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FUZZY SUB NEAR-FIELD SPACES OF A Γ-NEAR-FIELD SPACE OVER NEAR-FIELD (FSNF-Γ-NF-O NF)

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ABSTRACT

The object of this paper is to introduce notion of fuzzy left (respectively right) sub near-field spaces of a Γ-near-field space over a near-field and to study the related properties of fuzzy sub near-field spaces in a Γ-near-field space over a near-field.

Keywords: *Γ-near-field space; Fuzzy sub near-field space of Γ-near-field space; Fuzzy near-field space ofΓ-near-field space.*

2000 Mathematics Subject Classification: *43A10, 46B28, 46H25,6H99, 46L10, 46M20, 51 M 10, 51 F 15,03 B 30.*

SECTION-1: INTRODUCTION

In this paper we consider the fuzzification of left (resp. right) sub near-field spaces of Γ-near-filed spaces over a near-field, and we three Smt. Thurumella Madhavi Latha, Dr. T V Pradeep Kumar and Dr. N V Nagendram together investigate the related properties of left (resp. right) sub near-field spaces of Γ-near-filed spaces over a near-field.

In fact, Γ-near-rings were defined by Bhavanari Satyanarayana, who is the professor cum Head of the department of Mathematics, Acharya Nagarjuna University at present and also guide of Dr T V Pradeep Kumar and the ideal theory in Γ-near-rings was studied by Dr. Bhavanari Satyanarayana and G. L. Booth. Fuzzy ideals of rings were introduced by W. Liu, and it has been studied by several authors. The notion of fuzzy ideals and its properties were applied to various areas: semi-groups, BCK-algebras, and semi-rings.

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SECTION-2: PRELIMINARIES

A near-field space is a triple (N, +,.) such that (N , +) is a group, if (N, .) is a semi group, and "." Is left distributive over "+" i.e. w(x + z) = wx + wz for each w, x, y, z $\in$ N. A near-field space N is d.g. if there exists T $\subset$ N such that (T, .) is a semi sub near-field space of (N, .) each element of T is right distributive and T is an additive generating space for (N, +). The near-field space generated additively by all the endomorphism of a (not necessarily commutative) sub near-field space (G, +) is d.g. T being the space of endomorphisms. Such a near-field space will be called an endomorphism near-field space and will be denoted by E(G).

Dr N V Nagendram has shown that the near-field space generated by all the inner automorphisms of a finite simple, non-commutative, sub near-field space (G, +) is E(G). In fact, this near-field space generated by the inner automorphisms consists of all the mappings of G into G which leave 0 fixed and also has given a necessary and sufficient condition that the near-field space generated by the inner automorphisms of a sub near-field space of a near-field space be a near-field space. However, the more general endomorphism near-field space has not been studied.

If α is an endomorphism of (G, +) and g $\in$ G, the image of g under α is denoted by gα. Addition of functions on G is done point-wise and multiplication of such functions is composition.

Definition 2.1: A sub near-field space H of the near-field space N is a N-sub near-field space over a near-field if HN $\subset$ H. The radical sub near-field space J(N) is the intersection of the right sub near-field spaces of N which are maximal N-sub near-field spaces over a near-field.

Definition 2.2: closed (or open) sub near-field space. More generally, for positive integers m, n we define M to be an (m, n)-closed (or open) sub near-field space of N if $x^m \in M$ for $x \in N \Rightarrow x^n \in M$.

Definition 2.3: radical sub near-field space. M is a radical sub near-field space if and only if M is a (2, 1)-closed (or open) sub near-field space. In fact, an n-absorbing sub near-field space is (m, n)-closed (or open) sub near-field space for every positive integer m.

Note 2.4: clearly, a proper radical sub near-field space of N is (m, n)-closed (or open) radical sub near-field space for $1 \le m \le n$. So we often assume that $1 \le n \le m$.

Definition 2.5: A sub near-field space *A* of a Γ-near-filed spaces over a near-field *M* is called a *left (*resp. *right) sub near-field space* of *M* if (i) (*A;* +) is a normal divisor of (*M;* +), (ii) $u\alpha(x + v) - u\alpha v \in A$ (resp. $x\alpha u \in A$) for all $x \in A$, $\alpha \in \Gamma$ and $u; v \in M$.

We now review some fuzzy logic concepts.

Definition 2.6: A fuzzy sub near-field space in a Γ-near-filed space *M* is a function $\mu : M \to [0; 1]$.

Definition 2.7: We shall use the notation μ_t, called a level sub near-field space of μ, for $\{x\in M / \mu(x) \ge t\}$ where $t \in [0; 1]$. If μ_t is a fuzzy sub near-field space in a Γ-near-filed spaces over a near-field *M* and *f* is a function defined on *M*, then the fuzzy sub near-field space v in *f*(*M*) defined by $v(y) = \sup [\mu(x)]\ \forall x \in f^{-1}(y)$ and $\forall\ y \in f(M)$ is called the *image* of μ under *f* .

Definition 2.8: if *v* is a fuzzy sub near-field space in *f*(*M*), then the fuzzy sub near-field space $\mu = v \circ f$ in a Γ-near-filed space *over a near-field M* (that is, the fuzzy sub near-field space defined by $\mu(x) = v(f(x))$ for all $x \in M$) is called the preimage of *v* under *f* . We say that a fuzzy sub near-field space μ in a Γ-near-filed space *over a near-field M* has the *sup property* if, for any sub near-field space *T* of a Γ-near-filed space over a near-field *M*, there exists $t_0 \in T$ such that $\mu(t_0) = \sup\ [\mu(t)]$ for all $t \in T$.

SECTION-3: FUZZY SUB NEAR-FIELD SPACES OF Γ-NEAR-FILED SPACE OVER A NEAR-FIELD

Definition 3.1 A Fuzzy sub near-field space μ in a Γ-near-filed space over a near-field M is called a fuzzy left (resp. right) sub near-field space of a Γ-near-filed space over near-field M if (i) μ is a fuzzy normal divisor with respect to the addition, (ii) $\mu(u\alpha(x + v) - u\alpha v) \ge \mu(x)$ (resp. $\mu(x \circ u) \ge \mu(x)$) for all x; u; v $\in$ M and $\alpha \in \Gamma$. The condition (i) of Definition 3.1 means that μ satisfies: (i) $\mu(x - y) \ge \min\{\mu(x); \mu(y)\}$, and (ii) $\mu(y + x - y) \ge \mu(x)$, $\forall$ x; y $\in$ M.

Note 3.2: If μ is a fuzzy left (resp. right) sub near-field space of a Γ-near-filed space over a near-field *M*, then $\mu(0) \ge \mu(x)$ for all $x \in M$, where 0 is the zero sub near-field space of *M*.

Theorem 3.3: Let M be a fuzzy left (resp. right) sub near-field space of a Γ-near-filed space over a near-field M. Then the set $M_\mu = \{x \in M / \mu(x) = \mu(0)\}$ is a fuzzy left (resp. right) sub near-field space of a Γ-near-filed space over a near-field M.

Proof: Let μ be a fuzzy left ideal and let x; y ∈ M_μ. Then $\mu(x - y) \geq \min\{\mu(x); \mu(y)\} = \mu(0)$; and so $\mu(x - y) = \mu(0)$ or $x - y \in M_\mu$.

For every $y \in M$ and $x \in M_\mu$, we have $\mu(y + x - y) \geq \mu(x) = \mu(0)$. Therefore, we have, $y + x - y \in M_\mu$, which shows that M_μ is a fuzzy normal divisor left (resp. right) sub near-field space of a Γ-near-filed space *M* with respect to the addition. Let $x \in M_\mu$, $\alpha \in \Gamma$ and *u; v* ∈ *M*.

Then $\mu(u\alpha(x + v) - u\alpha v) \geq \mu(x) = \mu(0)$; and hence $\mu(u\alpha(x + v) - u\alpha v) = \mu(0)$; i.e., $u\alpha(x + v) - u\alpha v \in M_\mu$. Therefore M_μ is a fuzzy left (resp. right) sub near-field space of a Γ-near-filed space over a near-field *M*. Similarly we have the desired result for the right case. This completes the proof of the theorem.

Theorem 3.4: Let A be a non-empty fuzzy sub near-field space of a Γ-near-field space over an near-field M and μ_A be a fuzzy sub near-field space of a Γ-near-field space over an near-field in M defined by $\mu_A(x) = \begin{cases} s & if\ \ x \in A \\ t & Otherwise \end{cases}$, for all x ∈ M and s, t ∈ [0; 1] with s > t. Then μ_A is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over an near-field of M if and only A is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over an near-field of M. Moreover $M_{\mu A} = A$.

Proof: Let μ_A be a fuzzy sub near-field space of a Γ-near-field space over an near-field of M and let x, y ∈ A. Then $\mu_A(x - y) \geq \min\{\mu_A(x)/\ \mu_A(y)\} = s$; and so $\mu_A(x - y) = s$.

⇒ x − y 2 A. For any y ∈ M and x ∈ A, we have $\mu_A(y + x - y) \geq \mu_A(x) = s$ and so y + x − y ∈ A.

Now let x ∈ A, α ∈ Γ and u, v ∈ M.

Then $\mu_A(u\alpha(x + v) - u\alpha v) \geq \mu_A(x) = s$ (resp. $\mu_A(x\alpha u) \geq \mu_A(x) = s$), and therefore we have, $\mu_A(u\alpha(x + v) - u\alpha v) = s$ (resp. $\mu_A(x\alpha u) = s$).

Thus uα(x + v) − uαv ∈ A (resp. xαu ∈ A). This shows that A is a fuzzy sub near-field space of a Γ-near-field space over an near-field of M.

Conversely assume that A is a fuzzy sub near-field space of a Γ-near-field space over an near-field of M. Let x, y ∈ M. If at least one of x and y does not belong to A, then $\mu_A(x - y) \geq t = \min\{\mu_A(x) : \mu_A(y)\}$: If x, y ∈ A, then x − y ∈ A and so $\mu_A(x - y) = s = \min\{\mu_A(x) : \mu_A(y)\}$: If x ∈ A, then y + x − y ∈ A and hence $\mu_A(y + x - y) = s = \mu_A(x)$.

Clearly $\mu_A(y + x - y) \geq t = \mu_A(x)$ for all x ∉ A and y ∈ M. This shows that μ_A is a fuzzy normal divisor fuzzy sub near-field space of a Γ-near-field space over an near-field of M with respect to the addition.

Now let x, u, v ∈ M and α ∈ Γ. If x ∈ A, then uα(x + v) − uαv ∈ A (resp. xαu ∈ A) and thus $\mu_A(u\alpha(x + v) - u\alpha v) = s = \mu_A(x)$ (resp. $\mu_A(x\alpha u) = s = \mu_A(x)$). If x ∉ A, then clearly $\mu_A(u\alpha(x + v) - u\alpha v) \geq t = \mu_A(x)$ (resp. $\mu_A(x\alpha u) \geq t = \mu_A(x)$). Hence μ_A is a fuzzy sub near-field space of a Γ-near-field space over an near-field of M. Moreover $M_{\mu A} = \{x \in M / \mu_A(x) = \mu_A(0)\} = \{x \in M / \mu_A(x) = s\} = \{x \in M / x \in A\} = A$. This completed the proof of the theorem.

Corollary 3.5: Let M be a fuzzy sub near-field space of a Γ-near-field space over a near-field and χ_A be the characteristic function of a sub near-field space of a Γ-near-field space over an near-field A ⊂ M. Then χA is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over an near-field of M if and only if A is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over an near-field of M.

Theorem 3.6: Let μ be a fuzzy sub near-field space of a Γ-near-field space over a near-field in M. Then μ is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field of M if and only if each level subset μ_t, t ∈ Img(μ), of μ is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field of M. Here let us call μ_t a *level fuzzy left* (resp. *right*) sub near-field space of a Γ-near-field space over a near-field *μ*.

Proof: Let μ be a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field of M and let $t \in Im(\mu)$. For any x, y ∈ μ_t, we have $\mu(x - y) \geq \min\{\mu(x); \mu(y)\} \geq t$ and so $x - y \in \mu_t$. Let y ∈ M and x ∈ μ_t. Then $\mu(y + x - y) \geq \mu(x) \geq t$; whence $y + x - y \in \mu_t$. Now let x ∈ μ_t , μ ∈ Γ and u; v∈M. Then $\mu(u\alpha(x + v) - u\alpha v) \geq \mu(x) \geq t$ (resp. $\mu(x\alpha u) \geq \mu(x) \geq t$), which implies that $u\alpha(x + v) - u\alpha v \in \mu_t$ (resp. $x\alpha u \in \mu_t$).

Hence μ_t is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field of M. Conversely assume that μ_t is a fuzzy left (resp. right) a fuzzy sub near-field space of a Γ-near-field space over a near-field M for every t ∈ Img (μ). If $\mu(x_0 - y_0) < \min\{\mu(x_0), \mu(y_0)\}$ for some $x_0, y_0 \in M$, then by taking $t_0 = ½[(\mu(x_0 - y_0) + \min\{ \mu(x_0); \mu(y_0)\})]$ we have $\mu(x_0 - y_0) < t_0$, $\mu(x_0) > t_0$ and $\mu(y_0) > t_0$. Hence $x_0 - y_0 \notin \mu_{t0}$, $x_0 \in \mu_{t0}$ and $y_0 \in \mu_{t0}$. This is a contradiction, ⊗ and so $\mu(x - y) \geq \min \{ \mu(x), \mu(y)\}$ for all x, y ∈ M.

Assume that $\mu(y_0 + x_0 - y_0) < \mu(x_0)$ for some $x_0, y_0 \in M$. Putting $s_0 = 1/2 (\mu(y_0 + x_0 - y_0) + \mu(x_0))$; then $\mu(y_0 + x_0 - y_0) < s_0 < \mu(x_0)$: It follows that $x_0 \in \mu_{s0}$ and $y_0 + x_0 - y_0 \notin \mu_{s0}$ which is impossible. Hence, $\mu(y + x - y) \geq \mu(x)$ for all x, y∈M. If the condition (ii) of Definition 3.1 is not true, then for a fixed μ ∈ Γ there exist x, u, v ∈ M such that $\mu(u\alpha(x + v) - u\alpha v) < \mu(x)$ (resp. $\mu(x\alpha u) < \mu(x)$). Let $p_0 = 1/ 2 (\mu(u\alpha(x + v) - u\alpha v) + \mu(x))$ (resp. $q_0 = 1/2 (\mu(x\alpha u) + \mu(x))$. Then $u\alpha(x + v) - u\alpha v \notin \mu_{p0}$ and $x \in \mu_{p0}$ (resp. $x\alpha u \notin \mu_{q0}$ and $x \in \mu_{q0}$). This is a contradiction, and we are done. This completes the proof of the theorem.

Theorem 3.7: Let A be a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field M. Then for any t ∈ (0; 1] there exists a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field μ of M such that μ_t = A.

Proof: Let μ : M → [0; 1] be a fuzzy sub near-field space of a Γ-near-field space over a near-field defined by

$\mu(x) = \begin{cases} t & if \ \ x \in A \\ 0 & otherwise \end{cases}$ for all x ∈ M, where t ∈ (0; 1]. Then clearly μ_t = A.

It is easy to prove that $\mu(x - y) \geq \min \{\mu(x), \mu(y)\}$; x, y ∈ M:

Assume that $\mu(y + x - y) < \mu(x)$ for some x; y ∈ M. Since μ is two-valued, i.e., |Img(μ)| = 2, $\mu(y + x - y) = 0$ and μ(x) = t and hence $y + x - y \notin A$ and x ∈ A.

This contradicts the fact that (A, +) is a normal divisor sub near-field space of a Γ-near-field space over a near-field of (M, +).

Hence $\mu(y + x - y) < \mu(x)$ for all x, y ∈ M. Now assume that $\mu(u\alpha(x + v) - u\alpha v) < \mu(x)$ (resp. $\mu(x\alpha u) < \mu(x)$) for some x, u, v ∈ M and μ ∈ Γ.

Since |Img(μ)| = 2, we have $\mu(u\alpha(x + v) - u\alpha v) = 0$ and μ(x) = t (resp. $\mu(x\alpha u) = 0$ and μ(x) = t); whence $u\alpha(x+v)-u\alpha v \notin A$ and x ∈ A (resp. $x\alpha u \notin A$ and x ∈ A). This is impossible because A is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field of M, which proves the theorem.

Theorem 3.8: If μ is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field of M, then $\mu(x) = \sup\{t \in [0; 1]/x \in \mu_t\}$; ∀ x ∈ M:

Proof: Let $s := \sup\{ t \in [0; 1]/ x \in \mu_t \}$ and let ε > 0 be given. Then s − ε < t for some t ∈ [0; 1] such that x ∈ μ_t, and so s − ε < μ(x). Since ε is arbitrary, it follows that s ≤ μ(x). Now let μ(x) = u. Then x ∈ μ_u and so $u \in \{t \in [0; 1]/x \in \mu_t\}$: Hence $\mu(x) = u \leq \sup\{t \in [0; 1]/ x\in\mu_t\} = s$. Therefore μ(x) = s, as desired. 2 We now consider the converse of Theorem 3.8. Let A be a non-empty subset of [0; 1]. Without loss of generality, we may use A as an index set in the following:

Theorem 3.9: Let $\{A_t /t \in A\}$ be a collection of fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field M such that

(i) $M = \bigcup_{t \in A} A_t$, (ii) s > t if and only if $A_s \subset A_t$ for all s; t ∈ A. Define a fuzzy set μ in M by $\mu(x) = \sup\{t \in A/x \in A_t\}$; ∀ x ∈ M: Then μ is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field of M.

Proof: By theorem 3.6, It is sufficient to show that μ_p (≠ 0) is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field of M for every p ∈ [0; 1] . We consider the following two cases:

(1) $p = \sup\{t \in \mu/ t < p\}$ and (2) $p \neq \sup\{t \in \mu/ t < p\}$:

Case (1) implies that $x \in \mu p$, $x \in A_t$ for all $t < p$, $x \in \cap A_t$ for all $t<p$, whence $\mu_p = \cap A_t$ for all $t<p$ which is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field of M. For the case (2), there exists $\varepsilon > 0$ such that $(p-\varepsilon; p)\cap A = \phi$. We claim that $\mu_p = \bigcup_{t \geq p} A_t$.

If $x \in \bigcup_{t \geq p} A_t$, then $x \in A_t$ for some $t \geq p$. It follows that $\mu(x) \leq p - \varepsilon$ so that $x \notin \mu_p$ Conversely if $x \notin \mu_p$, $\mu_p = \bigcup_{t \geq p} A_t$ then $x \notin A_t$ for all $t \geq p$, which implies that $x \notin A_t$ for all $t > p - \varepsilon$, that is, if $x \in A_t$ then $t \leq p - \varepsilon$. Thus $\mu(x) \leq p - \varepsilon$ and so $x \notin \mu_p$. Consequently $\mu_p = \mu_p = \bigcup_{t \geq p} A_t$ At which is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field of M. This completes the proof.

Definition 3.10: Let M and N be Γ-near-field spaces over near-field. A mapping $\theta: M\to N$ is called a Γ-near-field space over a near-field homomorphism if $\theta(x + y) = \theta(x) + \theta(y)$ and $\theta(x\alpha y) = \theta(x)\alpha\theta(y)$ $\forall$ $x, y \in M$ and $\alpha\in\Gamma$.

Theorem 3.11: A fuzzy Γ-near-field space over near-field homomorphic preimage of a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field.

Proof: Let $\theta : M \to N$ be a fuzzy Γ-near-field space over near-field homomorphism, v a fuzzy left (resp. right) fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field of N and μ the preimage of v under θ.

Then $\mu(x - y) = v(\theta(x - y)) = v(\theta(x) - \theta(y)) \geq \min\{v(\theta(x)); v(\theta(y)) = \min\{\mu(x); \mu(y);$
$\mu(y + x - y) = v(\theta(y + x - y)) = v(\theta(y) + \theta(x) - \theta(y)) \geq v(\theta(x)) = \mu(x)$; and
$\mu(u\alpha(x + v) - u\alpha v) = v(\theta(u\alpha(x + v) - u\alpha v)) = v(\theta(u)\alpha(\theta(x) + \theta(v)) - \theta(u)\alpha\theta(v)) \geq v(\theta(x)) = \mu(x)$ (resp. $\mu(x\alpha u) = v(\theta(x\alpha u)) = v(\theta(x)\alpha\theta(u)) \geq v(\theta(x)) = \mu(x)$) $\forall$ $x, y, u, v \in M$ and $\mu \in \Gamma$. Hence μ is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field of M.

Let $\theta : M \to N$ be a fuzzy Γ-near-field space over near-field homomorphism. Assume that μ is a fuzzy left sub near-field space of a Γ-near-field space over a near-field of M with the sup property and let v be the image of μ under θ. Given $\theta(x); \theta(y) \in \theta(M)$, let $x_0 \in \theta^{-1}(\theta(x))$, $y_0 \in \theta^{-1}(\theta(y))$, $u_0 \in \theta^{-1}(\theta(u))$ and $v_0 \in \theta^{-1}(\theta(v))$ be such that $\mu(x_0) = \left\{\begin{matrix} Sup \ \ \mu(z) \\ z \in \theta^{-1}(\theta(x)) \end{matrix}, \mu(y_0) = \begin{matrix} Sup \ \ \mu(z) \\ z \in \theta^{-1}(\theta(y)) \end{matrix}\right\}$,
$\mu(u_0) = \begin{matrix} Sup \ \ \mu(z) \\ z \in \theta^{-1}(\theta(u)) \end{matrix}$ and $\mu(v_0) = \begin{matrix} Sup \ \ \mu(z) \\ z \in \theta^{-1}(\theta(v)) \end{matrix}$ respectively.

Then

$$v(\theta(x) - \theta(y)) = \begin{matrix} Sup \ \ \mu(z) \\ z \in \theta^{-1}(\theta(x) - \theta(y)) \end{matrix} \geq \mu(x_0) - \mu(y_0) \geq \min\{\mu(x_0), \mu(y_0)\}$$

$$= \min \left\{\begin{matrix} Sup \ \ \mu(z) \\ z \in \theta^{-1}(\theta(x)) \end{matrix}, \begin{matrix} Sup \ \ \mu(z) \\ z \in \theta^{-1}(\theta(y)) \end{matrix}\right\}$$

$$= \min \{v(\theta(x)); v(\theta(y));$$

$$v(\theta(y) + \theta(x) - \theta(y)) = \begin{matrix} Sup \ \ \mu(z) \\ z \in \theta^{-1}(\theta(y) + \theta(x) - \theta(y)) \end{matrix} \geq \mu(y_0 + x_0 - y_0) \geq \mu(x_0) = \begin{matrix} Sup \ \ \mu(z) \\ z \in \theta^{-1}(\theta(x)) \end{matrix} = v(\theta(x));$$

and for any $\alpha \in \Gamma$,

$$v(\theta(u)\alpha(\theta(x) + \theta(v)) - \theta(u)\alpha\theta(v)) = \begin{matrix} Sup \ \ \mu(z) \\ z \in \theta^{-1}(\theta(u)\alpha(x) + \theta(v) - \theta(u)) \end{matrix}$$

$$\geq \mu(u_0\alpha(x_0 + v_0) - u_0\alpha v_0) \geq \mu(x_0) = \left\{\begin{matrix} Sup \ \ \mu(z) \\ z \in \theta^{-1}(\theta(x)) \end{matrix}\right. = v(\theta(x)):$$

This proves that v is a fuzzy left sub near-field space of a Γ-near-field space over a near-field of N. Similarly if μ is a fuzzy right sub near-field space of a Γ-near-field space over a near-field of M with the sup property, then the image v of μ under θ is a fuzzy right sub near-field space of a Γ-near-field space over a near-field of N. This completes the proof of the theorem.

Theorem 3.12: A fuzzy Γ-near-field space over near-field homomorphic image of a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field of M possessing the sup property is a fuzzy left (resp. right) sub near-field space of a Γ-near-field space over a near-field.

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