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ANALYSIS OF A SINGLE SERVER QUEUE WITH FUZZY SERVICE RATE AND PRIORITY DISCIPLINE

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ABSTRACT

This paper gives a new approach for a queuing system in a fuzzy environment. A model with poisson interarrival time and fuzzy deterministic service rate is studied. The optimum selection for priority discipline (preemptive and non-preemptive) is done. The DSW algorithm is used to describe the membership functions of the performance measures of queueing system with priority.The numerical example is also done.

Keywords: *Fuzzy set theory, Priority discipline, DSW algorithm, Triangular fuzzy number.*

I. INTRODUCTION

In the recent decades many articles are published on queuing theory. The related studies are based on queuing theory in which the inter arrival times and service times are assumed to follow probability distributions. But in practice there are cases where parameters are not probabilistic and may be deterministic. Hence fuzzy queues are realistic than the commonly used queues.

If the usual priority queues are extended to the priority queues with fuzzy, the queuing models would have broader applications. When the arrival rate or the service rate is fuzzy the system performance measures of the priority queue will also be fuzzy. In priority scheme customers with the higher priority are served first then the low priority customers are served. The preemptive and non preemptive are two categories in priority discipline. In preemptive case the customer with high priority is permitted to enter service immediately even if low priority customer is already in service.

In the case of non preemptive the service of the lower priority customer is not broken up when a high priority customer arrives the system. After the service of the low priority customers is completed, the server begins servicing the high priority customers. Queuing system in a fuzzy situation is considered to develop a better study. Numerous authors have studied on fuzzy queuing system and few on priority fuzzy queues.

The fuzzy queuing model is studied by the authorities like Li and Lee [15], Buckley [2, 3], Negi and Lee [16], Chen [4, 5, 6] and Ke and lin [13]. The queue models with priority discipline are described by using fuzzy set theory in this paper and we optimise a priority queue with Poisson inter arrival time and the service rate is taken to be fuzzy.

The basic idea of this paper is cost analysis of preemptive, non-preemptive and no priority queues and to choose the best one which gives the minimum average total cost.

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II. PRIORITY DISCIPLINE FUZZY QUEUES

We consider a priority queuing system with single server, infinite calling population in which the arrival rate λ is assumed to be Poisson and the average fuzzy service time $\tilde{b}$ is just approximately known and are represented by a possibility distribution $\pi(b_i) = \mu_{\tilde{b}_i}(\tilde{b}_i)$. The objective of studying this model is to reduce the waiting time of the customers and also the cost of the system. To determine the priority discipline fuzzy queuing model we compare the average total cost of the system for the three cases. No priority, preemptive priority and non-preemptive priority which are denoted by c, c^1, c^2 respectively.

III. CRISP RESULTS

(a) No priority queuing model:
Average total cost of the system when there is no priority discipline

$$C = (C_1\lambda_1 + C_2\lambda_2)W$$

Where $W = \frac{\lambda b^2}{2(1-\rho)}$ with $b = \alpha_1 b_1 + \alpha_2 b_2, \rho = \lambda b;$

(b) Preemptive priority queuing model:
Average total cost of the system when there is preemptive priority

$$C^1 = C_1\lambda_1 W_{q,1} + C_2\lambda_2 W_{q,2}$$

With $W_{q,1} = \frac{\lambda_1 {b_1}^2 + \lambda_2 {b_2}^2}{2(1-\sigma_1)(1-\sigma_2)}, W_{q,2} = \frac{\lambda_1 {b_1}^2 + \lambda_2 {b_2}^2}{2(1-\sigma_2)}, \sigma_1 = \lambda_1 b_1 + \lambda_2 b_2 = \lambda b \; and \; \sigma_2 = \lambda_2 b_2$

(c) Non-preemptive priority queuing model:
Average total cost of the system when there is non preemptive priority

$$C^2 = C_1\lambda_1 W_{q,1} + C_2\lambda_2 W_{q,2}$$

With $W_{q,1} = \frac{b_1(1-\sigma_1) + \frac{\lambda_1 {b_1}^2 + \lambda_2 \, {b_2}^2}{2}}{(1-\sigma_1)(1-\sigma_2)} - b_1$, $W_{q,2} = \frac{b_2(1-\sigma_2) + \frac{\lambda_2 {b_2}^2}{2}}{(1-\sigma_2)} - b_2$, $\sigma_1 \; and \; \sigma_2$ defined as in previous model.

IV. DEFINITIONS

A. Fuzzy set

A fuzzy set is characterized by a membership function mapping elements of a domain space, or universe of discourse X to the unit interval [0, 1]. (i.e.) A = {(x,$\mu_A(x)$); x Є X}. Here μ_A: X → [0, 1] is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of x Є X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

B. α-cut of a fuzzy number

The α-cut of a fuzzy number A(x) is defined as $A(\alpha) = \{x: \mu(x) \geq \alpha, \alpha \in [0,1]\}$ Addition of two Triangular fuzzy numbers can be performed as

$$(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$

C. Triangular fuzzy number

For a Triangular number A(x), it can be represented by A(a, b, c,; 1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x = b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & otherwise \end{cases}$$

D. Interval Analysis Arithmetic

Let I_1 and I_2 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

$$I_1 = [a, b], a \leq b; I_2 = [c, d], c \leq d.$$

Define a general arithmetic property with the symbol *, where * = [+, -, ×, ÷] symbolically the operation.

$$I_1 * I_2 = [a, b] * [c, d]$$

represents another interval. The interval calculation depends on the magnitudes and signs of the element a, b, c, d.

$$[a, b] + [c, d] = [a + c, b + d]$$
$$[a, b] - [c, d] = [a - d, b - c]$$
$$[a, b] \bullet [c, d] = [min(ac, ad, bc, bd), max(ac, ad, bc, bd)]$$

$$[a,b] \div [c,d] = [a,b] \bullet \left[\frac{1}{d}, \frac{1}{c}\right] \; provided\ that\ \ [c,d]\ \neq 0$$
$$\alpha[a,b] = \begin{cases} [\alpha\, a, \alpha\, b]\ for\ \alpha > 0 \\ [\alpha\, b, \alpha\, a]\ for\ \alpha < 0 \end{cases}$$
$$where\ ac, bc, bd, are\ arithmetic\ products\ and\ \frac{1}{d}, \frac{1}{c}\ are\ quotients.$$

E. DSW Algorithm

Any continuous membership function can be represented by a continuous sweep of $\propto cut$ in term from $\alpha = 0\ to$ $\alpha = 1$. Suppose we have single input mapping given by $y = f(x)$ that is to be extended for fuzzy sets $\tilde{B} = f(\tilde{A})$ and we want to decompose $\tilde{A}$ in to the series of $\alpha\ cut$ intervals say $I_{\alpha.}$ It uses the full $\alpha - cut$ intervals in a standard interval analysis. The DSW algorithm consists of the following steps:

1. Select a $\alpha\ cut\ value\ where\ 0 \leq \alpha \leq 1$.
2. Find the intervals in the input membership functions that correspond to this α.
3. Using standard binary interval operations, compute the interval for the output membership function for the selected $\alpha\ cut$ level.
4. Repeat steps 1 -3 for different values of α to complete a $\alpha\ cut$ representation of the solution.

V. MATHEMATICAL FORMULATION

Decisions concerning the optimum selection of a priority discipline are based on the cost function

$$C = \sum_{i=1}^{n} C_i L_i$$

where C_i is the unit cost of system for units in class i and L_i is the average length in the system for units of class i and C is the total average cost of the system. We study a queuing model with two unit classes where α_1 of the units arriving belongs to one class and α_2 are in other class. Let the arrival rate is Poisson and is taken as the triangular fuzzy number $\tilde{\lambda}$, the possibility distributions of service time for both classes are given as the triangular fuzzy number $\tilde{b}_A$ and $\tilde{b}_B$ respectively.The possibility distribution of unit cost of system for units of the same class is taken as the triangular fuzzy number $\tilde{C}_A$ and $\tilde{C}_B$ and the membership function is as follows.

$$\mu_{\tilde{\lambda}} = \begin{cases} \dfrac{\lambda - a}{b - a} & a \leq \lambda \leq b \\ \dfrac{c - \lambda}{c - b} & b \leq \lambda \leq c \\ 0 & elsewhere \end{cases}$$

$$\mu_{\tilde{b}_A} = \begin{cases} \dfrac{b_A - a_1}{b_1 - a_1} & a_1 \leq b_A \leq b_1 \\ \dfrac{c_1 - b_A}{c_1 - b_1} & b_1 \leq b_A \leq c_1 \\ 0 & elsewhere \end{cases}$$

$$\mu_{\tilde{b}_B} = \begin{cases} \dfrac{b_B - a_2}{b_2 - a_2} & a_2 \leq b_B \leq b_2 \\ \dfrac{c_2 - b_B}{c_2 - b_2} & b_2 \leq b_B \leq c_2 \\ 0 & elsewhere \end{cases}$$

$$\mu_{\tilde{C}_A} = \begin{cases} \dfrac{C_A - a_3}{b_3 - a_3} & a_3 \leq C_A \leq b_3 \\ \dfrac{c_3 - C_A}{c_3 - b_3} & b_3 \leq C_A \leq c_3 \\ 0 & elsewhere \end{cases}$$

$$\mu_{\tilde{C}_B} = \begin{cases} \dfrac{C_B - a_4}{b_4 - a_4} & a_4 \leq C_B \leq b_4 \\ \dfrac{c_4 - C_B}{c_4 - b_4} & b_4 \leq C_B \leq c_4 \\ 0 & elsewhere \end{cases}$$

We choose three values of α viz, 0, 0.5,1. When $\alpha = 0$, we obtain 5 intervals as follows.

$$\tilde{\lambda}_0 = [a,c]; \tilde{b}_{A,0} = [a_1, c_1]; \tilde{b}_{B,0} = [a_2, c_2]; \tilde{C}_{A,0} = [a_3, c_3]; \tilde{C}_{B,0} = [a_4, c_4]$$

Similarly when $\alpha = 0.5,1$, we obtain 10 intervals.

(a) Average total cost of inactivity when there is no priority discipline

$$\tilde{C}_0 = [\tilde{c}_{A,0}\tilde{\lambda}_{1,0} + \tilde{c}_{B,0}\tilde{\lambda}_{2,0}]\left[\frac{\tilde{\lambda}_0(\alpha_1\tilde{b}_{A,0} + \alpha_2\tilde{b}_{B,0})^2}{2[1-(\tilde{\lambda}_0\alpha_1\tilde{b}_{A,0} + \tilde{\lambda}_0\alpha_2\tilde{b}_{B,0})]}\right]$$

$$\tilde{C}_{0.5} = [\tilde{c}_{A,0.5}\tilde{\lambda}_{1,0.5} + \tilde{c}_{B,0.5}\tilde{\lambda}_{2,0.5}]\left[\frac{\tilde{\lambda}_{0.5}(\alpha_1\tilde{b}_{A,0.5} + \alpha_2\tilde{b}_{B,0.5})^2}{2[1-(\tilde{\lambda}_{0.5}\alpha_1\tilde{b}_{A,0.5} + \tilde{\lambda}_{0.5}\alpha_2\tilde{b}_{B,0.5})]}\right]$$

$$\tilde{C}_1 = [\tilde{c}_{A,1}\tilde{\lambda}_{1,1} + \tilde{c}_{B,1}\tilde{\lambda}_{2,1}]\left[\frac{\tilde{\lambda}_1(\alpha_1\tilde{b}_{A,1} + \alpha_2\tilde{b}_{B,1})^2}{2[1-(\tilde{\lambda}_1\alpha_1\tilde{b}_{A,1} + \tilde{\lambda}_1\alpha_2\tilde{b}_{B,1})]}\right]$$

(b) Average total cost of inactivity when there is a preemptive discipline

$$\tilde{C}_0{}^1 = \tilde{c}_{A,0}\alpha_1\tilde{\lambda}_0\left[\frac{\alpha_1\tilde{\lambda}_0\tilde{b}_{A,0}{}^2 + \alpha_2\tilde{\lambda}_0\tilde{b}_{B,0}{}^2}{2(1-\tilde{\sigma}_{1,0})(1-\tilde{\sigma}_{2,0})}\right] + \tilde{c}_{B,0}\alpha_2\tilde{\lambda}_0\left[\frac{\alpha_1\tilde{\lambda}_0\tilde{b}_{A,0}{}^2 + \alpha_2\tilde{\lambda}_0\tilde{b}_{B,0}{}^2}{2(1-\tilde{\sigma}_{2,0})}\right]$$

$$\tilde{C}_{0.5}{}^1 = \tilde{c}_{A,0.5}\alpha_1\tilde{\lambda}_{0.5}\left[\frac{\alpha_1\tilde{\lambda}_{0.5}\tilde{b}_{A,0.5}{}^2 + \alpha_2\tilde{\lambda}_{0.5}\tilde{b}_{B,0.5}{}^2}{2(1-\tilde{\sigma}_{1,0.5})(1-\tilde{\sigma}_{2,0.5})}\right] + \tilde{c}_{B,0.5}\alpha_2\tilde{\lambda}_{0.5}\left[\frac{\alpha_1\tilde{\lambda}_{0.5}\tilde{b}_{A,0.5}{}^2 + \alpha_2\tilde{\lambda}_{0.5}\tilde{b}_{B,0.5}{}^2}{2(1-\tilde{\sigma}_{2,0.5})}\right]$$

$$\tilde{C}_1{}^1 = \tilde{c}_{A,1}\alpha_1\tilde{\lambda}_1\left[\frac{\alpha_1\tilde{\lambda}_1\tilde{b}_{A,1}{}^2 + \alpha_2\tilde{\lambda}_1\tilde{b}_{B,1}{}^2}{2(1-\tilde{\sigma}_{1,1})(1-\tilde{\sigma}_{2,1})}\right] + \tilde{c}_{B,1}\alpha_2\tilde{\lambda}_1\left[\frac{\alpha_1\tilde{\lambda}_1\tilde{b}_{A,1}{}^2 + \alpha_2\tilde{\lambda}_1\tilde{b}_{B,1}{}^2}{2(1-\tilde{\sigma}_{2,1})}\right]$$

(c) Average total cost of inactivity when there is a non-preemptive discipline.

$$\tilde{C}_0{}^2 = \tilde{c}_{A,0}\alpha_1\tilde{\lambda}_0\left[\frac{\tilde{b}_{A,0}(1-\tilde{\sigma}_{1,0}) + \frac{\alpha_1\tilde{\lambda}_0\tilde{b}_{A,0}{}^2 + \alpha_2\tilde{\lambda}_0\tilde{b}_{B,0}{}^2}{2}}{(1-\tilde{\sigma}_{1,0})(1-\tilde{\sigma}_{2,0})} - \tilde{b}_{A,0}\right] + \tilde{c}_{B,0}\alpha_2\tilde{\lambda}_0\left[\frac{\tilde{b}_{B,0}(1-\tilde{\sigma}_{2,0}) + \frac{\alpha_2\tilde{\lambda}_0\tilde{b}_{B,0}{}^2}{2}}{(1-\tilde{\sigma}_{2,0})} - \tilde{b}_{B,0}\right]$$

$$\tilde{C}_{0.5}{}^2 = \tilde{c}_{A,0.5}\alpha_1\tilde{\lambda}_{0.5}\left[\frac{\tilde{b}_{A,0.5}(1-\tilde{\sigma}_{1,0.5}) + \frac{\alpha_1\tilde{\lambda}_{0.5}\tilde{b}_{A,0.5}{}^2 + \alpha_2\tilde{\lambda}_{0.5}\tilde{b}_{B,0.5}{}^2}{2}}{(1-\tilde{\sigma}_{1,0.5})(1-\tilde{\sigma}_{2,0.5})} - \tilde{b}_{A,0.5}\right]$$
$$+ \tilde{c}_{B,0.5}\alpha_2\tilde{\lambda}_{0.5}\left[\frac{\tilde{b}_{B,0.5}(1-\tilde{\sigma}_{2,0.5}) + \frac{\alpha_2\tilde{\lambda}_{0.5}\tilde{b}_{B,0.5}{}^2}{2}}{(1-\tilde{\sigma}_{2,0.5})} - \tilde{b}_{B,0.5}\right]$$

$$\tilde{C}_1{}^2 = \tilde{c}_{A,1}\alpha_1\tilde{\lambda}_1\left[\frac{\tilde{b}_{A,1}(1-\tilde{\sigma}_{1,1}) + \frac{\alpha_1\tilde{\lambda}_1\tilde{b}_{A,1}{}^2 + \alpha_2\tilde{\lambda}_1\tilde{b}_{B,1}{}^2}{2}}{(1-\tilde{\sigma}_{1,1})(1-\tilde{\sigma}_{2,1})} - \tilde{b}_{A,1}\right] + \tilde{c}_{B,1}\alpha_2\tilde{\lambda}_1\left[\frac{\tilde{b}_{B,1}(1-\tilde{\sigma}_{2,1}) + \frac{\alpha_2\tilde{\lambda}_1\tilde{b}_{B,1}{}^2}{2}}{(1-\tilde{\sigma}_{2,1})} - \tilde{b}_{B,1}\right]$$

VI. NUMERICAL EXAMPLE

Comparison of the three total costs shows which priority discipline is preferable. Consider a queuing model in which two unit classes arrive with utilization of 30% and 70%.The arrival rate are triangular fuzzy numbers represented by $\tilde{\lambda} = (0.05,0.06,0.07)$, the possibility distribution of service time are given by the triangular fuzzy numbers $\tilde{b}_A = (12,15,16)$ and $\tilde{b}_B = (9,10,12)$ and the possibility distribution of unit cost of inactivity for units of the two classes are triangular numbers $\tilde{C}_A = (10,16,18)$ and $\tilde{C}_B = (5,6,9)$ respectively.

When there is no priority discipline:

$$C_0 = (1.5768,65.7182), C_{0.5} = (3.2613,16.9065), C_1 = (6.9111,6.9111)$$

When there is Preemptive discipline:

$$\tilde{C}_0{}^1 = (1.7209,30.0275), \widetilde{C}_{0.5}{}^1 = (3.2333,21.1283), \tilde{C}_1{}^1 = (8.3995,8.3995)$$

When there is Non Preemptive discipline:

$$\tilde{C}_0{}^2 = (2.0924,189.3681), \tilde{C}_{0.5}{}^2 = (2.3127,39.2217), \tilde{C}_1{}^2 = (6.0749,6.0749)$$

Comparing the three total costs the average total cost function of no priority discipline is minimum. Even though overlapping of fuzzy numbers is found, the total cost is minimum for the fuzzy queuing models without priorities. Hence the optimum value is given by queuing models without priorities.

VII. CONCLUSION

Fuzzy queuing models are more realistic and practical than classical ones. In this paper, the fuzzy theory has been applied to optimize the priority queues with fuzzy arrival rate. Interval analysis arithmetic is used for computational efficiency. The method proposed here can be extended to different queuing models with possibilistic or deterministic arrival and service rate.

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