

NUMERICAL SOLUTION  
 OF LAMINAR BOUNDARY LAYER FLOW ON AN ISOTHERMAL WALL

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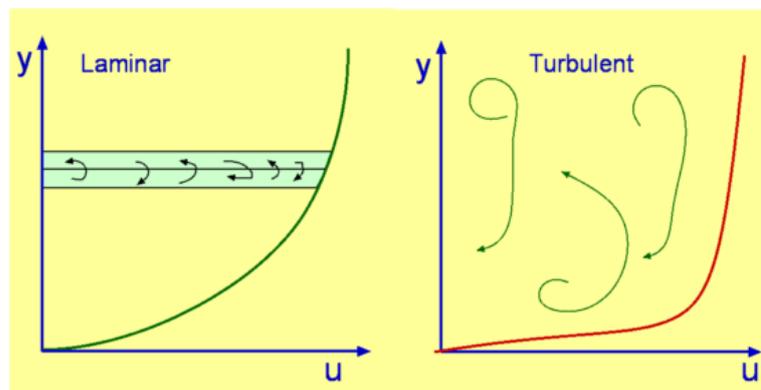
ABSTRACT

The aim of this paper is that the study of laminar boundary layer flow on an Isothermal wall/adabatic wall. The present study of this paper deals with two dimensional laminar boundary layer equation with steady state motion using similar transformation. The equation has been solved by Runge-Kutta method and Shooting technique. Results have been discussed by numerical solution and graphical representation.

**Key ward:** Renold's Number, Prandtl's Number, Pressure Gradient, Viscous Dissipation etc.

INTERODUCTION

Boundary layer may be defined by two type's firstly laminar boundary layer secondly turbulent boundary layer. A laminar boundary layer is one where the flow takes place in each layers slide past the adjacent layers, these is in contrast to turbulent boundary layers are an intense agitation. In a laminar boundary layer any exchange of mass or momentum takes place only between adjacent layers on a microscope scale which is not visible to see from eyes. Laminar boundary layer are founded only when Renold's numbers are small only other hand turbulent boundary layer is marked by mixing across several layers of it. The mixing is now on a microscopic scale there is an exchange of mass, momentum and energy on a much bigger scale composed by laminar boundary layer. Laminar boundary layer has large Renold's number.



Laminar boundary layer come in various forms and can be loosely classified according to their structure and the circumstance under which they are created. The thin shear layer which develops on an oscillating body is an example of Stoke layer. While the Blasius boundary layer refers to the well known similarity solution of the study boundary layer attached with plain wall. The pervious related to this paper “N. Kafasstas & A. Karbis” “Numerical study of two dimensional laminar boundary layer compressible flows with presser gradient and heat mass transfer” and other study “S. P. Mishra & G. C. Das” “Numerical solution of boundary layer MHD flow with viscous dissipation.”

Equation of Momentum

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

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**Equation of Energy**

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

Where  $\alpha = \frac{K}{\rho \mu c_p}$

It will be noted that the viscous dissipation term is omitted from the energy equation then the boundary conditions are

$$\begin{aligned} U &= 0 \text{ at } y = 0 \\ V &= 0 \text{ at } y = 0 \\ \frac{\partial u}{\partial y} &= 0 \text{ at } y = \infty \\ T &= T_\infty \text{ at } y = \infty \end{aligned} \tag{4}$$

The equation of continuity (1) is identically satisfied. Now it take the stream function  $\varphi$  then

$$u = \frac{\partial \varphi}{\partial y} \text{ and } v = \varphi \frac{\partial \varphi}{\partial x} \tag{5}$$

The momentum and energy equation (2) and (3) Transformed into the corresponding following similar transform

$$\begin{aligned} \varphi(x,y) &= \sqrt{k} x f(\eta) \\ \frac{T - T_w}{T_\infty - T_w} &= \theta(\eta) \end{aligned} \tag{6}$$

Where  $\eta = y \sqrt{\frac{k}{\nu}}$  and k = constant

Momentum and Energy equations are transferred into following equations

$$f f'' + f''' = 0 \tag{7}$$

$$\theta'' + P_r \theta' = 0 \tag{8}$$

where  $\alpha = \frac{K}{\rho \mu c_p}$  and  $P_r = \frac{\mu c_p}{\rho}$  (Prandtl Number)

Solving Equation (7) and (8) find the value of  $\theta$  and f. The corresponding boundary conditions are

$$\eta = 0, f' = 0 \text{ and } f = 0, \eta = 0, \theta = 1 \text{ and } \eta = \infty$$

$$\theta' = 0 \text{ at } y = 0, \eta = 0$$

$$\theta = 0 \text{ at } y = \infty, \eta = \infty$$

**RESULTS AND DISCUSSION**

For results and discussion we solve the complete solution of equation (7) and (8) using Runge-Kutta Method and Shooting technique. The value of (f) depends on the value of  $\eta$  and also the value of temperature depends on the value of  $\eta$ . The computations were done by a programme which uses a symbolic and computational computer language Matlab. Now we take the graphical solution.

f	0.2	0.4	0.6	0.8	1.0
$\eta = y \sqrt{\frac{k}{\nu}}$	1	2	3	4	5

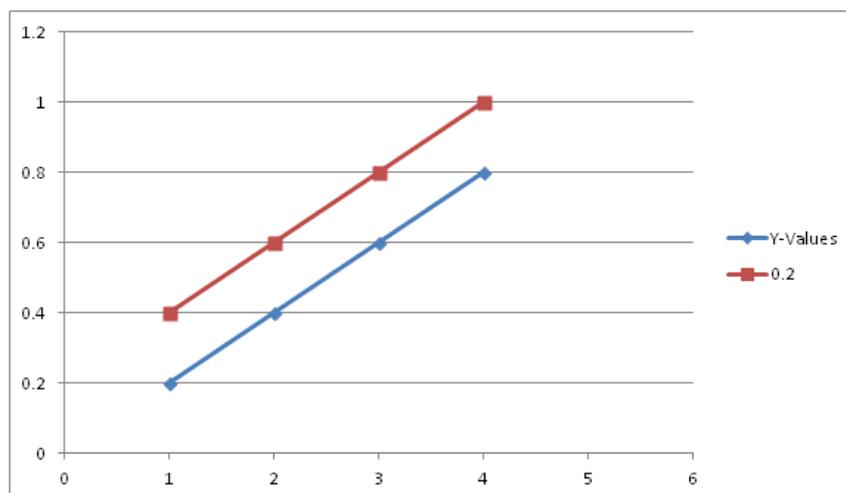
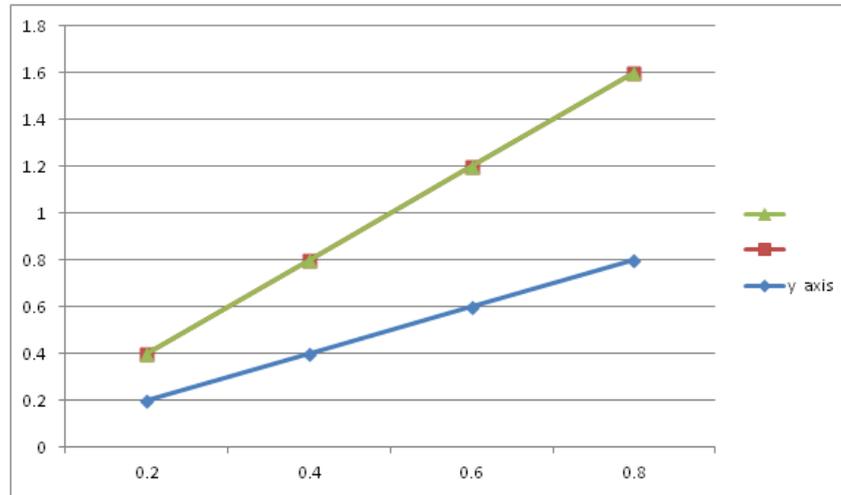


Figure-1

$\eta$	0.2	0.4	0.6	0.8	0	0	0	0	0
$\theta$	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8



**Figure-2**

## CONCLUSION

The effect of temperature and the effect of velocity both are depend on the value of  $\eta$ . If the value of  $\eta$  increase it's also increase the temperature. Parameter  $\alpha$  and the momentum boundary layer thickness decreases. Also, the dimensionless temperature profile as well as the thermal boundary layer thickness quickly reduces as increasing Pr. Similarly thermal boundary layer thickness decreases for some higher values of heat source parameter heat absorption occurs at the sheet. The rate of heat transfer increases with Prandtl number.

## REFERENCES

1. White, F. M.: Viscous Fluid Flow. New York: McGraw-Hill 1974 Harpole, G. M.; Berger, S. A.; Aroesty, J.: Approximate methods for calculating heated water laminar boundary-layer properties. J. of Appl. Mech. 46 (1979) 9–14
2. Brown, S. N.; Stewartson, K.: Laminar separation. Ann. Rev. Fluid Mech. 1 (1969) 45–72
3. Evans, H. L.: Laminar boundary-layer theory. Massachusetts: Addison-Wesley Publ. Comp. 1968
4. Ibrahim S. Y. and Makinde O. D., “Chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction”. Scientific Research and Essays, Vol. 5(19), 2010, pp.2875-2882.
5. Kao T. T., “Locally nonsimilar solution for laminar free convection adjacent to vertical wall”. Trans. ASME, Journal of Heat Transfer, Vol.98, 1976, pp.321-322.
6. Makinde O.D. “Similarity solutions for a natural convection from a moving vertical plate with internal heat generation and a convective boundary condition”. Thermal Science, Vol.15, Suppl.1, 2011, pp.S137-S143.
7. Sarmal U. and Hazarika G.C., “Effects of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence of a magnetic field”. Lat. Am. J. Phys. Educ., Vol.5 (1), 2011, pp.100-106.
8. Sharma P.R., “Preconvection effects on the flow of an ordinary viscous fluid past and infinite vertical porous plate with constant suction and constant heat flux”. J. Ult. Scientist. Phyl. Sciences, Vol.3, 1991, pp.88-91.
9. Sharma P.R. and Singh G., “Numerical solution of transient MHD free convection flow of an incompressible viscous fluid along an inclined plate with ohmic dissipation”. Int. J. of Appl. Math. and Mech., Vol.5 (5), 2009, pp.57-65.
10. Takhar H. S. and Beg O.A., “Effects of transverse magnetic field, prandtl number and reynolds number on non-darcy mixed convective flow of an incompressible viscous fluid past a porous vertical flat plate in a saturated porous medium”. Int. J. Energy Res., 21, 1997, pp. 87-100.
11. S. N. Antontsev, J. I. Díaz and S. I. Shmarev. Energy methods for free boundary problems, Progr. Nonlinear Differential Equations Appl. 48, Birkhäuser, 2002.
12. S. N. Antontsev and H. B. de Oliveira, Navier-Stokes equations with absorption under slip boundary conditions: existence, uniqueness and extinction in time. RIMS Kôkyûroku Bessatsu B1, Kyoto University (2007), pp. 21-42.
13. S. N. Antontsev, J. I. Díaz and S. I. Shmarev. Energy methods for free boundary problems, Progr. Nonlinear Differential Equations Appl. 48, Birkhäuser, 2002.
14. S. N. Antontsev, A. V. Kazhikhov and V. N. Monakhov. Boundary value problems in mechanics of nonhomogeneous fluids. Studies in Mathematics and its Applications 22, North-Holland, 1990.
15. Kao T. T., “Locally nonsimilar solution for laminar free convection adjacent to vertical wall”. Trans. ASME, Journal of Heat Transfer, Vol.98, 1976, pp.321-322.

16. Makinde O.D. "Similarity solutions for a natural convection from a moving vertical plate with internal heat generation and a convective boundary condition". *Thermal Science*, Vol.15, Suppl.1, 2011, pp.S137-S143.
17. Sarmal U. and Hazarika G.C., "Effects of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence of a magnetic field". *Lat. Am. J. Phys. Educ.*, Vol.5 (1), 2011, pp.100-106.
18. Sharma P.R., "Preconvection effects on the flow of an ordinary viscous fluid past and infinite vertical porous plate with constant suction and constant heat flux". *J. Ult. Scientist. Phyl. Sciences*, Vol.3, 1991, pp.88-91.
19. Sharma P.R. and Singh G., "Numerical solution of transient MHD free convection flow of an incompressible viscous fluid along an inclined plate with ohmic dissipation". *Int. J. of Appl. Math. and Mech.*, Vol.5 (5), 2009, pp.57-65.
20. Takhar H. S. and Beg O.A., "Effects of transverse magnetic field, prandtl number and reynolds number on non-darcy mixed convective flow of an incompressible viscous fluid past a porous vertical flat plate in a saturated porous medium". *Int. J. Energy Res.*, 21, 1997, pp. 87-100. 2348.

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