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$\lambda_{(m, n)}$-w-Closed sets in Bigeneralized Topological Spaces

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ABSTRACT

In this paper, we introduce the concept of $\lambda_{(m.n)}$ - w- closed sets in bigeneralized topological spaces and study some of their properties. We apply the notion of these sets to obtain a class of $wg_{(m, n)}$- continuous functions and investigate some of their characterizations.

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***Keywords**: bigeneralized topological spaces, $\lambda_{(m.n)}$-w-closed sets, $wg_{(m,n)}$- continuous functions.*

1.Iintroduction

Á. Császár [3] initiated the concepts of generalized neighborhood systems and he further developed the concepts of continuous functions and associated interior and closure operators on generalized neighborhood systems and on generalized topological spaces. In particular, he investigated characterizations for generalized continuous functions (= (g,g')- continuous functions). In [4], he introduced and studied the notions of g- α - open sets, g - semi-open sets, g – pre open sets and g - β open sets in generalized topological spaces. C.Boonpok [9] extended the concept of bigeneralized topological spaces and studied (m, n) - closed sets and (m, n)- open sets in bigeneralized topological spaces.

Kelly [7] introduced the concept of bitopological spaces. The concept of generalized closed sets was introduced by Levine[8] in a topological spaces and was further extended by several authors. S. Benchalli [2] defined the concept of w-closed sets in topological spaces..Since then many mathematicians generalized the topological concepts into bitopological setting. In this paper, we introduce the notion of $\lambda_{(m, n)}$- w - closed sets in bigeneralized topological spaces and discuss some of their properties. We further develope the concept of these set to obtain $wg_{(m, n)}$- continuous functions and investigate some of their characterizations.

2 Preliminaries

Let X be a non – empty set and λ be a collection of subset of X. Then λ is called a generalized topology (briefly GT) on X iff $\phi \in \lambda$ and $G_i \in \lambda$ for $i \in I \neq \phi$ implies $G = \bigcup_{i \in I} G_i \in \lambda$ .We call the pair (X,λ), a generalized topological space (briefly GTS) on X. The elements of λ are called λ- open sets and the complements are called λ- closed sets. The generalized closure of a subset S of X, denoted by $c_\lambda(S)$, is the intersection of generalized closed sets including S and the interior of S, denoted by $i_\lambda(S)$, is the union of generalized open sets contained in S.

Definitions: 2.1 Let (X, λ) be a generalized topological space and $A \subseteq X$, then A is said to be
(1) λ - semi open if $A \subseteq c_\lambda (i_\lambda (A))$
(2) λ - pre open if $A \subseteq i_\lambda(c_\lambda(A))$
(3) λ - α- open if $A \subseteq i_\lambda(c_\lambda (i_\lambda (A)))$
(4) λ - β open if $A \subseteq c_\lambda (i_\lambda (c_\lambda(A)))$
The complement of λ - semi open (resp λ - pre open, λ - α- open, λ - β open) is said to be λ - semi closed (resp λ - pre closed , λ - α- closed, λ - β closed). The class of all λ - semi open sets on X is denoted by $\sigma(\lambda_x)$ (briefly σ_x or σ). The class of λ - pre open (λ - α- open and λ - β open) sets on X as $[\pi(gx), \alpha(gx), \beta(gx)]$ or briefly $[[\pi,\alpha,\beta]$.

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Definition: 2.2 [1] Let X be non-empty set and let λ_1 and λ_2 be generalized topologies on X. A triple $(X, \lambda_1, \lambda_2)$ is said to be a bigeneralized topological space (briefly BGTS).

Let $(X, \lambda_1, \lambda_2)$ be a bigeneralized topological space and A be a subset of X. The closure of A and the interior of A with respect to λ_m are denoted by $c_{\lambda m}(A)$ and $i_{\lambda m}(A)$, respectively, for m = 1, 2.

Definition: 2.3 [1] A subset A of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is called (m, n)- closed if $c_{\lambda m}(c_{\lambda n}(A)) = A$, where m, n = 1,2 and $m \neq n$. The complement of (m, n) – closed set is called (m, n) – open.

Definition: 2.4 [9] A subset A of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is called (m, n)- generalized closed if $c_{\lambda n}(A) \subseteq U$, whenever $A \subseteq U$ and U is λ_m –open, where m, n = 1,2 and $m \neq n$. The complement of (m, n) – generalized closed set is called (m, n) – generalized open set.

Proposition: 2.5 [9] Let λ_1 and λ_2 be generalized topologies on X. If $\lambda_1 \subseteq \lambda_2$, then $\lambda_{(m,\,n)} – C(X) \subseteq \lambda_{(n,\,m)} – C(X)$.

Definition: 2.6 [9] Let $(X,\lambda_X^1, \lambda_X^2)$ and $(Y,\lambda_Y^1, \lambda_Y^2)$ be generalized topological spaces.

A function f: $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ is said to be (m, n) -generalized continuous (briefly $g_{(m,\,n)}$- continuous) if $f^{-1}(F)$ is $\lambda_{(m,\,n)}$- closed in X for every λ_n- closed F of Y, where m, n = 1,2 and $m\neq n$.

A function f: $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ is said to be pairwise -generalized continuous(briefly pairwise g-continuous) if f is $g_{(1,2)}$-continuous and $g_{(2,1)}$-continuous.

Definition: 2.7 [9] Let $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ be generalized topological spaces.

A function f: $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ is said to be g_m- continuous if $f^{-1}(F)$ is λ_m -closed in X for every λ_m – closed F of Y, for m = 1,2.

Definition: 2.8 [9] Let $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ be generalized topological spaces.

A function f: $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ is said to be g_m- closed(resp. g_m- open) if f(F) is λ_m –closed (resp λ_m - open) of Y for every λ_m – closed (resp. λ_m – open) F of X, for m = 1,2.

Definition: 2.9 [9] A bigeneralized topological space $(X,\lambda_X^1, \lambda_X^2)$ is said to be $\lambda_{(m,\,n)} – T_{1/2}$ – space if, for every $\lambda_{(m,\,n)}$-closed set is λ_n – closed, where m, n = 1,2 and $m\neq n$.

Definition: 2.10 [9]A bigeneralized topological space $(X,\lambda_X^1, \lambda_X^2)$ is said to be pairwise λ -$T_{1/2}$ space if it is both $\lambda_{(1,\,2)}$ –$T_{1/2}$ space and $\lambda_{(2,\,1)}$ – $T_{1/2}$ space.

3. $\lambda_{(m,\,n)}$ - w- closed sets

Definition: 3.1 A Subset A of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is said to be *$\lambda_{(m,\,n)}$ – w-Closed set* if $c_{\lambda n}(A) \subseteq U$, whenever $A \subseteq U$ and U is λ_m - semi open, where m, n = 1,2 and $m\neq n$. The complement of $\lambda_{(m,\,n)}$ – w - closed set is said to be *$\lambda_{(m,\,n)}$ – w-open set*.

Remark: 3.2 Every (m, n) - closed set is $\lambda_{(m,\,n)}$ – w- closed.

The converse is not true as can be seen from the following example.

Example: 3.3 Let X = {a, b, c}. Consider two generalized topologies $\lambda_1 = \{\phi,\{a\},\{a, b\}, X\}$, and $\lambda_2 = \{\phi,\{c\}, \{b, c\}\}$,.Let A = {a, b}, then A is $\lambda_{(m,\,n)}$ – w- closed but not (m, n) - closed.

Proposition: 3.4 Every $\lambda_{(m,\,n)}$ – w- closed set is $\lambda_{(m,\,n)}$ – generalized closed.

Proof: Let U be a λ_m - open set such that $A \subseteq U$. Since A is $\lambda_{(m,\,n)}$ – w- closed set, $c_{\lambda n}(A) \subseteq U$, but U is λ_m - open. Therefore A is $\lambda_{(m,\,n)}$ – closed.

Example: 3.5 Let X = {a, b, c}. Consider two generalized topologies $\lambda_1 = \{\phi,\{a\},\{a, b\}\}$, and $\lambda_2 = \{\phi,\{c\},\{b, c\}\}$. Let A = {a}, then A is $\lambda_{(m,\,n)}$ – generalized closed but not $\lambda_{(m,\,n)}$ – w- closed.

Proposition: 3.6 Let $(X, \lambda_1, \lambda_2)$ be a bigeneralized topological space and A be a subset of X. If A is λ_n – closed, then

A is $\lambda_{(m, n)}$ – w- closed, where m, n = 1, 2 and m ≠ n.

Remark: 3.7 The union of two $\lambda_{(m, n)}$ – w- closed need not be $\lambda_{(m, n)}$ – w - closed as can be seen from the following example.

Example: 3.8 Let X = {a, b, c, d}. λ_1 = {ϕ, {a, b}, {b, c}{a ,b,c}} and λ_2 = {ϕ, {a, b, d}, {b, c, d}, X}. Then {a} and {c} are $\lambda_{(m, n)}$ – w- closed sets but A ∪ B = {a, c} is not $\lambda_{(m, n)}$ – w- closed.

Remark: 3.9 The intersection of two $\lambda_{(m, n)}$ – w- closed sets need not be $\lambda_{(m, n)}$ – w - closed as can be seen from the following example.

Example: 3.10 Let X = {a, b, c}. λ_1 = {ϕ, {a}, {a, b}, X} and λ_2 = {ϕ, {c}, {b, c}.Then {a} and {b} are $\lambda_{(m, n)}$ – w - closed, but A ∩ B = ϕ is not - $\lambda_{(m, n)}$ – w closed set.

Proposition: 3.11 Let (X, λ_1, λ_2) be a bigeneralized topological space. If A is $\lambda_{(m, n)}$ – w - closed and F is (m, n) – semi closed, then A ∩ F is $\lambda_{(m, n)}$ – w – closed, where m, n = 1, 2 and m ≠ n.

Proof: Let U be a λ_m – semi open set such that A ∩ F ⊆ U. Then A ⊆ U ∪ (X - F) and so $c_{\lambda n}$ (A) ⊆U∪(X - F). Therefore $c_{\lambda n}$ (A) ∩ F ⊆ U .Since F is (m, n) - semi closed, $c_{\lambda n}$ (A ∩ F) ⊆ $c_{\lambda n}$ (A) ∩ $c_{\lambda n}$ (F) ⊆ U. Hence A ∩ F is $\lambda_{(m, n)}$ – w – closed.

Proposition: 3.12 For each element x of a bigeneralized topological space (X, λ_1, λ_2), {x} is λ_m –semi closed or X – {x} is $\lambda_{(m, n)}$ - w - closed, where m, n = 1, 2 and m ≠ n.

Proof: Let x ∈ X and the singleton {x} be not λ_m – semi closed. Then X – {x} is not λ_m –semi open, if X ∈ λ_m, then X is the only λ_m – semi open set which contains X – {x}, hence X – {x} is $\lambda_{(m, n)}$ – w – closed and if X ∉ λ_m, then X – {x} is $\lambda_{(m, n)}$ – w – closed.

Proposition: 3.13 Let (X, λ_1, λ_2) be a bigeneralized topological space. Let A ⊆ X be a $\lambda_{(m, n)}$ – w - closed subset of X, then $c_{\lambda n}$ (A) \ A does not contain any non- empty λ_m - semi closed set, where m, n = 1, 2 and m ≠ n.

Proof: Let A be a $\lambda_{(m, n)}$ – w - closed set and F ≠ ϕ is λ_m – semi closed such that F ⊆ $c_{\lambda n}$ (A) \ A. Then F ⊆ X \ A and hence A ⊆ X \ F. Since A is $\lambda_{(m, n)}$ – w - closed, $c_{\lambda n}$ (A) ⊆ X \ F and hence F ⊆ X \ $c_{\lambda n}$ (A).

So, F ⊆ $c_{\lambda n}$ (A) ∩ (X \ $c_{\lambda n}$ (A)) =ϕ. Therefore $c_{\lambda n}$ (A) \ A does not contain any non- empty λ_m - semi closed set.

Proposition: 3.14 Let λ_1 and λ_2 be generalized topologies on X. If A is $\lambda_{(m, n)}$ - w - closed set, then $c_{\lambda m}$ ({x}) ∩ A ≠ ϕ holds for each x ∈ $c_{\lambda n}$ (A), where m, n = 1, 2 and m ≠ n.

Proof: Let x ∈ $c_{\lambda n}$ (A). Suppose that $c_{\lambda m}$ ({x}) ∩ A = ϕ. Then A ⊆ X - $c_{\lambda m}$ ({x}). Since A is $\lambda_{(m, n)}$ – w – closed and X - $c_{\lambda m}$ ({x}) is λ_m – semi-open. Thus $c_{\lambda n}$ (A) ⊆ X - $c_{\lambda m}$ ({x}). Hence $c_{\lambda n}$ (A) ∩ $c_{\lambda m}$ ({x}) = ϕ. This is a contradiction.

Proposition: 3.15 If A is a $\lambda_{(m, n)}$ – w - closed set of (X, λ_1, λ_2) such that A ⊆ B ⊆ $c_{\lambda n}$ (A) then B is $\lambda_{(m, n)}$ – w - closed set, where m, n = 1, 2 and m ≠ n.

Proof: Let A be a $\lambda_{(m, n)}$ – w - closed set and A ⊆ B ⊆ $c_{\lambda n}$ (A). Let B ⊆ U and U is λ_m – semi-open. Then A ⊆ U. Since A is $\lambda_{(m, n)}$ – w – closed, we have $c_{\lambda n}$ (A) ⊆ U. Since B ⊆ $c_{\lambda n}$ (A), then $c_{\lambda n}$ (B) ⊆ $c_{\lambda n}$ (A) ⊆ U.

Hence B is $\lambda_{(m, n)}$ – w – closed.

Remark: 3.16 $\lambda_{(1,2)}$ – WC(X) is generally not equal to $\lambda_{(2,1)}$ – WC(X) as can be seen from the following example.

Example: 3.17 Let X = {a, b, c}. Consider two generalized topologies λ_1 = {ϕ, {a}, {a, b}, X} and λ_2 = {ϕ, {c}, {b, c}}. Then $\lambda_{(1,2)}$ – WC(X) = {{a}, {b},{a, b}, {b, c},X} and $\lambda_{(2,1)}$ – WC(X) = {{b}, {c},{a, b}, {b, c},ϕ, X }.

Thus $\lambda_{(1,2)}$ – WC(X) ≠ $\lambda_{(2,1)}$ – WC(X).

Proposition: 3.18 Let λ_1 and λ_2 be generalized topologies on X. if λ_1 ⊆ λ_2, then $\lambda_{(2,1)}$ – WC(X) ⊆ $\lambda_{(1,2)}$ – WC(X)

Proposition: 3.19 Let (X, λ_1, λ_2) be a bigeneralized topological space. Let A ⊆ X be a $\lambda_{(m, n)}$ – w - closed subset of (X, λ_1, λ_2). Then A is λ_m - semi closed if and only if $c_{\lambda n}$ (A) \ A is λ_m - semi closed, where m, n = 1, 2 and m ≠ n.

Proof: Let A be a $\lambda_{(m,n)}$ – w - closed set. If A is λ_m – semi closed, then $c_{\lambda n}(A) \setminus A = \phi$, but ϕ is λ_m – semi closed.

Therefore $c_{\lambda n}(A) \setminus A$ is λ_m - semi closed.

Conversely, Suppose that $c_{\lambda n}(A) \setminus A$ is λ_m - semi closed, As A is $\lambda_{(m,n)}$ – w – closed, $c_{\lambda n}(A) \setminus A = \phi$, Consequently $c_{\lambda n}(A) = A$.

Proposition: 3.20 Let $(X, \lambda_1, \lambda_2)$ be a bigeneralized topological space. If A is $\lambda_{(m,n)}$ – w - closed and $A \subseteq B \subseteq c_{\lambda n}(A)$, then $c_{\lambda n}(B) \setminus B$ has no non empty λ_m - semi closed subset.

Proof: $A \subseteq B$ implies $X – B \subseteq X – A$ and $B \subseteq c_{\lambda n}(B)$ implies $c_{\lambda n}((c_{\lambda n}(A)) = c_{\lambda n}(A)$.

Thus $c_{\lambda n}(B) \cap (X - B) \subseteq c_{\lambda n}(A) \cap (X - A)$ which yields $c_{\lambda n}(B) / B \subseteq c_{\lambda n}(A) / A$. As A is $\lambda_{(m,n)}$ – w – closed, $c_{\lambda n}(A) \setminus A$ has no non empty λ_m - semi closed subset. Therefore $c_{\lambda n}(B) \setminus B$ has no non empty λ_m - semi closed subset.

Remark: 3.21 The intersection of two $\lambda_{(m,n)}$ – w- open sets need not be $\lambda_{(m,n)}$ – w - open as can be seen from the following example.

Example: 3.22 Let X = {a, b, c, d}. λ_1 = {ϕ, {a, b}, {b, c}{a ,b,c}} and λ_2 = {ϕ, {a, b, d}, {b, c, d}, X}.Then {a, b, d} and {b, c, d} are $\lambda_{(m,n)}$ – w- open sets but {a, b, d} $\cap${b, c, d} = {b, d} is not $\lambda_{(m,n)}$ – w- open .

Remark: 3.23 The union of two $\lambda_{(m,n)}$ – w- open sets need not be $\lambda_{(m,n)}$ – w - open as can be seen from the following example.

Example: 3.24 Let X = {a, b, c}. λ_1 = {ϕ, {a}, {a, b}, X} and λ_2 = {ϕ, {c}, {b, c}}. Let A= {a} and B = {b, c} are $\lambda_{(m,n)}$ – w - open, but $A \cup B = X$ is not $\lambda_{(m,n)}$ – w- open.

Proposition: 3.25 A subset of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is $\lambda_{(m,n)}$ - w - open iff every subset of F of X, $F \subseteq i_{\lambda n}(A)$ whenever F is λ_m – semi closed and $F \subseteq A$, where m, n = 1, 2 and m ≠ n.

Proof: Let A be a $\lambda_{m,n}$ – w- open. Let $F \subseteq A$ and F is λ_m – semi closed. Then $X – A \subseteq X – F$ and X – F is λ_m – semi open, we have X – A is $\lambda_{(m,n)}$ – w – closed, then $c_{\lambda n}(X - A) \subseteq X - F$. Thus $X - i_{\lambda n}(A) \subseteq X – F$ and hence $F \subseteq i_{\lambda n}(A)$.

Conversely, let $F \subseteq i_{\lambda n}(A)$ whenever F is a λ_m - semi closed set such that $F \subseteq A$. Let $X - A \subseteq U$ where U is a λ_m - semi open. Then $X - U \subseteq A$ and X - U is λ_m - semi closed. By the assumption, $X - U \subseteq i_{\lambda n}(A)$, then $X - i_{\lambda n}(A) \subseteq U$.

Therefore, $c_{\lambda n}(X - A) \subseteq U$. Hence X - A is $\lambda_{(m,n)}$ - w - closed and hence A is $\lambda_{(m,n)}$ - w - open.

Proposition: 3.26 Let A and B be subsets of a bigeneralized topological space$(X, \lambda_1, \lambda_2)$ such that $i_{\lambda n}(A) \subseteq B \subseteq A$. If A is $\lambda_{(m,n)}$ - w - open then B is also $\lambda_{(m,n)}$ - w- open, where m, n = 1, 2 and m ≠ n.

Proof: Suppose that $i_{\lambda n}(A) \subseteq B \subseteq A$. Let F be a λ_m - semi closed set such that $F \subseteq B$. Since A is $\lambda_{(m,n)}$ -w - open, $F \subseteq i_{\lambda n}$. Since $i_{\lambda n} \subseteq B$, we have $i_{\lambda n}(i_{\lambda n}(A)) \subseteq i_{\lambda n}(B)$. Consequently $i_{\lambda n}(A) \subseteq i_{\lambda n}(B)$. Hence $F \subseteq i_{\lambda n}(B)$. Therefore, B is $\lambda_{(m,n)}$ - w -open.

Proposition: 3.27 If A be a subset of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is $\lambda_{(m,n)}$ - w- closed , then $c_{\lambda n}(A) - A$ is $\lambda_{m,n}$ – w – open, where m, n = 1, 2 and m ≠ n.

Proof: Suppose that A is $\lambda_{m,n}$ – w – closed. Let $X – (c_{\lambda n}(A) - A) \subseteq U$ and U is λ_m - semi open.

Then $X – U \subseteq X – (c_{\lambda n}(A) - A)$ and X – U is λ_m - semi closed. Thus we have $c_{\lambda n}(A) - A$ does not contain non – empty λ_m - semi closed by Proposition 3.14 . Consequently, $X – U = \phi$, then U = X. Therefore, $c_{\lambda n}(X - (c_{\lambda n}(A) – A)) \subseteq U$, so we obtain $X – (c_{\lambda n}(A) – A)$ is $\lambda_{m,n}$ – w- closed.

Hence $c_{\lambda n}(A) - A$ is $\lambda_{m,n}$ - w - open.

4. $wg_{(m,n)}$ - continuous functions

Definition: 4.1 Let $(X,\lambda_X^1, \lambda_X^2)$ and $(Y,\lambda_Y^1, \lambda_Y^2)$ be generalized topological spaces. A function f: $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ is said to be (m, n) - w- generalized continuous (briefly $wg_{(m,n)}$- continuous) if $f^{-1}(F)$ is $\lambda_{(m,n)}$- w-closed in X for every λ_n- closed F of Y, where m, n = 1,2 and m≠ n.

A function f: $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ is said to be pairwise w-generalized continuous(briefly pairwise wg-continuous) if f is $wg_{(1,2)}$-continuous and $wg_{(2,1)}$-continuous.

Theorem: 4.2 For an injective function f: $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$, the following properties are equivalent:

(1) f is $wg_{(m,n)}$- continuous,
(2) For each $x \in X$ and for every λ_n- open set V containing f(x), there exists a $\lambda_{(m,n)}$-w-open set U containing x such that $f(U) \subseteq V$:
(3) $f(c_{\lambda_X^n}(A)) \subseteq c_{\lambda_Y^n}(f(A))$ for every subset A of X;
(4) $c_{\lambda_X^n}(f^{-1}(B)) \subseteq f^{-1}(c_{\lambda_Y^n}(B)$ for every subset B of Y.

Proof: (1) $\Rightarrow$(2): Let $x \in X$ and V be a λ_n-open subset of Y containing f(x). Then by (1), $f^{-1}(V)$ is $\lambda_{(m,n)}$-w-open of X containing x. If $U = f^{-1}(V)$, then $f(U) \subseteq V$.

(2)$\Rightarrow$(3): Let A be a subset of X and $f(x) \notin (c_{\lambda_Y^n}(f(A))$. Then, there exists a λ_n – open subset V of Y containing f(x) such that $V \cap f(A) = \phi$.Then by(2), there exist a $\lambda_{(m,n)}$-w-open set such that $f(x) \in f(U) \subseteq V$. Hence, $f(U) \cap f(A) = \phi$ implies $U \cap A = \phi$. Consequently, $x \notin c_{\lambda_X^n}(A)$ and $f(x) \notin f(c_{\lambda_X^n}(A))$.

(3) $\Rightarrow$(4): Let B be a subset of Y. By (3) we obtain $f(c_{\lambda_X^n}(f^{-1}(B)) \subseteq c_{\lambda_Y^n}(f(f^{-1}(B))$. Thus $c_{\lambda_X^n}(f^{-1}(B)) \subseteq f^{-1}(c_{\lambda_Y^n}(B))$.

(4) $\Rightarrow$(1): Let F be a λ_n – closed subset of Y. Let U be a λ_m – semi open subset of X such that $f^{-1}(F) \subseteq U$. Since $c_{\lambda_Y^n}(F) = F$ and by (4), $c_{\lambda_X^n}(f^{-1}(F)) \subseteq U$. Hence f is $wg_{(m,n)}$ continuous.

Definition: 4.3 Let $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ be generalized topological spaces. A function f: $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ is said to be wg_m- continuous if $f^{-1}(F)$ is λ_m –semi-closed in X for every λ_m – closed F of Y, for m = 1,2.

Definition: 4.4 Let $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ be generalized topological spaces.

A function f: $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ is said to be wg_m- closed(resp. wg_m- open) if f(F) is λ_m – semi closed (resp λ_m –w- open) of Y for every λ_m – closed (resp. λ_m – open) F of X, for m = 1,2.

Proposition: 4.5 If f: $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ is wg_m- continuous and wg_m- closed , then f(A) is $\lambda_{(m,n)}$- w-closed subset of Y for every $\lambda_{(m,n)}$- w-closed subset A of X, where m, n = 1,2 and $m \neq n$.

Proof: Let U be λ_m – semi open subset of Y such that $f(A) \subseteq U$. Then $A \subseteq f^{-1}(U)$ and $f^{-1}(U)$ is λ_m – semi open subset of X. Since A is $\lambda_{(m,n)}$- w-closed, $c_{\lambda_X^n}(f(A)) \subseteq f^{-1}(U)$ and hence $f(c_{\lambda_X^n}(A)) \subseteq U$.

Therefore we have $c_{\lambda_Y^n}(f(A)) \subseteq c_{\lambda_Y^n}(f(c_{\lambda_X^n}(A))) = f(c_{\lambda_X^n}(A)) \subseteq U$. Therefore, f(A) is $\lambda_{(m,n)}$- w-closed subset of Y.

Lemma: 4.6 If f: $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ is wg_{m-}closed, then for each subset S of Y and each λ_m – semi open subset U of X containing $f^{-1}(S)$, there exists a λ_m – semi open subset V of Y such that $f^{-1}(V) \subseteq U$.

Proposition: 4.7 If f: $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ is injective, wg_m-closed and $wg_{(m,n)}$-continuous, then $f^{-1}(B)$ is $\lambda_{(m,n)}$- w-closed subset of X for every $\lambda_{(m,n)}$- w-closed subset of B of Y, where m, n = 1,2 and $m \neq n$.

Proof: Let B be a $\lambda_{(m,n)}$- w-closed subset of Y. Let U be a λ_m – semi open subset of X such that $f^{-1}(B) \subseteq U$. Since f is wg_m-closed and by lemma 4.6, there exists a λ_m – semi open subset of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$. Since B is $\lambda_{(m,n)}$- w-closed set and $B \subseteq V$, then $c_{\lambda_Y^n}(B) \subseteq V$. Consequently, $f^{-1}(c_{\lambda_Y^n}(B)) \subseteq f^{-1}(V) \subseteq U$.

By theorem 4.2, $c_{\lambda_X^n}(f^{-1}(B)) \subseteq f^{-1}(c_{\lambda_Y^n}(B) \subseteq U$ and hence $f^{-1}(B)$ is $\lambda_{(m,n)}$- w-closed subset of X.

Definition: 4.8 A bigeneralized topological space $(X,\lambda_X^1, \lambda_X^2)$ is said to be $\lambda_{(m,n)}$ – $wT_{1/2}$ – space if, for every $\lambda_{(m,n)}$- w-closed set is λ_n – closed, where m, n = 1,2 and $m \neq n$.

Definition: 4.9 A bigeneralized topological space $(X,\lambda_X^1, \lambda_X^2)$ is said to be pairwise λ – $wT_{1/2}$ – space if it is both $\lambda_{(1,2)}$ –$wT_{1/2}$ – space and $\lambda_{(2,1)}$ – $w_{1/2}$ – space.

Theorem: 4.10 A bigeneralized topological space is a $\lambda_{(m, n)}$- $wT_{1/2}$-space if and only if $\{x\}$ is λ_n - open or λ_m -semi closed for each $x \in X$, where m, n = 1,2 and m≠ n.

Proof: Suppose that $\{x\}$ is not λ_m -semi closed. Then $X - \{x\}$ is $\lambda_{(m, n)}$ -w-closed by proposition 3.13. Since X is $\lambda_{(1, 2)}$ -$T_{1/2}$ -space, $X - \{x\}$ is λ_n -closed. Hence, $\{x\}$ is λ_n - open.

Conversely, let F be a $\lambda_{(m, n)}$ - w-closed set. By assumption , $\{x\}$ is λ_n - open or λ_m -semi closed for any $x \in c_{\lambda n}(F)$.Case (i) Suppose that $\{x\}$is λ_n - open. Since $\{x\} \cap F \neq \phi$, we have $x \in F$. Case (ii) Suppose that $\{x\}$ is λ_m - semi closed.

If $x \notin F$, then $\{x\} \subseteq c_{\lambda n}(F) - F$, which is a contradiction to proposition 3.15, Therefore, $x \in F$. Thus in both case, we conclude that F is λ_n - closed. Hence, $(X,\lambda_X^1, \lambda_X^2)$ is a $\lambda_{(m, n)}$ -$wT_{1/2}$ -space.

Definition: 4.11 Let $(X,\lambda_X^1, \lambda_X^2)$ and $(Y,\lambda_Y^1, \lambda_Y^2)$ be generalized topological spaces.

A function f: $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ is said to be $wg_{(m, n)}$ - irresolute if $f^{-1}(F)$ is $\lambda_{(m, n)}$-w- closed in X for every $\lambda_{(m, n)}$ –w-closed F of Y, where m, n = 1,2 and m≠ n.

Proposition: 4.12 Let f: $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ and g: $(Y,\lambda_Y^1, \lambda_Y^2) \rightarrow (Z,\lambda_Z^1, \lambda_Z^2)$ be functions, the following properties hold:

(i) If f is $wg_{(m, n)}$ - irresolute and $wg_{(m, n)}$ -continuous, then $g \circ f$ is $wg_{(m, n)}$ -continuous.

(ii) If f and g are $wg_{(m, n)}$ - irresolute, then $g \circ f$ is $wg_{(m, n)}$ - irresolute;

(iii) Let $(Y,\lambda_Y^1, \lambda_Y^2)$ be a $\lambda_{(m, n)}$ -$wT_{1/2}$ -space. If f and g are $wg_{(m, n)}$ -continuous, then $g \circ f$ is $wg_{(m, n)}$ -continuous.

Proof: (i) Let F be a λ_n -closed subset of Z. Since g is $wg_{(m, n)}$ - continuous, then $g^{-1}(F)$ is $\lambda_{(m, n)}$ -w-closed subset of Y. Since f is $wg_{(m, n)}$ - irresolute, then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\lambda_{(m, n)}$ -w-continuous.

(ii)) Let F be a $\lambda_{(m, n)}$ -w- closed subset of Z. Since g is $wg_{(m, n)}$ - irresolute, then $g^{-1}(F)$ is $\lambda_{(m, n)}$ -w-closed subset of Y. Since f is $wg_{(m, n)}$ - irresolute, then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\lambda_{(m, n)}$ –w- closed subset of X. Hence, then $g \circ f$ is $wg_{(m, n)}$ - irresolute.

(iii) Let F be a λ_n -closed subset of Z. Since g is $wg_{(m, n)}$ -continuous, then $g^{-1}(F)$ is $\lambda_{(m, n)}$ -w- closed subset of Y. Since $(Y,\lambda_Y^1, \lambda_Y^2)$ is a $\lambda_{(m, n)}$ -$wT_{1/2}$ -space, then $g^{-1}(F)$ is λ_n -closed subset of Y. Since f is $wg_{(m, n)}$ -continuous,

Then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\lambda_{(m, n)}$ –w-closed subset of X. Hence then $g \circ f$ is $wg_{(m, n)}$ -continuous.

Proposition: 4.13 Let: $(X,\lambda_X^1, \lambda_X^2)$ be a $\lambda_{(m, n)}$ -$wT_{1/2}$ -space. If f: $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ is surjective, g_n -closed and $wg_{(m, n)}$ -irresolute, then $(Y,\lambda_Y^1, \lambda_Y^2)$ is a $\lambda_{(m, n)}$ -$wT_{1/2}$ -space, where m, n = 1,2 and m≠ n.

Proof: Let F be a $\lambda_{(m, n)}$ -w-closed subset of Y. Since f is $wg_{(m, n)}$ - irresolute, we have $f^{-1}(F)$ is a $\lambda_{(m, n)}$ -w-closed subset of X. Since $(X,\lambda_X^1, \lambda_X^2)$ is a $\lambda_{(m, n)}$ -$wT_{1/2}$ -space, $f^{-1}(F)$ is a λ_n -closed subset of X. It follows by assumption that F is a λ_n -closed subset of Y. Hence $(Y,\lambda_Y^1, \lambda_Y^2)$ is a $\lambda_{(m, n)}$ -$wT_{1/2}$ space.

Proposition: 4.14 Let: $(X,\lambda_X^1, \lambda_X^2)$ be a $\lambda_{(m, n)}$ -$wT_{1/2}$ space. If f: $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ is bijective, g_n -open and $wg_{(m, n)}$ – irresolute, then $(Y,\lambda_Y^1, \lambda_Y^2)$ is a $\lambda_{(m, n)}$ -$wT_{1/2}$space, where m, n = 1,2 and m≠ n.

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