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On Pre-generalized c*-open sets and Pre-generalized c*-open maps in topological spaces

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ABSTRACT

The aim of this paper is to introduce the notion of pre-generalized c-open sets in topological spaces and study their basic properties. Further the notion of pre-generalized c*-open maps are introduced and their basic properties are discussed.*

Key words: *pgc*-open sets and pgc*-open maps.*

1. INTRODUCTION

In 1963, Norman Levine introduced semi-open sets in topological spaces. Also in 1970, he introduced the concept of generalized closed sets. Bhattacharya and Lahiri introduced and study semi-generalized closed (briefly, sg-closed) sets in 1987. Palaniappan and Rao introduced regular generalized closed (briefly, rg-closed) sets in 1993. In the year 1996, Andrijevic introduced and studied b-open sets. Gnanambal introduced generalized pre-regular closed (briefly gpr-closed) sets in 1997. In this paper we introduce pre-generalized c*-open sets and pre-generalized c*-open maps in topological spaces and study their basic properties.

Section 2 deals with the preliminary concepts. In section 3, pre-generalized c*-open sets are introduced and their basic properties are discussed. The pre-generalized c*-open maps in topological spaces are introduced in section 4.

2. PRELIMINARIES

Throughout this paper X denotes a topological space on which no separation axiom is assumed. For any subset A of X, cl(A) denotes the closure of A, int(A) denotes the interior of A, pcl(A) denotes the pre-closure of A and bcl(A) denotes the b-closure of A. Further X\A denotes the complement of A in X. The following definitions are very useful in the subsequent sections.

Definition 2.1: A subset A of a topological space X is called

i. a semi-open set [8] if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.
ii. a pre-open set [16] if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$.
iii. a regular-open set [18] if $A = int(cl(A))$ and a regular-closed set if $A = cl(int(A))$.
iv. A γ-open set [10] (b-open set[1]) if $A \subseteq cl(int(A)) \cup int(cl(A))$ and a γ-closed set (b-closed set) if $int(cl(A)) \cap cl(int(A)) \subseteq A$.
v. a π-open set [22] if A is the finite union of regular-open sets and the complement of π-open set is said to be π-closed.

Definition 2.2: [12] A subset A of a topological space X is said to be a c*-open set if $int(cl(A)) \subseteq A \subseteq cl(int(A))$.

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Definition 2.3: A subset A of a topological space X is called

i. a generalized closed set (briefly, g-closed) [9] if cl(A)⊆H whenever A⊆H and H is open in X.
ii. a regular-generalized closed set (briefly, rg-closed) [17] if cl(A)⊆H whenever A⊆H and H is regular-open in X.
iii. a generalized pre-regular closed set (briefly, gpr-closed) [6] if pcl(A)⊆H whenever A⊆H and H is regular-open in X.
iv. a regular generalized b-closed set (briefly, rgb-closed) [15] if bcl(A)⊆H whenever A⊆H and H is regular-open in X.
v. a regular weakly generalized closed set (briefly, rwg-closed) [20] if cl(int(A))⊆H whenever A⊆H and H is regular-open in X.
vi. a semi-generalized b-closed set (briefly, sgb-closed) [7] if bcl(A)⊆H whenever A⊆H and H is semi-open in X.
vii. a weakly closed (briefly, w-closed) set [19] (equivalently, ĝ-closed set [21]) if cl(A)⊆H whenever A⊆H and H is semi-open in X.
viii. a semi-generalized closed set (briefly, sg-closed) [3] if scl(A)⊆H whenever A⊆H and H is semi-open in X.
ix. a generalized semi-closed (briefly, gs-closed) set [2] if scl(A)⊆H whenever A⊆H and H is open in X.
x. a (gs)*-closed set [5] if cl(A)⊆H whenever A⊆H and H is gs-open in X.

The complements of the above mentioned closed sets are their respectively open sets.

Definition 2.4: [12] A subset A of a topological space X is said to be a generalized c*-closed set (briefly, gc*-closed set) if cl(A)⊆H whenever A⊆H and H is c*-open. The complement of the gc*-closed set is gc*-open [13].

Definition 2.5: A function f: X → Y is said to be

i. a g-open map [11] if f(U) is g-open in Y for every open set U of X.
ii. a semi-generalized open (briefly, sg-open) [4] map if f(U) is sg-open in Y for every open set U of X.
iii. a ĝ-open map [21] if f(U) is ĝ-open in Y for every open set U of X.
iv. gc*-open map [13] if f(U) is gc*-open in Y for every open set U of X.

Definition 2.6: [14] A subset A of a space X is said to be pre-generalized c*-closed (briefly, pgc*-closed) if pcl(A)⊆H whenever A⊆H and H is c*-open.

3. Pre-generalized c*-open sets

The complement of a pgc*-closed set need not be pgc*-closed. This leads to the definition of pgc*-open sets. In this section we introduce pre-generalized c*-open sets and study their basic properties.

Definition 3.1: A subset A of a space X is said to be pre-generalized c*-open (briefly, pgc*-open) if its complement is pgc*-closed.

Example 3.2: Let X={a, b, c} with topology τ={ϕ,{b},{c},{b, c}, X}. Then the pgc*-open sets are ϕ, {a}, {b}, {c}, {b, c}, X.

Proposition 3.3: Let X be a topological space. Then

1. Every w-open (resp. open, (gs)*-open, π-open, regular open, gc*-open) set is pgc*-open.
2. Every pgc*-open set is gpr-open (resp. rgb-open).

The converse of the Proposition 3.3 need not be true as seen from the following example.

Example 3.4:

1. Let X={a, b, c, d} with topology τ={ϕ,{a},{b},{a, b},{a, c},{a, b, c}, X}. Then the subset {a, b, d} is pgc*-open but not w-open (ĝ-open), (gs)*-open, open, π-open, regular-open and gc*-open.
2. Let X={a, b, c, d, e} with topology τ={ϕ,{a},{d},{e},{a, d},{a, e},{d, e},{a, d, e}, X}. Then the subset {a, c, d} is gpr-open and rgb-open but not pgc*-open.

Proposition 3.5: Let X be a topological space. Then for any element p ∈ X, the set {p} is either pgc*-open or c*-open.

Proof: Suppose {p} is not a c*-open set. Then X\ {p} is not a c*-open set. By Proposition 3.20 [14], X\ {p} is pgc*-closed. Hence {p} is a pgc*-open.

The intersection of two pgc*-open subsets of a space X need not be pgc*-open. For example, let X={a, b, c, d, e} with topology τ={ϕ,{a, b},{c, d},{a, b, c, d},X}. Then the subsets {b, c, d, e} and {a, c, d, e} are pgc*-open but their intersection {c, d, e} is not a pgc*-open set.

The union of two pgc*-open subsets of a space X need not be pgc*-open. For example, let X={a, b, c, d, e} with topology τ={ϕ,{a, b},{c, d},{a, b, c, d}, X}. Then the subsets {a, d} and {a, e} are pgc*-open but their union {a, d, e}is not a pgc*-open set.

Proposition 3.6: Let A be a subset of a space X. Then the following are equivalent.

i. A is pgc*-open.
ii. H⊆p-int(A) whenever H⊆A and H is c*-open.

Proof:
(i)⟹(ii): Assume that A is pgc*-open. Then X\A is pgc*-closed. Let H be a c*-open set and H⊆A. Then X\H is a c*-open set containing X\A. Since X\A is pgc*-closed, we have pcl(X\A)⊆X\H. This implies, X\(pcl(X\A))⊇H. That is, H⊆p-int(A).

(ii)⟹(i): Assume that H is a c*-open set containing X\A. Then X\H is a c*-open set and A⊇X\H. By hypothesis, X\H ⊆p-int(A). This implies, X\(p-int(A)) ⊆H. That is, pcl(X\A)⊆H. Therefore, X\A is pgc*-closed. Hence, A is pgc*-open.

Proposition 3.7: Let X be a topological space. If A is a pgc*-open subset of X such that p-int(A)⊆B⊆A, then B is pgc*-open.

Proof: Let A be a pgc*-open set and p-int(A)⊆B⊆A. Then X\A is a pgc*-closed set and X\A ⊆X\B⊆pcl(X\A). Therefore, by Proposition 3.22 [14], X\B is pgc*-closed. Therefore, B is pgc*-open.

Proposition 3.8: A subset A of X is pgc*-open if and only if for each H⊆A and H is c*-open, there exists a pre-open set G such that H⊆G⊆A.

Proof: Suppose that A is pgc*-open. Assume that H⊆A and H is c*-open. Then, by Proposition 3.6, H⊆p-int(A). If we put G = p-int(A), then H⊆G⊆A. Conversely, assume that H is a c*-open set contained in A. Then by hypothesis, there exists a pre-open set G such that H⊆G⊆A. Since p-int(A) is the largest pre-open set contained in A, we have G⊆p-int(A). Also, since H⊆G, we have H⊆p-int(A). Therefore, by Proposition 3.6, A is pgc*-open.

4. Pre-generalized c*-open maps

In this section, we introduce pre-generalized c*-open maps in topological spaces. Also, we derive some of their basic properties.

Definition 4.1: Let X and Y be two topological spaces. A function f : X → Y is said to be pre-generalized c*-open map (briefly, pgc*-open map) if f(U) is pgc*-open in Y for every open set U in X.

Example 4.2: Let X={a, b, c} with topology τ={ϕ, {a}, X} and Y={1, 2, 3} with topology σ={ϕ, {1},{1,2},Y}. Define f : X → Y by f(a)=2, f(b)=3, f(c)=1. Then f is pgc*-open.

Proposition 4.3: Let X, Y be two topological spaces. A function f: X → Y is a pgc*-open if and only if the image of each closed subset of X is pgc*-closed in Y.

Proof: Assume that f: X → Y is a pgc*-open map. Let V be a closed set in X. Then X\V is open in X. Therefore, by our assumption, f(X\V) is pgc*-open in Y. This implies, Y\f(V) is pgc*-open in Y. Hence, f(V) is pgc*-closed in Y. Conversely, assume that the image of each closed subset of X is pgc*-closed in Y. Let U be an open set in X. Then X\U is closed in X. Therefore, by our assumption, f(X\U) is pgc*-closed in Y. This implies, Y\f(U) is pgc*-closed in Y. This implies, f(U) is pgc*-open in Y. Therefore, f is a pgc*-open map.

Proposition 4.4: Let X, Y be two topological spaces. Then every open map is pgc*-open.

Proof: Let f : X → Y be an open map and U be an open set in X. Then f(U) is open in Y. By Proposition 3.3, f(U) is a pgc*-open set. Therefore, f is a pgc*-open map.

The following example shows that the converse of the Proposition 4.4 is not true.

Example 4.5: Let X={a, b, c} and Y={1, 2, 3}. Then, clearly τ={ϕ,{b},{c},{b, c}, X} is a topology on X and σ={ϕ,{1},{1,2},{1,3},Y} is a topology on Y. Define f : X→ Y by f(a)=2, f(b)=3, f(c)=1. Clearly, f is a pgc* -open map. But f is not an open map, since the image of an open set {b} under f is {3}, which is not open in Y.

Proposition 4.6: Let X, Y be two topological spaces. Then every ĝ-open map is pgc*-open.

Proof: Let f: X → Y be a ĝ-open map. Let U be an open set in X. Then f(U) is ĝ-open in Y. Therefore, by Proposition 3.3, f(U) is a pgc*-open set. Therefore, f is a gc*-open map.

The converse of the Proposition 4.6 need not be true, which can be verified from the following example.

Example 4.7: Let X={a, b, c} and Y={1, 2, 3}. Then, clearly τ={ϕ,{b},{c},{b, c},X} is a topology on X and σ={ϕ,{1},{1,2},{1,3},Y} is a topology on Y. Define f : X→ Y by f(a)=2, f(b)=3, f(c)=1. Then f is a pgc* -open map. Consider the open set {b} in X. Then f({b})={3}, which is not a ĝ-open set in Y. Therefore, f is not a ĝ-open map.

The g-open and pgc*-open maps are independent. For example, let X={a, b, c, d} and Y={1,2,3,4,5}. Then, clearly τ={ϕ,{a},{b},{a, b},{a, c},{a, b, c},X} is a topology on X and σ={ϕ,{1},{4},{5},{1,4},{1,5},{4,5},{1,4,5}, Y} is a topology on Y. Define f : X → Y by f(a)=1, f(b)=2, f(c)=f(d)=3. Then f is a g-open map. Consider the open set {a, c} in X. Then f({a, c})= {1,3}, which is not a pgc*-open set in Y. Hence f is not a pgc*-open map. Define g : X→ Y by g(a)=g(b)=2, g(c)=3, g(d)=5. Then g: X → Y is a pgc* -open map. Consider the open set {a, c} in X. Then g({a, c})={2,3}, which is not a g-open set in Y. Therefore, f is not a g-open map.

The sg-open and pgc*-open maps are independent. For example, let X={a, b, c, d} and Y={1,2,3,4,5}. Then, clearly τ={ϕ,{a},{b},{a, b},{a, c},{a, b, c}, X} is a topology on X and σ={ϕ,{1},{4},{5},{1,4},{1,5},{4,5},{1,4,5}, Y} is a topology on Y. Define f : X→ Y by f(a)=1, f(b)=4, f(c)=f(d)=3. Then f is a sg -open map. Consider the open set {a, c} in X. Then f({a, c})= {1,3}, which is not a pgc*-open set in Y. Hence f is not a pgc*-open map. Define g : X→ Y by g(a)=g(b)=2, g(c)=3, g(d)=5. Then g : X→ Y is a pgc* -open map. Consider the open set {b} in X. Then g({b})={2}, which is not a sg-open set in Y. Therefore, g is not a sg-open map.

Proposition 4.8: Let X, Y and Z be topological spaces. If f : X→ Y is an open map and g : Y→ Z is a gc* -open map, then g∘f is pgc*-open map.

Proof: Let U be an open set in X. Since f is an open map, f(U) is open in Y. Then g(f(U)) is a gc*-open set in Z. That is, (g∘f)(U) is a gc*-open set in Z. Therefore, by Proposition 3.3, (g∘f)(U) pgc*-open set in Z. Therefore, g∘f is a pgc*-open map.

Proposition 4.9: Let X, Y and Z be topological spaces. If f : X → Y and g : Y → Z are open maps, then g∘f : X → Z is a pgc*-open map.

Proof: Let U be an open set in X. Since f is an open map, f(U) is open in Y. Also, since g is an open map, g(f(U)) is open in Z. That is, (g∘f)(U) is a open set in Z. By Proposition 3.3, (g∘f)(U) is a pgc*-open set in Z. Therefore, g∘f is a pgc*-open map.

Proposition 4.10: Let X, Y and Z be topological spaces. If f : X→Y is an open map and g : Y→Z is a pgc*-open map, then g∘f is pgc*-open map.

Proof: Let U be an open set in X. Since f is an open map, f(U) is open in Y. Then g(f(U)) is a pgc*-open set in Z. That is, (g∘f)(U) is a pgc*-open set in Z. Therefore, g∘f is a pgc*-open map.

Proposition 4.11: Let X, Y and Z be topological spaces. If f: X→ Y is an open map and g: Y→ Z is a ĝ-open map, then g∘f : X → Z is a pgc*-open map.

Proof: Let U be an open set in X. Since f is an open map, f(U) is open in Y. Then g(f(U)) is a ĝ-open set in Z. That is, (g∘f)(U) is a ĝ-open set in Z. Therefore, by Proposition 3.3, (g∘f)(U) is a pgc*-open set in Z. Hence g∘f : X → Z is a pgc*-open map.

Proposition 4.12: Let X, Y be two topological spaces. A surjective function f : X→ Y is a pgc* -open map if and only if for each subset B of Y and for each closed set U containing $f^{-1}(B)$, there is a pgc*-closed set V of Y such that B⊂V and $f^{-1}(V)$⊂U.

Proof: Suppose f : X→ Y is a surjective pgc*-open map and B is a subset of Y. Let U be a closed set in X such that $f^{-1}(B)$⊂U. Then V=Y\f(X\U) is a pgc*-closed subset of Y containing B and $f^{-1}(V)$⊂U. Conversely, suppose F is an open subset of X. Then X\F is closed in X. Also, $f^{-1}(Y\setminus f(F))=X\setminus f^{-1}(f(F))\subseteq X\setminus F$. Therefore, by hypothesis, there exists a pgc*-closed set V of Y such that Y\f(F)⊂V and $f^{-1}(V)\subset X\setminus F$. This implies, $F\subset X\setminus f^{-1}(V)$.Therefore, $f(F)\subset f(X\setminus f^{-1}(V))$ ⊂ Y\V. Also, Y\V⊂f(F). This implies, f(F)=Y\V, which is pgc*-open in Y. Therefore, f is a pgc*-open map.

CONCLUSION

In this paper we have introduced pgc*-open sets and pgc*-open maps in topological spaces and studied some of their basic properties. Also, we have studied the relationship between pgc*-open sets with some generalized sets in topological spaces.

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