MULTIOBJECTIVE FRACTIONAL TRANSPORTATION PROBLEM IN FUZZY ENVIRONMENT

SHEEMA SADIA*, NEHA GUPTA2 AND QAZI M. ALI1

1Department of Statistics & Operations Research, A.M.U., Aligarh-202002, India.
2Faculty of Commerce and Management, SGT University, Gurugram, India.

(Received On: 13-10-17; Revised & Accepted On: 11-12-17)

ABSTRACT

The fractional programming is a generalization of linear programming where the objective function is a ratio of two linear functions. Similarly, in fractional transportation problem the objective is to optimize the ratio of two cost functions or damage functions or demand functions. As the ratio of two functions is considered, the fractional programming models become more suitable for real life problems. Keeping in view the complexities associated with real life transportation problem like vagueness and uncertainty involved with the parameters. The implementation of fuzzy techniques can be very useful. Therefore, in this article a Fully Fuzzy Multi-objective Fractional Transportation Problem (FFMOFTP) is considered. All the coefficients of the parameters, demands and supplies are considered as fuzzy numbers. The purpose of using fuzzy numbers is to deal with the uncertainties and vagueness associated with the parameters. Two cases are considered, one with triangular and other with trapezoidal fuzzy numbers. The fuzzy problem is converted into crisp form by two ranking methods; Yager’s ranking method and Maleki ranking method respectively. The converted crisp problem is linearized by using Taylor’s series. Finally the compromise solution of the converted problem is obtained using fuzzy goal programming technique. The aim of this paper is not only to provide a method for solving FFMOFTP but also to evaluate the efficiency of the two ranking methods under the same circumstances. Two numerical problems for each case are also solved at the end to demonstrate the efficiency of the proposed approach.

Keywords: Fractional transportation problem; Fuzzy goal programming; Multiobjective programming; Fuzzy numbers; Ranking method.

1. INTRODUCTION

The distribution of goods from manufacturer to customer is a common problem and is described as Transportation Problem (TP). It was originally formulated by Hitchcock (1941). The TP can be solved by simplex method. Some shortcuts have been developed to solve TPs. However, the real life problems are quite complex and difficult to solve by conventional methods. For instance, the traditional TPs generally deal with a single objective function of minimizing cost, but in real world, one has to deal with many objectives other than minimizing cost such as minimizing time, damage charges etc. Such problems are called as Multiobjective Transportation Problems (MOTP). Several authors worked on MOTP like Lee and Moore (1973) studied multiobjective optimization transportation problems. Wahed and Waiel (2001) worked on multi-objective transportation problem under fuzziness. Pramanik and Roy (2008) formulated MOTP with fuzzy parameters.

In real life, a situation may arise where the objective is a ratio of two linear functions, e.g. ratio of transporting costs one with the travelled route and other by the preferred route. Such type of problem is an example of Fractional Transportation Problem (FTP), was originally proposed by Swarup (1966). The TP with fractional objective functions has been extensively used by several authors like Verma and Puri (1991), Khurana and Arora (2006), Joshi and Gupta (2011), Gupta and Arora (2012). In FTP, when more than one objective is taken into consideration, then such problems become Multiobjective Fractional Transportation Problems (MOFTP). Real life transportation problems generally have multiple objectives. Mukherjee and Basu (2010) solved an assignment problem with fuzzy cost by ranking method described in Yager (1981) that transforms the fuzzy assignment problem into a crisp assignment problem. Biswas and Pramanik (2011) presented multi-objective assignment problem with fuzzy costs.

In traditional TP, the decision maker is aware about the values of transportation cost, availability and demand of the product. But in real world situations, all the parameters of the transportation problems may not be known precisely, due to many uncontrollable factors. In such situations fuzzy set theory, introduced by Zadeh (1965), is quite useful and the imprecise data can be represented by fuzzy numbers.

The fuzzy concept is used in an application, when its boundaries are not fixed. Or in other words, when the information provided regarding the application is vague or imprecise, fuzzy numbers can be used for the representation of such imprecise data. Moreover, the fuzzy logic is easy to understand and it is based on natural approach. So, it becomes quite effective when applied on real life problems. In past, many researchers used this theory to deal with imprecise or vague data such as Dubois and Prade (1980), Verdagay (1984). Bit et.al (1992) also used application of the method given in Zimmerman (1978, 1985, 1987). Chanas et al. (1984) analysed TP with fuzzy supply values and fuzzy demand values. Bit et al. (1993) applied fuzzy programming on MOTP. Ammar and Youness (2005) used the concept of fuzzy numbers on MOTP. Recently, Sadia et al. (2016) used fuzzy programming approach to solve MOFTP. The reason behind using fuzzy numbers is the uncertainties and vagueness associated with the coefficients of the parameters and with demands and supplies.

In this article, a fully fuzzy MOFTP is formulated. The objective functions are fractional and obtained as a ratio of two linear functions. And each coefficient in the numerator and denominator of all the objective functions, supply and demand are taken as triangular and trapezoidal fuzzy numbers respectively. Fuzzy numbers are converted into crisp form by using Yager’s Ranking function and Maleki ranking each time. The non-linear objective functions obtained are linearized by using Taylor series. And the linearized problem is solved by fuzzy goal programming approach.

The paper is divided into various sections; the first section gives the introduction with brief literature review. The second section describes some important terms and definitions which are used in the paper. Section 3 gives the statement of the problem. The algorithm of the problem is given in section four. The numerical problems with their solutions are presented in section 5.1 and 5.2 respectively. Results are summarised in section 6. The conclusions and discussion of the results obtained are presented in section 7.

2. SOME PRELIMINARIES

In this section, some definitions and notations related to fuzzy set theory are discussed.

**Definition 2.1 (Fuzzy Set):** Let \( X \) be a universal set then a fuzzy subset \( \tilde{A} \) of \( X \) is defined by its membership function \( \mu_{\tilde{A}}(x) : X \rightarrow [0,1] \) which assigns to each element \( x \in X \) a real number \( \mu_{\tilde{A}}(x) \) in the interval \([0,1] \), where the value of \( \mu_{\tilde{A}}(x) \) at \( x \) represents the grade of membership of \( x \) in \( A \). Thus, nearer the value of \( \mu_{\tilde{A}}(x) \) is unity, the higher is the grade of membership of \( x \) in \( A \).

A fuzzy subset \( \tilde{A} \) can be characterized as a set of ordered pairs of element \( x \) and grade \( \mu_{\tilde{A}}(x) \). This is often written as, \( \tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\} \)

where, \( \mu_{\tilde{A}}(x) \) is called the membership function or grade of membership (also degree of compatibility or degree of truth) that maps to the membership space. When this space contains only the two points 0 and 1, \( \tilde{A} \) is non-fuzzy and \( \mu_{\tilde{A}}(x) \) is identical to the characteristic function of a non-fuzzy set, [see Sakawa (1993)]

**Definition 2.2: (Support):** The support of a fuzzy set \( A \) on \( X \), denoted by supp(\( \tilde{A} \)), is the set of points in \( X \) at which \( \mu_{\tilde{A}}(x) \) is positive, i.e. supp(\( \tilde{A} \)) = \{x \in X | \mu_{\tilde{A}}(x) > 0\} [see Sakawa (1993)]

**Definition 2.3: (\( \alpha \)-cut):** The \( \alpha \)-cut of a fuzzy subset \( \tilde{A} \) of \( X \) can be defined by \( \tilde{A}(\alpha) = \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha, \alpha \in [0,1] \} \forall \alpha \in [0,1] \) [see Tzeng and Huang (2013)]

**Fuzzy number:** A fuzzy set \( \tilde{A} \) on \( R \) must have the following properties to qualify as a fuzzy number.

1. \( \tilde{A} \) must be a normal fuzzy set.
2. \( \tilde{A} \) must be closed interval for every \( \alpha \in [0,1] \).
3. The supp (\( \tilde{A} \)) must be bounded in \( R \).

© 2017, IJMA. All Rights Reserved
Triangular Fuzzy number: It is fuzzy number represented with three points as follows in figure 1. This representation is interpreted as membership functions and holds the following conditions.

1. \( a_1 \) to \( a_2 \) is increasing function.
2. \( a_2 \) to \( a_3 \) is decreasing function.
3. \( a_1 \leq a_2 \leq a_3 \)

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
0 & \text{for } x < a_1 \\
\frac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2 \\
\frac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\
0 & \text{for } x > a_3 
\end{cases}
\]

Trapezoidal fuzzy number: A trapezoidal fuzzy number (Figure 2) can be completely specified by the foursome \( \tilde{R} = (r_1, r_2, r_3, \text{and } r_4) \) with membership function as-

\[
\mu_{\tilde{R}}(r) = \begin{cases} 
0, & r \leq r_1 \\
\frac{r - r_1}{r_2 - r_1}, & r_1 \leq r \leq r_2 \\
1, & r_2 \leq r \leq r_4 \\
0, & r \geq r_4
\end{cases}
\]

The Yager’s Ranking Function: For a fuzzy number \( \tilde{A} \), the Yager’s Ranking Index will be:

\[
Y(\tilde{A}) = \frac{1}{0.5(A^L_{\alpha} + A^U_{\alpha})} d\alpha
\]

where \([A^L_{\alpha}, A^U_{\alpha}]\) is the crisp interval \( \forall \alpha \in [0,1] \) obtained by \( \alpha - \text{cut} \). The \( \alpha - \text{cut} \) of triangular fuzzy number \( \tilde{A} \) is as follows:

\[
^\alpha \tilde{A} = [A^L_{\alpha}, A^U_{\alpha}] = [(a_2 - a_1) \alpha + a_1, -(a_3 - a_2) \alpha + a_3]
\]

The \( \alpha - \text{cut} \) of trapezoidal fuzzy number \( \tilde{R} \) is as follows:

\[
^\alpha \tilde{R} = [R^L_{\alpha}, R^U_{\alpha}] = [(r_2 - r_1) \alpha + r_1, -(r_4 - r_3) \alpha + r_4]
\]

Since, \( Y(\tilde{A}) \) is calculated from the values of \( \alpha - \text{cut} \) of \( \tilde{A} \), rather than its membership function, Yager’s ranking function is also applicable even if the explicit form of membership function is not known.

The Maleki Ranking Function


Let \( \tilde{T} \) be a trapezoidal fuzzy number of the form \( \tilde{T} = (t', t^*, \gamma, \beta) \) where \( (t' - \gamma, t^* + \beta) \) is the support of \( \tilde{T} \) and \( (t', t^*) \) is the core of it.
The Maleki ranking function of $\tilde{T}$ is:

$$
\mathcal{R}(\tilde{T}) = \int (\inf \tilde{T}_\alpha + \sup \tilde{T}_\alpha) \, d\alpha
$$

$$
\mathcal{R}(\tilde{T}) = t^* + t^* + \frac{1}{2}(\beta - \gamma)
$$

where $\gamma = \beta$ or $\gamma \neq \beta$

For our case, where $\tilde{R}$ is the trapezoidal number of the form $(r_1, r_2, r_3, r_4)$, the Maleki ranking function is:

$$
\mathcal{R}(\tilde{R}) = r_2 + r_3 + \frac{1}{2}(r_2 - r_1 - r_2 + r_4)
$$

Keeping in view the fact that trapezoidal fuzzy number becomes triangular fuzzy number if $r_2 = r_3$. So the Maleki Ranking function for triangular fuzzy number of the form $\tilde{A} = (a_1, a_2, a_3)$ is:

$$
\mathcal{R}(\tilde{A}) = 2a_2 + \frac{1}{2}(a_3 - a_2 - a_2 + a_1)
$$

$$
= 2a_2 + \frac{1}{2}(a_3 - 2a_2 + a_1)
$$

3. STATEMENT OF THE PROBLEM

Let us consider a Fully Fuzzy Multiobjective Fractional Transportation Problem (FFMOFTP) as:

Minimize $Z_k(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}^k x_{ij}$

subject to $\sum_{j=1}^{n} x_{ij} \leq a_i$; $k = 1, 2, \ldots, K.$

$$
\sum_{i=1}^{m} x_{ij} \leq b_j;
$$

$x_{ij} \geq 0$;

where $Z_k(x); k = 1, 2, \ldots, K$ is a vector of $K$ objective functions. The variable $X_{ij}$ represents the unknown quantity transported from $i^{th}$ origin to $j^{th}$ destination. $c_{ij}^k$ and $r_{ij}^k$ are the penalties associated with transporting a unit product from source $O_i$ to destination $D_j$ for the $k^{th}$ criterion. In general $\tilde{c}_{ij}^k$ represents the fuzzy transporting costs or fuzzy transportation time or fuzzy damage cost either due to loss of quality or quantity of transportation, whereas $\tilde{r}_{ij}^k$ represents the fuzzy transportation cost due to preferred route or fuzzy standard transportation time or fuzzy damage cost due to preferred route.

Here the fuzzy coefficients are considered as triangular and trapezoidal fuzzy numbers. The purpose of using fuzzy numbers is to overcome the uncertainty associated with parameters.

4. ALGORITHM

Step-1: The first step is to convert all the fuzzy numbers (triangular or trapezoidal) into crisp form by using Yager’s and Maleki ranking function.

Now, the problem (1) reduces to the crisp problem as:
Minimize \( Z_k(x) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} R(\tilde{c}_{ij}^{k}) x_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{m} R(\tilde{r}_{ij}^{k}) x_{ij}} \)
subject to
\[
\sum_{i=1}^{n} x_{ij} \leq a_j, \quad k = 1, 2, \ldots, K.
\]
\[
\sum_{j=1}^{m} x_{ij} \leq b_i
\]
\[
x_{ij} \geq 0
\]

(2)

**Step-2:** To obtain the fuzzy goal programming model of FFMOFTP as given in (2), we transform the objective functions \( K_k x Z_k \), \( k = 1, 2, \ldots, K \), into fuzzy goals by assigning an imprecise aspiration level to each of them.

Let \( Z_k^*(x) \), \( k = 1, 2, \ldots, K \), be the optimal solutions of the \( K \) objective functions \( Z_k(x) \) of FFMOFTP, when calculated in isolation subject to the system constraints. Then, the fuzzy goals appear in the form:

\( Z_k(x) \geq Z_k^*(x) \)

A payoff matrix is obtained using the individual best solutions as follows:

\[
\begin{bmatrix}
Z_1(x) & Z_2(x) & \cdots & Z_K(x) \\
x_{ij}^{(1)} & Z_1(x_{ij}^{(1)}) & Z_2(x_{ij}^{(1)}) & \cdots & Z_K(x_{ij}^{(1)}) \\
x_{ij}^{(2)} & Z_1(x_{ij}^{(2)}) & Z_2(x_{ij}^{(2)}) & \cdots & Z_K(x_{ij}^{(2)}) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_{ij}^{(K)} & Z_1(x_{ij}^{(K)}) & Z_2(x_{ij}^{(K)}) & \cdots & Z_K(x_{ij}^{(K)})
\end{bmatrix}
\]

where \( x_{ij}^{(K)} \) are the individual optimal points of the objective functions of FFMOFTP.

The maximum value of each column gives the upper bound for the objective functions whereas the minimum value of each column gives lower bound for the objective functions.

The objective values which are equal to or larger than \( Z_k^*(x) \) should be absolutely satisfactory to the problem. A satisfactory optimal solution of the problem is reached, if the individual best solutions are identical. However, this situation arises rarely because the objective functions are conflicting in general.

The membership function \( \mu_k(x); k = 1, 2, \ldots, K \), is not linear and is corresponding to the objective function \( Z_k^*(x) \). It can be formulated as:

\[
\mu_k(x) = \begin{cases} 
0, & \text{if } Z_k(x) \geq Z_k^*(x); \\
\frac{Z_k^*(x) - Z_k(x)}{Z_k^*(x) - Z_k^*(x)}, & \text{if } Z_k^*(x) \leq Z_k(x) \leq Z_k^*(x); \\
1, & \text{if } Z_k(x) \leq Z_k^*(x).
\end{cases}
\]

Where, \( Z_k^*(x) \) and \( Z_k^*(x) \) are respectively the upper and lower bounds of the fuzzy objective goals for FFMOFTP.

Before step 3, the problem (2) reduces to

Minimize \( \mu_k(x) \)
subject to
\[
\sum_{i=1}^{n} x_{ij} \leq a_j, \quad k = 1, 2, \ldots, K.
\]
\[
\sum_{j=1}^{m} x_{ij} \leq b_i
\]
\[
x_{ij} \geq 0
\]

(3)
Step-3: Let \( x_{ij}^{(k)r} \), \( k = 1,2,\ldots,K \), be the individual best solutions of the non-linear membership functions \( \mu_k(x) \) subject to constraints. Then, the above membership functions \( \mu_k(x) \) are transformed into equivalent linear form at individual best solution point by first order Taylor’s series as follows:

\[
\mu_k(x) \approx \mu_k\left(x_{ij}^{(k)r}\right) + \left(x_{ij} - x_{ij}^{(k)r}\right) \frac{\partial}{\partial x_{ij}} \mu_k\left(x_{ij}^{(k)r}\right) + \ldots + \left(x_{mn} - x_{mn}^{(k)r}\right) \frac{\partial}{\partial x_{mn}} \mu_k\left(x_{ij}^{(k)r}\right) = \lambda_k.
\]

Using the Taylor’s Series expansion, the FFMOFTP represented by (3) reduces to the following problem

\[
\begin{align*}
\text{Minimize} & \quad \lambda_k(x) \\
\text{subject to} & \quad \sum_{j=1}^{n} x_{ij} \leq a_i, \quad k = 1,2,\ldots,K. \\
& \quad \sum_{j=1}^{n} x_{ij} \leq b_j, \\
& \quad x_{ij} \geq 0.
\end{align*}
\]

(4)

Step-4: As the maximum value of a membership function is one, so for the defined membership functions in (3), the flexible membership goals having the aspiration level unity is presented as:

\[
\lambda_k(x) + \delta_k = 1; \quad k = 1,2,\ldots,K.
\]

Here \( \delta_k \geq 0; k = 1,2,\ldots,K \) are the deviational variables.

Finally, the fuzzy goal programming (FGP) model will be:

\[
\begin{align*}
\text{Minimize} & \quad \delta_k \\
\text{subject to} & \quad \lambda_k(x) + \delta_k = 1 \\
& \quad \sum_{j=1}^{n} x_{ij} \leq a_i, \quad k = 1,2,\ldots,K. \\
& \quad \sum_{j=1}^{n} x_{ij} \leq b_j \\
& \quad \delta_k, \quad x_{ij} \geq 0.
\end{align*}
\]

(5)

Step-5: The fuzzy goal programming model obtained in step 4 is solved to obtain the final compromise solution. LINGO 13 Software is used for doing all the calculations.

5.1 Problem description with data
In order to demonstrate the problem and the utility of the approach, discussed above, two numerical problems are presented. The first problem is considered in which data is of the form of triangular fuzzy numbers. And it is in the form of trapezoidal fuzzy number for second problem.

A MOFTP in fuzzy environment is considered. There are three origins and three destinations. The TP cost (ratio of transporting cost one with travelled route and the other by preferred route, time) (ratio of actual transportation time and standard transportation time, damage charges (ratio of damage charges one with travelled route and the other by preferred route), demands and supplies during the transportation are considered as fuzzy numbers (triangular or trapezoidal) and are presented in matrices below.

**Problem 1: FFMOFTP WITH TRIANGULAR FUZZY DATA**

**Fuzzy Cost Matrix**

<table>
<thead>
<tr>
<th></th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( b_3 )</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>(3,5,7)</td>
<td>(4,7,8)</td>
<td>(14,15,17)</td>
<td>( \leq (10,12,14) )</td>
</tr>
<tr>
<td></td>
<td>(13,5)</td>
<td>(14,5)</td>
<td>(11,13,15)</td>
<td></td>
</tr>
<tr>
<td>( a_2 )</td>
<td>(6,8,10)</td>
<td>(14,17,18)</td>
<td>(11,12,13)</td>
<td>( \leq (13,15,17) )</td>
</tr>
<tr>
<td></td>
<td>(11,12,15)</td>
<td>(12,14,16)</td>
<td>(5,7,8)</td>
<td></td>
</tr>
<tr>
<td>( a_3 )</td>
<td>(12,14,16)</td>
<td>(7,10,11)</td>
<td>(11,13,14)</td>
<td>( \leq (18,20,22) )</td>
</tr>
<tr>
<td></td>
<td>(12,15,16)</td>
<td>(14,6,8)</td>
<td>(8,9)</td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>(7,9,11)</td>
<td>(11,13,15)</td>
<td>(19,21,23)</td>
<td></td>
</tr>
</tbody>
</table>
Fuzzy Time Matrix

<table>
<thead>
<tr>
<th></th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>$\frac{(15,17,18)}{(7,9,12)}$</td>
<td>$\frac{(3,5,6)}{(1,2,4)}$</td>
<td>$\frac{(8,10,12)}{(0,3,4)}$</td>
<td>$(10,12,14)$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$\frac{(0,1,4)}{(1,2,5)}$</td>
<td>$\frac{(8,11,12)}{(3,4,5)}$</td>
<td>$\frac{(4,6,7)}{(3,5,6)}$</td>
<td>$(13,15,17)$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$\frac{(10,13,14)}{(6,8,10)}$</td>
<td>$\frac{(15,16,17)}{(10,12,13)}$</td>
<td>$\frac{(9,10,13)}{(9,11,14)}$</td>
<td>$(18,20,22)$</td>
</tr>
</tbody>
</table>

Fuzzy Damage Charge Matrix

<table>
<thead>
<tr>
<th></th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>$\frac{(10,13,14)}{(6,8,11)}$</td>
<td>$\frac{(13,15,16)}{(7,9,10)}$</td>
<td>$\frac{(6,8,9)}{(9,11,12)}$</td>
<td>$(10,12,14)$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$\frac{(13,15,16)}{(9,11,12)}$</td>
<td>$\frac{(13,14,15)}{(4,6,8)}$</td>
<td>$\frac{(15,19,21)}{(5,7,8)}$</td>
<td>$(13,15,17)$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$\frac{(4,7,8)}{(6,9,10)}$</td>
<td>$\frac{(13,15,16)}{(4,6,7)}$</td>
<td>$\frac{(15,17,18)}{(6,7,9)}$</td>
<td>$(18,20,22)$</td>
</tr>
</tbody>
</table>

5.1 Solution procedure

Using the above data, the FFMOFTP has been formulated which cannot be solved by any standard method. To solve the fuzzy problem, firstly we have to convert it into crisp problem by using ranking function.

Case-1 (Yager’s ranking function): To convert the triangular fuzzy data into crisp form. Yager’s ranking function is used as follows.

The $\alpha$-cut of fuzzy number $(3, 5, 7)$ is $[C^\alpha_n, C^\gamma_n] = (3+2\alpha, 7-2\alpha)$ for which,

$\mathcal{R}_\alpha(C^\alpha_n) = \mathcal{R}_\alpha(3,5,7) = \int_0^1 0.5(C^\alpha_n + C^\gamma_n) d\alpha = 5$

$\mathcal{R}_\alpha(1,3,5) = 3, \mathcal{R}_\alpha(4,7,8) = 6.5$.

Similarly, the Yager’s ranking index of each element can be obtained.

$\mathcal{R}_\alpha(T^\alpha_n) = 6.5, \mathcal{R}_\alpha(T^\gamma_n) = 3.5, \mathcal{R}_\alpha(T^\gamma_n) = 15.25, \mathcal{R}_\alpha(T^\gamma_n) = 10, \mathcal{R}_\alpha(T^\gamma_n) = 25, \mathcal{R}_\alpha(T^\gamma_n) = 7.5$.

Here $n$ and $d$ in the superscripts denote the numerator and denominator values.

Using the above data, the MOFTP with mixed constraints can be given as follows:
Minimize $Z_1 = \frac{(5x_{11} + 6.5x_{12} + 15.25x_{13} + 8x_{21} + 16.5x_{22} + 12x_{23} + 14x_{31} + 14.5x_{32} + 12.75x_{33})}{(3x_{11} + 3.5x_{12} + 13x_{13} + 12.5x_{21} + 14x_{22} + 6.75x_{23} + 14.5x_{31} + 6x_{32} + 7.75x_{33})}$

Minimize $Z_2 = \frac{(16.75x_{11} + 4.75x_{12} + 10x_{13} + 1.5x_{21} + 10.5x_{22} + 5.75x_{23} + 12.5x_{31} + 16.75x_{32} + 10.5x_{33})}{(9.25x_{11} + 2.25x_{12} + 2.5x_{13} + 2.5x_{21} + 4x_{22} + 4.75x_{23} + 8x_{31} + 11.75x_{32} + 11.25x_{33})}$

Minimize $Z_3 = \frac{(12.5x_{11} + 14.75x_{12} + 7.75x_{13} + 14.75x_{21} + 14x_{22} + 18.5x_{23} + 6.5x_{31} + 14.75x_{32} + 16.75x_{33})}{(8.25x_{11} + 8.75x_{12} + 10.75x_{13} + 10.75x_{21} + 6x_{22} + 6.75x_{23} + 8.5x_{31} + 5.75x_{32} + 7.5x_{33})}$

subject to

\[ x_{11} + x_{12} + x_{13} \leq 12 \]
\[ x_{21} + x_{22} + x_{23} \leq 15 \]
\[ x_{31} + x_{32} + x_{33} \leq 20 \]
\[ x_{11} + x_{21} + x_{31} = 9 \]
\[ x_{12} + x_{22} + x_{32} = 13 \]
\[ x_{31} + x_{32} + x_{33} = 21 \]
\[ x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33} \geq 0 \text{ and integers.} \]

The pay-off matrix is obtained after solving the above problem separately for each objective function using the optimizing software LINGO as follows:

\[
\text{Payoff Matrix} = \begin{pmatrix}
0.47661 & 0.185787 & 1.152935 \\
1.858025 & 1.379630 & 1.570192 \\
1.312642 & 1.858025 & 1.047661
\end{pmatrix}
\]

The non-linear membership functions for objective functions $C(x), T(x)$ and $D(x)$ are obtained as follows: -

\[ \mu_1 = \frac{1.312642 - Z_1}{1.312642 - 1.152935}, \quad \mu_2 = \frac{1.858025 - Z_2}{1.858025 - 1.379630}, \quad \mu_3 = \frac{1.815867 - Z_3}{1.815867 - 1.047661} \]

Then, the non-linear membership functions are transformed into linear at the individual best solution point by first order Taylor polynomial series. It will be as follows: -

\[ \mu_1 = 1 + (-0.0207)(x_{11} - 10) + (-0.03324)(x_{12} - 0) + (-0.0035316)(x_{13} - 12) + 0.08546(x_{21} - 6) + (-0.0048407)(x_{22} - 9) + (-0.05688)(x_{23} - 0) + 0.03665(x_{31} - 0) + (-0.034829)(x_{32} - 7) + (-0.051450)(x_{33} - 0) = \lambda_1 \]

\[ \mu_2 = 1 + (-0.032441)(x_{11} - 10) + (-0.010996)(x_{12} - 12) + (-0.033920)(x_{13} - 9) + 0.0051433(x_{21} - 9) + (-0.028995)(x_{22} - 1) + (-0.003555)(x_{23} - 0) + (-0.018928)(x_{31} - 0) + (-0.0204048)(x_{32} - 0) + (0.005911)(x_{33} - 20) = \lambda_2 \]

\[ \mu_3 = 1 + (-0.0066583)(x_{11} - 10) + (-0.015949)(x_{12} - 0) + 0.0421698(x_{13} - 12) + 0.000482(x_{21} - 0) + (-0.034077)(x_{22} - 13) + (-0.054714)(x_{23} - 0) + 0.031127(x_{31} - 9) + (-0.040598)(x_{32} - 0) + (-0.038131)(x_{33} - 9) = \lambda_3 \]

Then, the FGP model for solving the above problem will be as follows:
Minimize \[ \delta_1 + \delta_2 + \delta_3 \]
subject to
\[ \lambda_1 + \delta_1 = 1 \]
\[ \lambda_2 + \delta_2 = 1 \]
\[ \lambda_3 + \delta_3 = 1 \]
\[ x_{11} + x_{12} + x_{13} \leq 12; \quad x_{21} + x_{22} + x_{23} \leq 15; \quad x_{31} + x_{32} + x_{33} \leq 20 \]
\[ x_{11} + x_{12} + x_{13} = 9; \quad x_{21} + x_{22} + x_{23} = 13; \quad x_{31} + x_{32} + x_{33} = 21 \]
\[ \delta_1, \delta_2, \delta_3 \geq 0 \]
\[ x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33} \geq 0 \]
and integers.

By solving the above problem using LINGO-13, we get the following results:
\[(x^*, x^*, x^*, x^*, x^*, x^*, x^*, x^*, x^*) = (0, 0, 0, 0, 0, 0, 0, 0, 0)\] with minimum cost \( \mathcal{C} = 1.187085 \), Damage Charges, \( D = 1.560027 \), and Time = 1.486371, respectively.

Case-2 (Maleki ranking function): To convert the triangular fuzzy data into crisp form. Maleki ranking function is used as follows.

The crisp form of fuzzy number \((3, 5, 7)\) on applying Maleki Ranking will be
\[ \mathcal{R}_u((C^1_u, C^2_u, C^3_u)) = 9 \]

Similarly, the Yager’s ranking index of each element can be obtained.

Here \( n \) and \( d \) in the superscripts denote the numerator and denominator values.

Using the above data, the MOFTP with mixed constraints can be given as follows:

Minimize \[ Z_1 = (10x_{11} + 13x_{12} + 30.5x_{13} + 16x_{21} + 33x_{22} + 24x_{23} + 28x_{31} + 19x_{32} + 25.5x_{33}) \]
\[ (6x_{11} + 7x_{12} + 26x_{13} + 25x_{21} + 28x_{22} + 13.5x_{23} + 29x_{31} + 12x_{32} + 15.5x_{33}) \]

Minimize \[ Z_2 = \frac{(33.5x_{11} + 9.5x_{12} + 20x_{13} + 3x_{21} + 21x_{22} + 11.5x_{23} + 25x_{31} + 33.5x_{32} + 21x_{33})}{(18.5x_{11} + 4.5x_{12} + 5x_{13} + 5x_{21} + 8x_{22} + 9.5x_{23} + 16x_{31} + 23.5x_{32} + 22.5x_{33})} \]

Minimize \[ Z_3 = \frac{(25x_{11} + 29.5x_{12} + 15.5x_{13} + 29.5x_{21} + 28x_{22} + 37x_{23} + 13x_{31} + 29.5x_{32} + 33.5x_{33})}{(16.5x_{11} + 17.5x_{12} + 21.5x_{13} + 21.5x_{21} + 12x_{22} + 13.5x_{23} + 17x_{31} + 11.5x_{32} + 15x_{33})} \]

subject to
\[ x_{11} + x_{12} + x_{13} \leq 12 \]
\[ x_{21} + x_{22} + x_{23} \leq 15 \]
\[ x_{31} + x_{32} + x_{33} \leq 20 \]
\[ x_{11} + x_{21} + x_{31} = 9 \]
\[ x_{12} + x_{22} + x_{32} = 13 \]
\[ x_{31} + x_{32} + x_{33} = 21 \]
\[ x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33} \geq 0 \]
and integers.

We will follow the steps given in section (4) and proceeding in the same manner as above. We will obtain the final result as follows.
\[(x^*, x^*, x^*, x^*, x^*, x^*, x^*, x^*, x^*) = (0, 0, 12, 9, 0, 2, 0, 13, 7)\] with minimum cost \( \mathcal{C} = 1.187085 \), Damage Charges, \( D = 1.560027 \), and Time = 1.486371, respectively.
Problem 2: FFMOFTP WITH TRAPEZOIDAL FUZZY DATA

Fuzzy Cost Matrix

<table>
<thead>
<tr>
<th></th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>(4,6,8,10)</td>
<td>(3,6,9,11)</td>
<td>(12,13,15,17)</td>
<td>(10,11,13,16)</td>
</tr>
<tr>
<td>$a_2$</td>
<td>(4,6,8,10)</td>
<td>(12,15,17,18)</td>
<td>(11,13,15,17)</td>
<td>(6,9,12,14)</td>
</tr>
<tr>
<td>$a_3$</td>
<td>(12,13,16,18)</td>
<td>(8,10,12,14)</td>
<td>(10,13,15,18)</td>
<td>(8,10,11,12)</td>
</tr>
</tbody>
</table>

Fuzzy Time Matrix

<table>
<thead>
<tr>
<th></th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>(12,15,18,21)</td>
<td>(2,4,6,10)</td>
<td>(6,10,14,16)</td>
<td>(2,5,8,11)</td>
</tr>
<tr>
<td>$a_2$</td>
<td>(4,5,7,9)</td>
<td>(8,10,14,16)</td>
<td>(3,4,5,9)</td>
<td>(4,5,9,11)</td>
</tr>
<tr>
<td>$a_3$</td>
<td>(10,13,16,19)</td>
<td>(13,15,19,22)</td>
<td>(7,9,13,15)</td>
<td>(5,7,9,10)</td>
</tr>
</tbody>
</table>

Fuzzy Damage Matrix

<table>
<thead>
<tr>
<th></th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>(10,13,15,19)</td>
<td>(13,16,17,20)</td>
<td>(7,8,10,12)</td>
<td>(8,9,10,11)</td>
</tr>
<tr>
<td>$a_2$</td>
<td>(14,16,18,20)</td>
<td>(12,14,18,20)</td>
<td>(14,18,20,24)</td>
<td>(3,5,7,9)</td>
</tr>
<tr>
<td>$a_3$</td>
<td>(3,4,8,10)</td>
<td>(13,15,17,19)</td>
<td>(13,15,17,21)</td>
<td>(5,7,9,12)</td>
</tr>
</tbody>
</table>

Using the above trapezoidal data, the FFMOFTP has been formulated. It cannot be solved by any standard methods. To solve this problem, convert it into crisp problem by using ranking functions.

Case-1 (Yager’s ranking function): To convert the trapezoidal fuzzy data into crisp form. Yager’s ranking function is used as follows.

The $\alpha$-cut of fuzzy number $(4, 6, 8, 10)$ is $[C_{\alpha}^L, C_{\alpha}^U] = (4+2\alpha, 10-2\alpha)$ for which,

$\mathcal{R}_\alpha(C_{\alpha}^L) = \mathcal{R}_\alpha(4,6,8,10) = \int_0^1 0.5(C_{\alpha}^L + C_{\alpha}^U) d\alpha = 7$

$\mathcal{R}_\alpha(3,6,9,11) = 7.25, \mathcal{R}_\alpha(12,13,15,17) = 14.25.$

Similarly, the Yager’s ranking index of each element can be obtained.

$\mathcal{R}_\alpha(\tilde{C}_{\alpha}^L) = 7.25, \mathcal{R}_\alpha(\tilde{C}_{\alpha}^U) = 4.25, \mathcal{R}_\alpha(C_{\alpha}^L) = 14.25, \mathcal{R}_\alpha(C_{\alpha}^U) = 12.5, \mathcal{R}_\alpha(\tilde{C}_{\alpha}^L) = 7, \mathcal{R}_\alpha(\tilde{C}_{\alpha}^U) = 14, \mathcal{R}_\alpha(\tilde{C}_{\alpha}^L) = 15.5$ and so on.

$\mathcal{R}_\alpha(\tilde{T}_{\alpha}^L) = 16.5, \mathcal{R}_\alpha(\tilde{T}_{\alpha}^U) = 11.5, \mathcal{R}_\alpha(T_{\alpha}^L) = 5.5, \mathcal{R}_\alpha(T_{\alpha}^U) = 5, \mathcal{R}_\alpha(\tilde{T}_{\alpha}^L) = 11.5, \mathcal{R}_\alpha(\tilde{T}_{\alpha}^U) = 6.5, \mathcal{R}_\alpha(T_{\alpha}^L) = 6.25$ and so on.

$\mathcal{R}_\alpha(D_{\alpha}^L) = 14.25, \mathcal{R}_\alpha(D_{\alpha}^U) = 10.75, \mathcal{R}_\alpha(D_{\alpha}^L) = 16.5, \mathcal{R}_\alpha(D_{\alpha}^U) = 8.25, \mathcal{R}_\alpha(H_{\alpha}^L) = 9.25, \mathcal{R}_\alpha(H_{\alpha}^U) = 9.5$ and so on.

Here $n$ and $d$ in the superscripts denote the numerator and denominator values.
Using the above data, the MOFTP with mixed constraints can be given as follows:

Minimize \( Z_1 = \left( 7x_{11} + 7.25x_{12} + 12.5x_{13} + 14.25x_{21} + 11.5x_{22} + 14.5x_{23} + 14x_{31} + 1.75x_{32} + 14x_{33} \right) \)

Minimize \( Z_2 = \left( 16.5x_{11} + 5.5x_{12} + 12x_{13} + 6.25x_{21} + 12x_{22} + 9.25x_{23} + 14.5x_{31} + 17.25x_{32} + 10.5x_{33} \right) \)

Minimize \( Z_3 = \left( 14.25x_{11} + 11.5x_{12} + 9.25x_{13} + 17x_{21} + 16x_{22} + 19x_{23} + 6.25x_{31} + 16x_{32} + 16.5x_{33} \right) \)

subject to \( x_{11} + x_{12} + x_{13} = 16 \)
\( x_{21} + x_{22} + x_{23} \leq 20 \)
\( x_{31} + x_{32} + x_{33} \leq 24 \)
\( x_{11} + x_{21} + x_{31} = 12 \)
\( x_{12} + x_{22} + x_{32} = 12 \)
\( x_{13} + x_{23} + x_{33} = 14 \)
\( x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33} \geq 0 \) and integers.

The pay-off matrix is obtained after solving the above problem separately for each objective function using the optimizing software LINGO as follows:

\[ \begin{array}{ccc}
Z_1(x) & Z_2(x) & Z_3(x) \\
0.927899 & 0.980026 & 1.128099 \\
1.602083 & 1.211087 & 1.558484 \\
1.454420 & 1.888048 & 1.228748 \\
\end{array} \]

\( C_e = 0.927899, C_v = 1.128099, T_e = 1.228848, T_v = 1.888048, D_e = 1.211087, D_v = 1.602083. \)

The non-linear membership functions for objective functions \( C(x), T(x) \) and \( D(x) \) are obtained as follows:

\[ \mu_1 = \frac{1.228748 - Z_1}{1.228748 - 0.927899}, \quad \mu_2 = \frac{1.602083 - Z_2}{1.602083 - 1.211087}, \quad \mu_3 = \frac{1.888048 - Z_3}{1.888048 - 1.228748} \]

We will follow the steps given in section 4 and will obtain the final result as follows.

\( (x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) = (0, 2, 1, 4, 2, 8, 0, 0, 2, 0) \) with minimum cost \( C = 0.927899, \) Damage Charges, \( D = 1.454420, \) and Time = 1.602083, respectively.

Case-2 (Maleki ranking function): To convert the triangular fuzzy data into crisp form. Maleki ranking function is used as follows.

The crisp form of fuzzy number \((3, 5, 7)\) on applying Maleki Ranking will be

\( \mathcal{R}_{[3,5,7]}(C_1) = 9 \mathcal{R}_{[3,5,7]}(3) = 2 \times 5 + \frac{1}{2} (7 - 2 \times 5 + 3) = 10 \)

\( \mathcal{R}_{[3,5,7]}(13) = 6, \mathcal{R}_{[3,5,7]}(4,7,8) = 13. \)

Similarly, the Yager’s ranking index of each element can be obtained.

\( \mathcal{R}_{[3,5,7]}(C_2) = 13, \mathcal{R}_{[3,5,7]}(C_2) = 7, \mathcal{R}_{[3,5,7]}(C_1) = 30.5, \mathcal{R}_{[3,5,7]}(C_2) = 26, \mathcal{R}_{[3,5,7]}(C_3) = 16, \mathcal{R}_{[3,5,7]}(C_2) = 25, \mathcal{R}_{[3,5,7]}(C_2) = 33 and so on. \)

\( \mathcal{R}_{[3,5,7]}(D_1) = 33.5, \mathcal{R}_{[3,5,7]}(D_1) = 18.5, \mathcal{R}_{[3,5,7]}(D_1) = 9.5, \mathcal{R}_{[3,5,7]}(D_1) = 4.5, \mathcal{R}_{[3,5,7]}(D_1) = 20, \mathcal{R}_{[3,5,7]}(D_1) = 5, \mathcal{R}_{[3,5,7]}(D_1) = 3 and so on. \)

\( \mathcal{R}_{[3,5,7]}(D_1) = 25, \mathcal{R}_{[3,5,7]}(D_1) = 16.5, \mathcal{R}_{[3,5,7]}(D_1) = 29.5, \mathcal{R}_{[3,5,7]}(D_1) = 17.5, \mathcal{R}_{[3,5,7]}(D_1) = 15.5, \mathcal{R}_{[3,5,7]}(D_1) = 21.5 and so on. \)

Here \( n \) and \( d \) in the superscripts denote the numerator and denominator values.

Using the above data, the MOFTP with mixed constraints can be given as follows:
We will follow the steps given in section 4 and will obtain the final result as follows. 

\[ (x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) = (0, 2, 1, 0, 0, 2, 0, 0, 2, 0) \]

The results obtained for problem 1 are minimum cost \( C = 1.187085 \), damage charges, \( D = 1.560027 \), and time = 1.486371, respectively. The results are same for both the ranking functions.

5. RESULT

On solving the above problems using LINGO-13, we have obtained the following results of the given problem. The result will remain same on applying Yager as well as Maleki Ranking method. So, we can use any of the two ranking function. As Maleki function is comparatively easy to apply, so, it’s favourable to use it.

\[
\text{TABLE-1:}
\]

<table>
<thead>
<tr>
<th>Problem 1</th>
<th>( x_{11} )</th>
<th>( x_{12} )</th>
<th>( x_{13} )</th>
<th>( x_{21} )</th>
<th>( x_{22} )</th>
<th>( x_{23} )</th>
<th>( x_{31} )</th>
<th>( x_{32} )</th>
<th>( x_{33} )</th>
<th>Min. Cost</th>
<th>Min. Damage</th>
<th>Min. Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yager's method</td>
<td>0</td>
<td>0</td>
<td>12</td>
<td>9</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>13</td>
<td>7</td>
<td>1.187085</td>
<td>1.560027</td>
<td>1.486371</td>
</tr>
<tr>
<td>Maleki method</td>
<td>0</td>
<td>0</td>
<td>12</td>
<td>9</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>13</td>
<td>7</td>
<td>1.187085</td>
<td>1.560027</td>
<td>1.486371</td>
</tr>
</tbody>
</table>

The results obtained for problem 1 are minimum cost \( C = 1.187085 \), damage charges, \( D = 1.560027 \), and time = 1.486371, respectively. The results are same for both the ranking functions.

\[
\text{TABLE-2:}
\]

<table>
<thead>
<tr>
<th>Problem 2</th>
<th>( x_{11} )</th>
<th>( x_{12} )</th>
<th>( x_{13} )</th>
<th>( x_{21} )</th>
<th>( x_{22} )</th>
<th>( x_{23} )</th>
<th>( x_{31} )</th>
<th>( x_{32} )</th>
<th>( x_{33} )</th>
<th>Min. Cost</th>
<th>Min. Damage</th>
<th>Min. Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yager's method</td>
<td>0</td>
<td>2</td>
<td>14</td>
<td>12</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0.927899</td>
<td>1.454420</td>
<td>1.602083</td>
</tr>
<tr>
<td>Maleki method</td>
<td>0</td>
<td>2</td>
<td>14</td>
<td>12</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0.927899</td>
<td>1.454420</td>
<td>1.602083</td>
</tr>
</tbody>
</table>

The results obtained for problem 2 are minimum cost \( C = 0.927899 \), damage charges, \( D = 1.454420 \), and time = 1.602083, respectively. The results are same for both the ranking functions.

6. CONCLUSION AND DISCUSSIONS

In the present work, a fuzzy programming approach is used to find a compromise solution for the fully fuzzy multi-objective fractional transportation problem. The proposed approach has the following key features:

1. It provides the decision maker with a simple and easy approach to solve fully fuzzy MOFTP.
2. Two cases are discussed one with triangular fuzzy numbers and other with trapezoidal fuzzy numbers.
3. Two ranking methods i.e. Yagers and Maleki ranking methods are used to convert the fuzzy problem into crisp problem to check the efficiency of the two ranking methods from the obtained results.
4. The remarkable conclusion obtained from solution table 1 and 2 is that the results provided by Yagers ranking method and Maleki ranking methods are same. So, any of the two methods can be used for solving the problem. However, it is advisable to use Maleki method to avoid mathematical errors as it is comparatively easier.
5. The provided approach can be very useful while dealing with fully fuzzy MOFTP and similar real life problems involving uncertainty of the coefficients of the parameters or demands and supplies.
6. The approach can also be used for solving other fractional programming problems.
7. REFERENCES

15. Porchelvi, R. S., & Sheela, A. A Linear Fractional Interval Transportation Problem with and Without Budgetary Constraints.

Source of support: Nil, Conflict of interest: None Declared.

[Copy right © 2017. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]