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# TOTAL RESOLVING NUMBER OF EDGE CYCLE GRAPHS $G(C_3)$

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***ABSTRACT***

*Let G = (V, E) be a simple connected graph. An ordered subset W of V is said to be a resolving set of G if every vertex is uniquely determined by its vector of distances to the vertices in W. The minimum cardinality of a resolving set is called the resolving number of G and is denoted by r(G). Total resolving number as the minimum cardinality taken over all resolving sets in which ⟨W⟩ has no isolates and is denoted by tr(G). In this paper, we determine the exact values for the total resolving number of $T(C_3)$, $C_n(C_3)$ and $F_s(C_3)$. Also, we obtain bounds for the total resolving number of $G(C_3)$ and characterize the extremal graphs.*

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***Keywords:*** *resolving number, total resolving number, edge cycle graph.*

## 1. INTRODUCTION

Let G = (V, E) be a finite, simple, connected and undirected graph. The *degree* of a vertex v in a graph G is the number of edges incident with v and it is denoted by d(v). The maximum degree in a graph G is denoted by Δ(G) and the minimum degree is denoted by δ(G). The *distance* d(u, v) between two vertices u and v in G is the length of a shortest u- v path in G. The maximum value of distance between vertices of G is called its *diameter*. Let $P_n$ denote any *path* on n vertices, $C_n$ denote any *cycle* on n vertices and $K_n$ denote any *complete graph* on n vertices. A *complete bipartite graph* is denoted by $K_{s, t}$. $K_{1, n-1}$ is called a *star*. A tree containing exactly two vertices that are not end vertices is called a *bistar* and it is denoted by $B_{s, t}$. The *join* G + H consists of G ∪ H and all edges joining a vertex of G and a vertex of H. Let P denote the set of all pendant edges of G and |P| = p. Vertices which are adjacent to pendant vertices are called *support vertices*.

A graph H is called a subgraph of a graph G if V(H) ⊆ V(G) and E(H) ⊆ E(G). A subgraph F if a graph G is called an *induced subgraph* ⟨F⟩ of G if whenever u and v are vertices of F and uv is an edge of G, then uv is an edge of F as well. For a cut vertex v of a connected graph G, suppose that the disconnected graph G \ {v} has k components $G_1, G_2, \dots, G_k$ (k ≥ 2). The induced subgraphs $B_i = G[V(G_i) \cup \{v\}]$ are connected and referred to as the *brances* of G at v. The *complement* $G^c$ of a graph G is that graph whose vertex set is V(G) and such that for each pair u, v of vertices of G, uv is an edge of $G^c$ if and only if uv is not an edge of G. A vertex v in a graph G is called *complete vertex* if the subgraph by its neighborhood is complete. For an integer s ≥ 2, $sK_2 + K_1$ is called the *friendship graph* and is denoted by $F_s$.

If $W = \{w_1, w_2, \dots, w_k\} \subseteq V(G)$ is an ordered set, then the ordered k-tuple $(d(v, w_1), d(v, w_2), \dots, d(v, w_k))$ is called the representation of v with respect to W and it is denoted by r(v | W). Since the representation for each $w_i \in W$ contains exactly one 0 in the $i^{th}$ position, all the vertices of W have distinct representations. W is called a resolving set for G if all the vertices of V \ W also have distinct representations. The minimum cardinality of a resolving set is called the resolving number of G and it is denoted by r(G).

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In 1975, Slater [9] introduced these ideas and used *locating set* for what we have called *resolving set*. He referred to the cardinality of a minimum resolving set in G as its *location number*. In 1976, Harary and Melter [4] discovered these concepts independently as well but used the term metric dimension rather than location number. In 2003, Ping Zhang and Varaporn Saenpholphat [7, 8] studied *connected resolving number* and in 2015, we introduced and studied *total resolving number*. In this paper, we use the term *resolving number* to maintain uniformity in the current literature.

If W is a resolving set and the induced subgraph $\langle W \rangle$ has no isolates, then W is called a *total resolving set* of G. The minimum cardinality taken over all total resolving sets of G is called the *total resolving number* of G and is denoted by tr(G). We introduced edge cycle graph in [5] and studied the resolving number of edge cycle graph $G(C_k)$. An *edge cycle graph* of a graph G is the graph $G(C_k)$ formed from one copy of G and $|E(G)|$ copies of $P_k$, where the ends of the $i^{th}$ edge are identified with the ends of $i^{th}$ copy of $P_k$.

In this paper, we determine the exact values for the total resolving number of $T(C_3)$, $C_n(C_3)$ and $F_s(C_3)$. Also, we obtain bounds for the total resolving number of $G(C_3)$ and characterize the extremal graphs.

## 2. BUILDING BLOCKS

The following results are used in the subsequent sections.

**Theorem 2.1:** [6] *Let $\{w_1, w_2\} \subset V(G)$ be a total resolving set in G. Then the degrees of $w_1$ and $w_2$ are at most 3.*

**Lemma 2.2:** [6] *For $n \geq 3$, $tr(P_n) = 2$ and $tr(C_n) = 2$.*

**Observation 2.3:** [6] *Let G be a graph of order $n \geq 3$. Then $2 \leq tr(G) \leq n-1$.*

**Theorem 2.4:** [6] *Let G be a graph of order $n \geq 3$. Then $tr(G) = n - 1$ if and only if $G = K_n$ or $K_{1, n-1}$.*

**Definition 2.5:** *A block of G containing exactly one cut vertex of G is called an end block of G.*

**Lemma 2.6:** [5] *Let G be a 1-connected graph with $\delta(G) \geq 2$. Then every resolving set contains at least one non cut vertex of each end block.*

**Corollary 2.7:** [5] *If G contains b end blocks, then $r(G) \geq b$.*

**Definition 2.8:** *A cycle $C_r$ is called an end cycle if $C_r$ contains exactly one vertex of degree at least 3.*

**Notation 2.9:** *Let $e_c$ denote the number of end cycles of the graph G.*

**Theorem 2.11:** [6] *Let T be a tree of order $n \geq 3$. Then $r(T(C_3)) = p$.*

In this paper, we investigate the total resolving number of the edge cycle graphs $G(C_3)$.

## 3. TOTAL RESOLVING NUMBER OF EDGE CYCLE GRAPHS $G(C_3)$

In this section, we determine the exact values for the total resolving number of $T(C_3)$, $C_n(C_3)$ and $F_s(C_3)$.

**Observation 3.1:** *For $n = 3, 4, 5$, $tr(C_n(C_3)) = 3$*

**Theorem 3.2:** *For $n \geq 6$, $tr(C_n(C_3)) = 4$.*

**Proof:** Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$, $E(C_n) = \{v_1v_2, v_2v_3, \ldots, v_nv_1\}$ and $u_1, u_2, \ldots, u_n$ be the new vertices in $C_n(C_3)$ corresponding to the edges $v_1v_2, v_2v_3, \ldots, v_nv_1$. Then $V(C_n(C_3)) = V \cup U$, where $V = V(C_n)$, $U = \{u_1, u_2, \ldots, u_n\}$ and $E(C_n(C_3)) = E(C_n) \cup \{u_iv_i, u_iv_{i+1} / 1 \leq i \leq n-1\} \cup \{u_nv_n, u_nv_1\}$. Let W be a total resolving set of $C_n(C_3)$.

First, we claim that $tr(C_n(C_3)) \geq 4$. Suppose that $tr(C_n(C_3)) \leq 3$. By Theorem 2.1, $tr(C_n(C_3)) = 3$. Therefore $\langle W \rangle$ is $P_3$ or $K_3$. If $\langle W \rangle$ is $K_3$, then without loss of generality, let $W = \{v_1, u_2, v_2\}$. Then $r(v_n \mid W) = r(u_n \mid W) = (1, 2, 3)$, which is a contradiction. If $\langle W \rangle$ is $P_3$, then without loss of generality, let $W \subseteq \{v_1, v_2, v_3, u_1, u_2,\}$. Then $r(v_n \mid W) = r(u_n \mid W)$, which is a contradiction. Thus $tr(C_n(C_3)) \geq 4$.

Let W = $\left\{v_1,\ v_2,\ v_{\lfloor\frac{n}{2}\rfloor+1}, v_{\lfloor\frac{n}{2}\rfloor+2}\right\}$. Let x, y be two distinct vertices of $V(C_n(C_3)) \setminus W$. If $d(x, v_1) \neq d(y, v_1)$ or $d(x, v_2) \neq d(y, v_2)$, then $r(x \mid W) \neq r(y \mid W)$. So we may assume that $d(x, v_1) = d(y, v_1)$ or $d(x, v_2) = d(y, v_2)$. Then $x \in U$ and $y \in V$ or $x \in V$ and $y \in U$. Without loss of generality, let $x \in U$ and $y \in V$. But $d(x, v_{\lfloor\frac{n}{2}\rfloor+1}) = d(y, v_{\lfloor\frac{n}{2}\rfloor+1}) + 1$ and $d(x, v_{\lfloor\frac{n}{2}\rfloor+2}) = d(y, v_{\lfloor\frac{n}{2}\rfloor+2}) + 1$. It follows that $r(x \mid W) \neq r(y \mid W)$. Thus W is a resolving set of $C_n(C_3)$ and $\langle W\rangle$ has no isolates, $tr(C_n(C_3)) \leq 4$. Hence $tr(C_n(C_3)) = 4$.

**Lemma 3.3:** *Let G be a graph of order $n \geq 3$ and $\delta(G) = 1$. Then $tr(G(C_3)) \geq p + s$.*

**Proof:** Let W be a total resolving set of $G(C_3)$. Let $B_1, B_2, \ldots, B_p$ be the end blocks of $G(C_3)$. Then by Lemma 2.6, $W \cap V(B_i) \neq \emptyset$, for all $1 \leq i \leq p$. Since W is a total resolving set, $|W \cap V(B_i)| \geq 2$ for all $1 \leq i \leq p$. But some end blocks have the common vertex, $|W \cap V(G(C_3))| \geq p + s$ and hence $tr(G(C_3)) \geq p + s$.

**Theorem 3.4:** *Let T be a tree of order at least 3. Then $tr(T(C_3)) = p + s$.*

**Proof:** The proof follows from Theorem 2.11 and Lemma 3.3.

**Corollary 3.5:** *For $n \geq 4$, $tr(P_n(C_3)) = 4$.*

**Corollary 3.6:** *For $n \geq 2$, $tr(K_{1,n-1}(C_3)) = n$.*

**Corollary 3.7:** *For $s, t \geq 1$, $tr(B_{s,t}(C_3)) = s + t + 2$.*

**Theorem 3.8:** *For $s \geq 2$, $tr(F_s(C_3)) = 2s$.*

**Proof:** Let $V(F_s)=\{u, u_{11}, u_{12}, u_{21}, u_{22}, \ldots, u_{s1}, u_{s2}\}$ and

$$E(F_s) = \{uu_{ij} / 1 \leq i \leq s \text{ and } j = 1, 2\} \cup \{u_{11}u_{12}, u_{21}u_{22}, \ldots, u_{s1}u_{s2}\}.$$

For $1 \leq j \leq s$, let $v_i$ be the new vertex of the edge $u_{i1}u_{i2}$, $v_{j1}$ be the new vertex of the edge $uu_{j1}$ and $v_{j2}$ be the new vertex of the edge $uu_{j2}$ in $F_s(C_3)$. Then we have G contains exactly s blocks, say $B_1, B_2, \ldots, B_s$. Let W be a total resolving set of $F_s(C_3)$.

First, we claim that $tr(F_s(C_3)) \geq 2s$. Suppose that $tr(F_s(C_3)) \leq 2s - 1$. Then we have W contains at most three vertices from union of two blocks. Without loss of generality, let $B_1$ and $B_2$ be such blocks. Then we have $|W \cap (V(B_1) \cup V(B_2))| \leq 3$. By Lemma 2.6, $|W \cap (V(B_1) \setminus \{u\}| \neq \emptyset$ and $|W \cap (V(B_2) \setminus \{u\}| \neq \emptyset$. Let $u, x, y \in W$, where $x \in N(u) \cap V(B_1)$ and $y \in N(u) \cap V(B_2)$. Then $d(x) = 2$ or $4$ in $F_n(C_3)$. If $d(x) = 2$, then without loss of generality, let $x = v_{11}$. But we have $r(u_{12} \mid W) = r(v_{12} \mid W)$. If $d(x) = 4$, then without loss of generality, let $x = u_{11}$. But we have $r(v_{11} \mid W) = r(u_{12} \mid W)$, which is a contradiction. Hence $tr(F_s(C_3)) \geq 2s$.

Next, we claim that $tr(F_s(C_3)) \leq 2s$. Now, let $W = \{u_{11}, u_{21}, \ldots, u_{s1}\} \cup \{u_{12}, u_{22}, \ldots, u_{s2}\}$. Let x, y be two distinct vertices of $V(F_s(C_3)) \setminus W$. Then we consider the following two cases.

**Case-1:** $x, y \in V(B_i)$ for some $1 \leq i \leq s$.

Without loss of generality, let $x, y \in V(B_1)$. If $d(x, w) \neq d(y, w)$ for some $w \in W \cap (V(B_1)$, then $r(x \mid W) \neq r(y \mid W)$. So we may assume that $d(x, w) = d(y, w)$ for all $w \in W \cap (V(B_1))$. Then $x = v_1$ and $y = u$. But $3 = d(x, w) > d(y, w) = 1$. It follows that $r(x \mid W) \neq r(y \mid W)$.

**Case-2:** $x \in V(B_i)$, $y \in V(B_j)$ for some $1 \leq i \neq j \leq s$.

Then clearly, $d(x, w) < d(y, w)$ for all $w \in W \cap V(B_i)$. It follows that $r(x \mid W) \neq r(y \mid W)$.

Thus W is a resolving set and $\langle W\rangle$ has no isolates, $tr(F_3(C_3)) \leq 2s$. Hence $tr(F_3(C_3)) = 2s$.

**GENERAL BOUNDS AND EXTREMAL GRAPHS**

In this section, we obtain bounds for the total resolving number of $G(C_3)$ and characterize the extremal graphs.

**Theorem 4.1:** *Let G be a graph of order $n \geq 3$. Then $3 \leq tr(G(C_3)) \leq n$.*

**Proof:** By Theorem 2.1, $tr(G(C_3)) \geq 3$. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ and $v_{ij}$ be the new vertex of the edge $v_iv_j$ in $G(C_3)$, where $i, j \in \{1, 2, \ldots, n\}$ and $i \neq j$. Let $W = V(G)$. Then $i^{th}$ and $j^{th}$ coordinates of the representation of $v_{ij}$ are 1. Since $i \neq j$, representation of all $v_{ij}$ are distinct. Therefore $tr(G(C_3)) \leq n$. Hence $3 \leq tr(G(C_3)) \leq n$.

**Theorem 4.2:** *Let G be a graph of order $n \geq 3$. Then $tr(G(C_3)) = 3$ if and only if $G \cong P_3$* or $K_3$ or $C_4$ or $K_4 \setminus \{e\}$ or $K_4$ or $C_5$.

**Proof:** Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ and $tr(G(C_3)) = 3$. If $n = 3$, then $G \cong P_3$ or $K_3$. So we may assume that $n \geq 4$. For $i, j \in \{1, 2, \ldots, n\}$ and $i \neq j$, let $v_{ij}$ be the new vertex of the edge $v_iv_j$ in $G(C_3)$. Let $W = \{w_1, w_2, w_3\}$ be a total resolving set of $G(C_3)$.

Let $\langle W \rangle$ be $K_3$. If W is not a subset of V(G), then without loss of generality, let $W = \{v_1, v_2, v_{12}\}$. Let $X = V(G) \setminus \{v_1, v_2\}$. Since G is connected, a vertex of X, say $v_3$ is adjacent to $v_1$ or $v_2$ or both. If $v_3$ is adjacent to $v_1$ or $v_2$, say $v_1$, then $r(v_3 | W) = r(v_{13} | W) = (1, 2, 2)$, which is a contradiction. If no vertex of X is adjacent to exactly one vertex of $\{v_1, v_2\}$, then a vertex of X, say $v_3$ is adjacent to $v_1$ and $v_2$. Since G is connected and $n \geq 4$, $v_3$ is adjacent to a vertex of X, say $v_4$. But we have $r(v_4 | W) = r(v_{34} | W) = (2, 2, 3)$, which is a contradiction and hence $W \subset V(G)$.

Without loss of generality, let $W = \{v_1, v_2, v_3\}$ and $X = V(G) \setminus W$. Then $r(v_{12} | W) = (1, 1, 2)$, $r(v_{23} | W) = (2, 1, 1)$, $r(v_{31} | W) = (1, 2, 1)$ which shows that no vertex of X has exactly two neighbors in W. If a vertex $v_i \in X$ is adjacent to exactly one vertex of W, say $v_j$, $j \in \{1, 2, 3\}$, then $r(v_i | W) = r(v_{1j} | W)$, which is a contradiction. If there exists a vertex of X, say $v_i$ is adjacent to no vertex of W, then $r(v_i | W) = r(v_{ik} | W)$, where $v_iv_k \in E(G)$, which is a contradiction. Hence each vertex of X is adjacent to all the vertices of W. If $|X| > 1$, then $r(v_4 | W) = r(v_5 | W) = \ldots = r(v_n | W)$, which is a contradiction. Consequently, $|X| = 1$. Hence $X = \{v_4\}$ and $G \cong K_4$.

Let $\langle W \rangle$ be $P_3$. Then we consider the following two cases.

**Case-1:** W is a subset of V(G).

Then without loss of generality, let $W = \{v_1, v_2, v_3\}$, where $v_2$ is adjacent to $v_1$ and $v_3$. Then $r(v_{12} | W) = (1,1, 2)$ and $r(v_{23} | W) = (2, 1, 1)$. Let $X = V(G) \setminus W$. If there exists a vertex $v_i \in X$ which is adjacent to $v_2$ but not to $v_1$ and $v_3$, then $r(v_i | W) = r(v_{i2} | W) = (2, 1, 2)$ in $G(C_3)$, which is a contradiction. If there exist two distinct vertices $v_i, v_j \in X$ such that $v_i$ is adjacent to $v_1$ & $v_3$ and $v_j$ is adjacent to $v_1$, $v_2$ & $v_3$, then $r(v_{i1} | W) = r(v_{j1} | W) = (1, 2, 2)$ and $r(v_{3i} | W) = r(v_{3j} | W) = (2, 2, 1)$ in $G(C_3)$, which is a contradiction.

Now, we claim that $|N(W)| = 1$ or 2. Suppose $|N(W)| \geq 4$. Let $N(W) = \{v_4, v_5, \ldots, v_k\}$, $k \geq 7$. Without loss of generality, let $v_4$ be adjacent to $v_1$ but not to $v_2$ and $v_3$, $v_5$ be adjacent to $v_3$ but not to $v_1$ and $v_2$, $v_6$ be adjacent to $v_1$ & $v_2$ or $v_1$, $v_2$ & $v_3$. But a vertex of $\{v_7, v_8, \ldots, v_k\}$ is adjacent to $v_1$ or $v_3$ or $v_1$ & $v_3$. If $v_7$ is adjacent to $v_1$ or $v_3$, say $v_1$, then $r(v_{14} | W) = r(v_{17} | W) = (1, 2, 3)$, which is a contradiction. If $v_7$ is adjacent to $v_1$ and $v_3$, then $r(v_6 | W) = r(v_7 | W)$, which is a contradiction.

Suppose $|N(W)| = 3$. Then without loss of generality, let $N(W) = \{v_4, v_5, v_6\}$ and $v_4$ be adjacent to $v_1$, $v_5$ be adjacent to $v_3$ and $v_6$ be adjacent to either $v_1$ and $v_3$ or $v_1$, $v_2$ & $v_3$. If $\langle\{v_4, v_5, v_6\}\rangle$ is either $K_3^c$ or $K_2 \cup K_1$, then without loss of generality, let $v_4$ be not adjacent to $v_5$ and $v_6$. Then $r(v_4 | W) = r(v_{14} | W) = (1, 2, 3)$ in $G(C_3)$, which is a contradiction. If $\langle\{v_4, v_5, v_6\}\rangle$ is either $P_3$ or $K_3$, then $r(v_4 | W) = r(v_{16} | W) = (1, 2, 2)$ in $G(C_3)$, which is a contradiction. Hence $|N(W)| = 1$ or 2. Now, we consider the following two subcases.

**Subcase-1:** $|N(W)| = 1$.
Then without loss of generality, let $N(W) = \{v_4\}$. We claim that $|X| = 1$. Suppose $|V_1| \geq 2$. Then $v_4$ is a cut vertex of G. Then there are at least two branches at $v_4$ in $G(C_3)$, say $B_1$ and $B_2$. Let $\langle\{v_1, v_2, v_3, v_4\}\rangle = B_1$. Therefore $B_2$ contains at least one end block. But no vertex of $B_2$ belongs to W, which is a contradiction to Lemma 2.6 and hence $X = \{v_4\}$.

If $v_4$ is adjacent to $v_1$ but not to $v_2$ and $v_3$ in G, then $r(v_4 | W) = r(v_{14} | W) = (1, 2, 2)$ in $G(C_3)$, which is a contradiction. If $v_4$ is adjacent to $v_1$ and $v_3$ but not to $v_2$, then $G \cong C_4$ and if $v_4$ is adjacent to $v_1$, $v_2$ and $v_3$, then $G \cong K_4 \setminus \{e\}$.

**Subcase-2:** $|N(W)| = 2$.
Then without loss of generality, let $N(W) = \{v_4, v_5\}$. Then exactly one vertex of $\{v_4, v_5\}$ is adjacent to exactly one vertex of $\{v_1, v_3\}$. Without loss of generality, let $v_4$ be adjacent to $v_1$. If $v_4v_5 \notin E(G)$, then $r(v_4 | W) = r(v_{4i} | W)$, $i \in \{1, 2, 3\}$, which is a contradiction. Thus $v_4v_5 \in E(G)$.

If $v_5$ is adjacent to $v_3$, then we claim that $|V| = 5$. Suppose $|V| > 5$. Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$, $n \geq 6$. Let $\langle\{v_4, v_5, v_i\}\rangle \cong P_3$ for some $i \in \{6, 7, \ldots, n\}$. If $v_i$ is adjacent to $v_4$, then $r(v_i \mid W) = r(v_{4i} \mid W) = (2, 3, 3)$ in $G(C_3)$, which is a contradiction. If $\langle\{v_4, v_5, v_i\}\rangle \cong K_3$ for some $i \in \{6, 7, \ldots, n\}$, then $r(v_i \mid W) = r(v_{45} \mid W) = (2, 3, 2)$ in $G(C_3)$, which is a contradiction. Therefore $|V| = 5$ and hence $G \cong C_5$.

If $v_5$ is adjacent to $v_1$ and $v_3$ in G, then $r(v_{15}|W) = r(v_4|W) = (1, 2, 2)$ in $G(C_3)$, which is a contradiction.

**Case-2:** W is not a subset of V(G).
Then without loss of generality, let $v_1$, $v_2$ and $v_3$ be three vertices of G such that $\langle\{v_1, v_2, v_3\}\rangle \cong P_3$ or $K_3$, $v_3 \notin W$ and $v_2 \in W$. Let $V_1 = V(G) \setminus X$. Then clearly, no vertex of $V_1$ is adjacent to $v_2$ in G, for, if $v_i \in V_1$ is adjacent to $v_2$ in G, then $r(v_i \mid W) = r(v_{2i} \mid W) = (2, 1, 2)$ in $G(C_3)$, which is a contradiction.

Now, we claim that $|N(X)| = 1$. Suppose $|N(X)| \geq 4$. Let $N(X) = \{v_4, v_5, \ldots, v_k\}$, $k \geq 7$. Then without loss of generality, let $v_4$ be adjacent to exactly one vertex of $\{v_1, v_3\}$, say $v_1$, $v_5$ be adjacent to $v_3$ not to $v_1$ and $v_6$ be adjacent to $v_1$ and $v_3$. But a vertex of $\{v_7, v_8, \ldots, v_k\}$ is adjacent to $v_1$ or $v_3$ or both. Without loss of generality, let $v_7$ be adjacent to say $v_1$. Then $r(v_7 \mid W) = r(v_4 \mid W)$, which is a contradiction and hence $|N(X)| \leq 3$.

If $|N(X)| = 2$, then without loss of generality, let $v_4$ and $v_5$ be two vertices in N(X). If $v_4$ is adjacent to $v_1$, $v_5$ is adjacent to $v_3$ or $v_4$ is adjacent to $v_1$ and $v_5$ is adjacent to $v_1$ and $v_3$, then $r(v_4 \mid W) = r(v_{14} \mid W)$ in $G(C_3)$, which is a contradiction.

Let $v_4$ be adjacent to $v_3$ and $v_5$ is adjacent to $v_1$ and $v_3$. If W contains exactly one vertex of V, then $r(v_4 \mid W) = r(v_{14} \mid W) = (3, 2, 2)$ in $G(C_3)$, which is a contradiction. If W contains two vertices of V, then by our assumption $v_2 \in W$ and $v_3 \notin W$, $v_1 \in W$. If $\langle\{v_1, v_2, v_3\}\rangle \cong K_3$, then $r(v_4 \mid W) = r(v_{34} \mid W) = (2, 2, 2)$. If $\langle\{v_1, v_2, v_3\}\rangle \cong P_3$ and $v_4v_5 \notin E(G)$, then $r(v_4 \mid W) = r(v_{34} \mid W) = (3, 2, 2)$. If $\langle\{v_1, v_2, v_3\}\rangle \cong P_3$ and $v_4v_5 \in E(G)$, then $r(v_4 \mid W) = r(v_{35} \mid W) = (2, 2, 2)$, which is a contradiction.

If $N(X) = 3$, then without loss of generality, let $v_4$, $v_5$, $v_6$ be three vertices in N(X) and $v_4$ be adjacent to $v_1$, $v_5$ be adjacent to $v_3$, $v_6$ be adjacent to $v_1$ and $v_3$ in G. Then $r(v_4 \mid W) = r(v_{14} \mid W) = (2, 2, 3)$ in $G(C_3)$, which is a contradiction.

Without loss of generality, let $N(X) = \{v_4\}$. We claim that $V_1 = \{v_4\}$. Suppose $V_1 = \{v_4, v_5, \ldots, v_n\}$, $n \geq 5$. If H is $\langle V_1\rangle$, then $H(C_3)$ contains at least one end block. But no vertex of $H(C_3)$ belongs to W, which is a contradiction to Lemma 2.6. Therefore $X = \{v_4\}$. If $v_4$ is adjacent to either $v_1$ or $v_3$, say $v_1$, then $r(v_4 \mid W) = r(v_{14} \mid W) = (1, 2, 3)$ in $G(C_3)$, which is a contradiction and hence $v_4$ is adjacent to $v_1$ and $v_3$. But if $\langle\{v_1, v_2, v_3\}\rangle \cong K_3$, then $r(v_4 \mid W) = r(v_{13} \mid W)$ in $G(C_3)$, which is a contradiction. Therefore $\langle\{v_1, v_2, v_3\}\rangle \cong P_3$ and hence in this case, $G \cong C_4$.

Conversely, let $G \cong P_3$ or $K_3$ or $C_4$ or $K_4 \setminus \{e\}$ or $K_4$ or $C_5$. Let $W = \{v_1, v_2, v_3\}$ and $v_1v_2, v_2v_3 \in E(G)$. Then W is a total resolving set of $G(C_3)$.

Thus $tr(G(C_3)) \leq 3$. By Theorem 4.1, $tr(G(C_3)) \geq 3$ and hence $tr(G(C_3)) = 3$.

**Theorem 4.3:** *Let G be a graph of order $n \geq 3$. Then $tr(G(C_3)) = n$ if and only if each non support vertex is a complete vertex of degree 2.*

**Proof:** Assume that $tr(G(C_3)) = n$. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$. Let $v_{ij}$ be the new vertex of the edge $v_iv_j$ in $G(C_3)$. Then we claim that each non support vertex is a complete vertex of degree 2. Suppose not. Then we consider the following two cases.

**Case-1:** There exists a non support vertex $v_i$ for some i such that $d(v_i) \geq 3$ in G.
Then without loss of generality, let $v_1$ be such vertex and $N(v_1) = \{v_1, v_3, \ldots, v_{k+1}\}$, $k \geq 3$ in G. Let $W = \{v_2, v_3, \ldots, v_n\}$. Then for $2 \leq i \neq j \leq n$, $i - 1^{th}$ and $j - 1^{th}$ coordinates of the representation of $v_{ij}$ are 1, $1^{st}$ k coordinates of the representation of $v_1$ are 1 and $j - 1^{th}$ coordinate of the representation of $v_{1j}$, $2 \leq j \leq k + 1$ is 1 in $G(C_3)$. Therefore each vertex of $V(G(C_3)) \setminus W$ have distinct representations. Since $\langle W\rangle$ has no isolates, $tr(G(C_3)) \leq n - 1$, which is a contradiction.

**Case-2:** There exists a non support vertex $v_i$ for some i such that $d(v_i) = 2$ and $v_i$ is not a complete vertex in G.
Then without loss of generality, let $v_i$ be such vertex in G. Let $N(v_1) = \{v_2, v_3\}$ and $W = \{v_2, v_3, \ldots, v_n\}$. Then for $2 \leq i \neq j \leq n$, $i - 1^{th}$ and $j - 1^{th}$ coordinates of the representation of $v_{ij}$ are 1, $1^{st}$ and $2^{nd}$ coordinates of $v_1$ are 1, $1^{st}$ coordinate of $v_{12}$ is 1 and $2^{nd}$ coordinate of $v_{13}$ is 1 in $G(C_3)$. Thus each vertex of $V(G(C_3)) \setminus W$ have distinct representations. Since $\langle W\rangle$ has no isolates, $tr(G(C_3)) \leq n - 1$, which is a contradiction.

Hence each non support vertex is a complete vertex of degree 2.

 

Conversely, suppose that each non support vertex is a complete vertex of degree 2. By Theorem 4.1, $tr(G(C_3)) \leq n$. Let W be a total resolving set for G $(C_3)$. Let $d(v_i) = 2$, $v_i$ is a complete non support vertex and $N(v_i) = \{v_j, v_k\}$. Then $d(v_i, v) = d(v_{jk}, v)$ for all $v \in V(G(C_3)) \setminus v_i, v_{jk}$. Therefore $v_i$ or $v_{jk} \in W$ and by Lemma 3.3, $tr(G(C_3) \geq p + s + s' = n$, where s' denote the number of non support vertices of G. Thus $tr(G(C_3)) \geq n$ and hence $tr(G(C_3)) = n$.

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