})(x,q),\end{aligned}$$

$$\begin{aligned}(\cup\overline{\lambda}_{A_i})(y+x-y,q)&=\wedge\{\lambda_{A_i}(y+x-y,q)\mid i\in I\}\\&=\vee\{\overline{\lambda}_{A_i}(x,q)\mid i\in I\}\\&=(\cup\overline{\lambda}_{A_i})(x,q),\end{aligned}$$

(3)
$$\begin{aligned}(\cap\overline{\mu}_{A_i})(rx,q)&=\wedge\{\overline{\mu}_{A_i}(rx,q)\mid i\in I\}\\&\geq\wedge\{\overline{\mu}_{A_i}(x,q)\mid i\in I\}\\&=(\cap\overline{\mu}_{A_i})(x,q),\end{aligned}$$

$$(\cup\overline{\lambda}_{A_i})(rx,q)=\vee\{\overline{\lambda}_{A_i}(rx,q)\mid i\in I\}$$

$$\leq \vee\{\overline{\lambda}_{A_i}(x,q) \mid i \in I\}$$
$$= (\cup\overline{\lambda}_{A_i})(x,q),$$

(4) $(\cap\overline{\mu}_{A_i})((x+a)y-xy,q) \mid i \in I\} \geq \wedge\{\overline{\mu}_{A_i}(x,q) \mid i \in I\}$
$$= (\cap\overline{\mu}_{A_i})(x,q),$$
$$(\cup\overline{\lambda}_{A_i})((x+a)y-xy,q) = \vee\{\overline{\lambda}_{A_i}((x+a)y-xy,q) \mid i \in I\}$$
$$\leq \vee\{\overline{\lambda}_{A_i})(x,q) \mid i \in I\}$$
$$= (\cup\overline{\lambda}_{A_i})(x,q),$$

for all $x, y, a, r \in R$ and $q \in Q$.

Hence $\cap\{\overline{A}_i \mid i \in I\}$ is an intuitionistic Q-fuzzy ideal of N.

An intuitionistic Q-fuzzy ideal $\overline{A}$ of N induces another intuitionistic Q-fuzzy ideal of N is show in the following theorem.

Theorem 3.7: 12If $\overline{A} = (\overline{\mu}_A, \overline{\lambda}_A)$ is an intuitionistic Q-fuzzy ideal of N then

(i) $\circ\overline{A} = (\overline{\mu}_A, \overline{\mu}_A^c)$ is also an i-v intuitionistic Q-fuzzy ideal of N;

(ii) $\bullet\overline{A} = (\overline{\lambda}_A^c, \overline{\lambda}_A)$ is also an i-v intuitionistic Q-fuzzy ideal in N.

Proof: Let $x, y, a, r \in N$ and $q \in Q$.

(i) (1) $\overline{\mu}_A^c(x-y,q) = \bar{1} - \overline{\mu}_A(x-y,q) \leq \bar{1} - (\overline{\mu}_A(x,q) \wedge \overline{\mu}_A(y,q))$.
$$= (\bar{1} - \overline{\mu}_A(x,q)) \vee (\bar{1} - \overline{\mu}_A(y,q)) = \overline{\mu}_A^c(x,q) \vee \overline{\mu}_A^c(y,q).$$

(2) $\overline{\mu}_A^c(y+x-y,q) = \bar{1} - \overline{\mu}_A(y+x-y,q) = \bar{1} - \overline{\mu}_A(x,q) = \overline{\mu}_A^c(x,q)$.

(3) $\overline{\mu}_A^c(rx,q) = \bar{1} - \overline{\mu}(rx,q) \leq \bar{1} - \overline{\mu}_A(x,q) = \overline{\mu}_A^c(x,q)$.

(4) $\overline{\mu}_A^c((x+a)y-xy,q) = \bar{1} - \overline{\mu}((x+a)y-xy,q) \leq \bar{1} - \overline{\mu}_A(a,q) = \overline{\mu}_A^c(a,q)$.

(ii) (1) $\overline{\lambda}_A^c(x-y,q) = \bar{1} - \overline{\lambda}_A(x-y,q) \geq \bar{1} - (\overline{\mu}_A(x,q) \wedge \overline{\lambda}_A(y,q))$.
$$= (\bar{1} - \overline{\lambda}_A(x,q)) \wedge (\bar{1} - \overline{\lambda}_A(y,q)) = \overline{\lambda}_A^c(x,q) \wedge \overline{\lambda}_A^c(y,q).$$

(2) $\overline{\lambda}_A^c(y+x-y,q) = \bar{1} - \overline{\lambda}_A(y+x-y,q) = \bar{1} - \overline{\lambda}_A(x,q) = \overline{\lambda}_A^c(x,q)$.

(3) $\overline{\lambda}_A^c(rx,q) = \bar{1} - \overline{\lambda}(rx,q) \geq \bar{1} - \overline{\lambda}_A(x,q) = \overline{\lambda}_A^c(x,q)$.

(4) $\overline{\lambda}_A^c((x+a)y-xy,q) = \bar{1} - \overline{\lambda}((x+a)y-xy,q) \geq \bar{1} - \overline{\lambda}_A(a,q) = \overline{\lambda}_A^c(a,q)$.

Hence $\circ\overline{A} = (\overline{\mu}_A, \overline{\mu}_A^c)$ and $\bullet\overline{A} = (\overline{\lambda}_A^c, \overline{\lambda}_A)$ are i-v intuitionistic Q-fuzzy ideals of N. From the above theorem we obtain the following.

Theorem 3.8:13 $\overline{A} = (\overline{\mu}_A, \overline{\lambda}_A)$ is an i-v intuitionistic Q-fuzzy ideal of N if and only if $\circ\overline{A}$ and $\bullet\overline{A}$ are i-v intuitionistic Q-fuzzy ideals of N.

For a given $\overline{A} = (\overline{\mu}_A, \overline{\lambda}_A)$, we define

$N\overline{\mu}_A = \{x \in N \mid \overline{\mu}_A(x,q) = \overline{\mu}_A(0,q)\}$ and $N\overline{\lambda}_A = \{x \in R \mid \overline{\lambda}(x,q) = \overline{\lambda}_A(0,q)\}$. We prove an important property of $N\overline{\mu}_A$ and $N\overline{\lambda}_A$ in the following theorem.

Theorem 3.9:14 If an $IVIQFS$ $\overline{A} = (\overline{\mu}_A, \overline{\lambda}_A)$ is an i-v intuitionistic Q-fuzzy ideal of N then the sets $N\overline{\mu}_A$ and $N\overline{\lambda}_A$ are ideals of N for all $q \in Q$.

Proof: Let $\overline{A}=(\overline{\mu}_A,\overline{\lambda}_A)$ be an intuitionistic Q-fuzzy ideal of N. Let $x,y\in N\overline{\mu}_A$ and $q\in Q$. Then $\overline{\mu}_A(x,q)=\overline{\mu}_A(0\ q),\overline{\mu}(y,q)=\overline{\mu}_A(0\ q)$.

(i) Now $0\in N$ therefore $0\in N\overline{\mu}_A$. Hence $N\overline{\mu}_A$ is a non-empty subset of N.

(ii) Since $\overline{A}=(\overline{\mu}_A,\overline{\lambda}_A)$ is an i-v intuitionistic Q-fuzzy ideal of N,
$\overline{\mu}_A(x-y,q)\geq\overline{\mu}_A(x,q)\wedge\overline{\mu}_A(y,q)=\overline{\mu}_A(0\ q)$.
But $\overline{\mu}_A(0\ q)\geq\overline{\mu}_A(x-y,q)$. Therefore $\overline{\mu}_A(x-y,q)=\overline{\mu}_A(0\ q)$. Thus $x-y\in N\overline{\mu}_A$ for all $x,y\in N$. Hence $(N\overline{\mu}_A,+)$ is a subgroup of $(N,+)$.

(iii) Again since $\overline{A}=(\overline{\mu}_A,\overline{\lambda}_A)$ is an i-v intuitionistic Q-fuzzy ideal of N,
$\overline{\mu}_A(y+x-y,q)=\overline{\mu}(x,q)=\overline{\mu}(0\ q)$.
Therefore $y+x-y\in N\overline{\mu}_A$ for all $x,y\in N$. Hence $(N\overline{\mu}_A,+)$ is a normal subgroup of $(N,+)$.

(iv) Since $\overline{\mu}(rx,q)\geq\overline{\mu}_A(x,q)$ for all $x,r\in N$ and $q\in Q$, therefore $\overline{\mu}_A(rx,q)\geq\overline{\mu}_A(x,q)$ But $\overline{\mu}_A(0\ q)\geq(rx,q)$. Hence $\overline{\mu}_A(rx,q)=\overline{\mu}_A(0\ q)$ implies that $rx\in N\overline{\mu}_A$ for all $r\in N,x\in N\overline{\mu}_A$,
i.e., $NN\overline{\mu}_A\subseteq N\overline{\mu}_A$.
Hence $N\overline{\mu}_A$ is a left ideal of N.

(v) Next $\overline{\mu}_A((u+i)v-uv,q)\geq\overline{\mu}_A(i,q)=\overline{\mu}_A(0\ q)$ for all $u,v\in N,i\in N\overline{\mu}_A$. But $\overline{\mu}_A(0,q)\geq\overline{\mu}_A((u+i)v-uv,q)$. Therefore $\overline{\mu}_A((u+i)v-uv,q)=\overline{\mu}_A(0,q)$ for all $u,v\in N,i\in N\overline{\mu}_A$. Hence $(u+i)v-uv\in N\overline{\mu}_A$ for all $u,v\in N$ and $i\in N\overline{\mu}_A$. Thus $N\overline{\mu}_A$ is a right ideal of N.

Theorem 3.10:15 If an $IVIQFS\ \overline{A}=(\overline{\mu}_A,\overline{\lambda}_A)$ is an i-v intuitionistic Q-fuzzy ideal of N then the sets $U(\overline{\mu}_A,t)$ and $L(\overline{\lambda}_A,t)$ are ideals of N for all $q\in Q,t\in I_m(\overline{\mu}_A)\cap I_m(\overline{\lambda}_A)$.

Proof: Let $q\in Q$ and $t\in I_m(\overline{\mu}_A)\cap I_m(\overline{\lambda}_A)$. We have
$U(\overline{\mu},t)=\{x\in N\,|\,\overline{\mu}_A(x,q)\geq t,q\in Q\}$,
$L(\overline{\mu},t)=\{x\in N\,|\,\overline{\lambda}_A(x,q)\leq t,q\in Q\}$.

(i) Let $x,y\in U(\overline{\mu},t)$. Then $\overline{\mu}_A(x,q)\geq t$ and $\overline{\mu}_A(y,q)\geq t$. Since $\overline{\mu}_A(x-y,q)\geq\overline{\mu}_A(x,q)\wedge\overline{\mu}_A(y,q)\geq t$, therefore $x-y\in U(\overline{\mu},t)$ for all $x,y\in U(\overline{\mu},t)$. Hence $(U(\overline{\mu},t),+)$ is a subgroup of $(N,+)$.

(ii) Let $x\in U(\overline{\mu},t)$ and $y\in N$. Again since $\overline{\mu}_A(y+x-y,q)=\overline{\mu}_A(x,q)\geq t$, therefore $y+x-y\in U(\overline{\mu},t)$ for all $x\in U(\overline{\mu},t)$ and $y\in N$. Hence $(U(\overline{\mu},t),+)$ is a normal subgroup of $(N,+)$.

(iii) Let $x\in U(\overline{\mu},t)$ and $r\in N$. Then $\overline{\mu}_A(rx,q)\geq\overline{\mu}_A(x,q)\geq t$ implies $rx\in U(\overline{\mu},t)$ for all $x\in U(\overline{\mu},t)$ and $r\in N$, i.e., $NU(\overline{\mu},t)\subseteq U(\overline{\mu},t)$. Hence $U(\overline{\mu},t)$ is a left ideal of N.

(iv) Let $i\in U(\overline{\mu},t)$ and $x,y\in N$. Then $\overline{\mu}_A((x+i)y-xy,q)\geq\overline{\mu}_A(i,q)\geq t$.
Therefore $(x+i)y-xy\in U(\overline{\mu},t)$ for all $i\in U(\overline{\mu},t)$ and $x,y\in N$. Hence $U(\overline{\mu},t)$ is a right ideal of N.

Theorem 3.11:16 If $\overline{A}=(\overline{\mu}_A,\overline{\lambda}_A)$ is an i-v intuitionistic Q-fuzzy subset of N such that all non-empty level sets $U(\overline{\mu}_A,t)$ and $L(\overline{\lambda}_A,t)$ are ideals of N then $\overline{A}=(\overline{\mu}_A,\overline{\lambda}_A)$ is an i-v intuitionistic Q-fuzzy ideal of N.

Proof: Suppose $\overline{A}=(\overline{\mu}_A,\overline{\lambda}_A)$ is an i-v intuitionistic Q-fuzzy set of N such that all non-empty level sets $U(\overline{\mu}_A,t)$ and $L(\overline{\lambda}_A,t)$ are ideals of N.

CONFERENCE PAPER
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(i) Let $t_0 = \overline{\mu}_A(x,q) \wedge \overline{\mu}_A(y,q)$ and $t_1 = \overline{\lambda}_A(x,q) \vee \overline{\lambda}(y,q)$ for $x, y \in N, q \in Q$.
Then $x, y \in U(\overline{\mu}_A, t_0)$ also $x, y \in L(\overline{\mu}_A, t_1)$. Therefore $x - y \in U(\overline{\mu}_A, t_0)$ and $x - y \in L(\overline{\mu}_A, t_1)$. Thus $\overline{\mu}_A(x-y,q) \geq t_0 = \overline{\mu}_A(x,q) \wedge \overline{\mu}(y,q)$ and $\overline{\mu}_A(x-y,q) \leq t_1 = \overline{\mu}_A(x,q) \wedge \overline{\mu}_A(y,q)$.

(ii) Let $t_0 = \overline{\mu}_A(x,q)$ and $t_1 = \overline{\lambda}_A(x,q)$. Then $x \in U(\overline{\mu}_A, t_0)$ also $x \in L(\overline{\lambda}_A, t_1)$ and $y \in N$.
Since $U(\overline{\mu}_A, t)$ and $L(\overline{\lambda}_A, t)$ are ideals of N, they are normal. Therefore $y + x - y \in U(\overline{\mu}_A, t_0)$ and $y + x - y \in L(\overline{\lambda}_A)$ for all $x \in U(\overline{\mu}_A, t_0)$
Now $x \in L(\overline{\lambda}_A, t_1)$ and $y \in N$.
$\Rightarrow \overline{\mu}_A(y + x - y, q) \geq \overline{\mu}_A(x,q)$ and $\overline{\lambda}_A(y + x - y, q) \leq \overline{\lambda}_A(x,q)$.

(iii) Let $t_2 = \overline{\mu}_A(x,q)$ and $t_3 = \overline{\lambda}_A(x,q)$ for $x \in N$ and $q \in Q$. Then $x \in U(\overline{\lambda}_A, t_2)$ and $x \in L(\overline{\lambda}_A, t_3)$.
Since $U(\overline{\mu}_A, t_2)$ and $L(\overline{\lambda}_A, t_3)$ are left ideals of $N, rx \in U(\overline{\mu}_A, t_2)$, and $rx \in L(\overline{\lambda}_A, t_3)$ for $r \in N$.
Hence $\overline{\mu}_A(rx,q) \geq t_2 = \overline{\mu}_A(x,q)$ and $\overline{\lambda}_A(rx,q) \leq t_3 \overline{\lambda}_A(x,q)$.

(iv) Let $t_2 = \overline{\mu}_A(i,q)$ and $t_3 = \overline{\lambda}_A(i,q)$ for $x \in N$ and $q \in Q$. Then $i \in U(\overline{\mu}_A, t_2)$ and $i \in L(\overline{\lambda}_A, t_3)$.
Since $U(\overline{\mu}_A, t)$ is a right ideal of N,
$(x+i)y - xy \in U(\overline{\mu}_A, t_2)$ for all $x, y \in N$ and $i \in U(\overline{\mu}_A, t_2)$ and
$(x+i)y - xy \in L(\overline{\lambda}_A, t_3)$ for all $x, y \in N$ and $i \in L(\overline{\lambda}_A, t_3)$. Therefore
$\overline{\mu}_A((x+i)y - xy, q) \geq t_2 = \overline{\mu}_A(i,q)$ and $\overline{\lambda}_A((x+i)y - xy, q) \leq t_3 = \overline{\lambda}_A(i,q)$ for all $x, y, i \in N$ and $q \in Q$.
Hence $\overline{A} = (\overline{\mu}_A, \overline{\lambda}_A)$ is an i-v intuitionistic Q-fuzzy ideal of N.

CONCLUSION

In this paper we have presented the notion of interval valued intuitionistic Q-fuzzy ideals of near-rings and derived the properties of these ideals.

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