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A COMPARATIVE STUDY OF EARLY EGYPTIAN, BABYLONIAN AND MAYAN NUMBER SYSTEM

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ABSTRACT

We here discuss different number systems (so to say numerals) available in ancient times in particular in Egypt, Babylon and Mayan exhibiting the operations available in the literature. Finally, we give some light on these aspects to compare the various systems mentioned above.

***2000 Mathematics Subject Classification**: 01A16, 01A17, 01A99.*

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1. INTRODUCTION

Ancient Egypt was an ancient civilization of eastern North Africa, concentrated along the lower reaches of the Nile River.

Some of the achievements of the ancient Egyptians include the quarrying, surveying and construction techniques that facilitate the building of monumental pyramids, temples and obelisks, a system of mathematics, a practical and effective system of medicine, irrigation systems and agricultural production techniques, new forms of literature etc.

The Babylonians (2300 BC to 1600 BC) lived in Mesopotamia about 5,000 years ago, Babylonians began a numbering system. It is one of the oldest numbering system.

The Mayans(Indians may see this civilization as related our ancient scriptures like Ramayana where we find relative of great King Ravana, in Particular Maya-Ravana, in **PATAAL** (down the Earth) meaning America (3113BC to 900 AD) lived in Central America. The Mayans were highly skilled mathematicians, astronomers, artists and architects. The Maya civilization collapsed mysteriously around 900 AD.

The Mayans had several calendars. There was 360 days civil year, a 260 days religious year and the complicated Long Count Calendar which measured time from the start of Maya civilization (August 12, 3113 BC) and completes a full cycle on December 21, 2012.

The early Egyptian, Babylonian and Mayan were very rich in mathematics. They developed their own number system. But there were some differences between the number system of Egyptian, Babylonian and Mayan.

1.1 In ancient Egypt (2300 BC to 1600 BC) the numerical notation was very simple. They used some symbols to represent the numbers called hieroglyphics. Some Egyptian numbers were

English	1	2	3	4	5	6	7	8	9	10
Egyptian	I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII	∩

English	11	15	20	22	60	90	100
Egyptian	∩ I	∩IIIII	∩∩	∩∩II	∩∩∩ ∩∩∩	∩∩∩∩ ∩∩∩∩∩	𓍢 **Or** 𓍢

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English	1000	10,000	1,00,000	1,000,000
Egyptian	♣	or	or	

Therefore we have observed that in ancient Egyptian number system there was a special sign for every power of ten.

1 = I

10 = ∩ Heel bone or string.

100 = Or coil of rope.

1000 = ♣ flower.

10,000 = or Pointing finger.

100,000 = or Tad pole.

1,000,000 = surprised man.

Generally, the Egyptian numbers were written from left to right as the modern form of express a number. Such as

∩∩∩ ∩∩∩ = 360

♣♣ ∩∩ ∩∩ III = 2343

♣♣♣♣ ∩∩ III = 34023

The Egyptian numbers were also written from right to left i.e. the largest decimal order would be written first. In this case the symbols themselves were reversed. Below some of the Egyptian numbers written from right to left and their modern English form

∩∩ ∩∩∩ = 350

III ∩∩∩∩ ♣♣ = 2343

III ∩∩ ♣♣♣♣ = 34023

In Egyptian number system if we write a number from left to right (i.e. the largest decimal order would be written last) or we write the same number from right to left (i.e. the largest decimal order would be written first) the value of the number remain same.

It is possible because in ancient Egyptian number system there was a special sign for every power of ten.

1.2 The Babylonians lived in Mesopotamia about 5,000 years ago. Babylonians began a number system. It is one of the oldest numbering system. The Babylonians developed a form of writing based on cuneiform in between 2300 BC to 1600 BC .In Latin the meaning of cuneiform is "wedge shape". They used only two symbols to represent their all numbers.. These two symbols are $\nabla=1, <=10$. Some of the Babylonian numbers were

English	1	2	5	9	10	14	15
Babylonian	∇	∇ ∇	∇ ∇ ∇ ∇∇	∇ ∇∇ ∇ ∇ ∇∇ ∇∇	<	< ∇ ∇ ∇ ∇	< ∇ ∇ ∇ ∇∇

English	20	24	30	36	40	42	59
Babylonian	<<	<< ∇∇ ∇∇	<< <	<< ∇∇∇ ∇∇∇	<<<<	<<<<∇∇	<<<<< ∇∇∇∇∇ ∇∇∇∇

1.3 The Maya number system used a combination of two symbols. A dot (.) was used to represent the units and a dash (—) was used to represent five.

Some of Mayans numbers were

English	1	2	3	4	5	6	7	8	9	10
Mayan	•	••	•••	••••	—	• —	•• —	••• —	•••• —	— —

English	11	12	13	1 4	15	16	17	18	19
Mayan	• — —	•• — —	••• — —	•••• — —	— — —	• — — —	•• — — —	••• — — —	•••• — — —

2. THE DECIMAL, SEXAGESIMAL AND VIGESIMAL SYSTEM

The Egyptian number system was decimal base, Babylonian number system was sexagesimal and Mayan number system was vigesimal base. Before going to discuss about decimal, sexagesimal and vigesimal system we must know about place value notation. Place value notation is the use of numerals in different position to represent numbers.

2.1 A number represents a quantity. Number is an abstract idea of collection of things but numerals are man-made symbols that represent the numbers. Numbers (Quantity) are always the same value, no matter what symbol or word is used to represent them. As for example, 1, I, i all three numerals represent the same number we know as "one".

Our system uses place value notation, for example, 43 means "four tens and three ones."

Tens	Ones
4	3

In place value notation, the places are the exponents of the base. In our decimal (base 10) system we have places for 1, 10, 10 squared (100), 10 cubed (1000) and so on. In a sexagesimal (base 60) system, the places are 1, 60, 60 squared (3600) etc. In vigesimal (base 20) system, 1, 20, 20 squared (400), 20 cubed (800) and so on.

Value "Two hundred and twelve" in Base 10 & Base 60

DECIMAL	10 squared place (100)	Tens place (10)	Ones place (1)
Value = 212	2	1	2
SEXAGESIMAL	60 Squared place (60^2 =3600)	Sixties place (60)	Ones place (1)
Value=212		3	32

2.2 The Egyptian number system was decimal base. Some of the Egyptian numbers with their place values and decimal values are given below.

Place Value							Decimal Value
10,00000 𓁨	100,000 𓆐	10000 𓂭	1000 ♣	100 𓍢	10 ∩	1 |	
				𓍢𓍢𓍢𓍢	∩∩∩ ∩∩∩		460
			♣♣♣	𓍢𓍢	∩∩∩	||||	3234
		𓂭 𓂭	♣♣		∩∩	||	22022
	𓆐	𓂭 𓂭	♣♣♣	𓍢𓍢𓍢𓍢𓍢	∩∩∩ ∩∩∩	||||	1,23,564
𓁨𓁨	𓆐	𓂭	♣♣	𓍢𓍢𓍢𓍢𓍢	∩∩	||	2, 11, 2522

2.3 In between 2300 BC and 1600 BC the Babylonian had a very advanced number system. It was the base 60 system or sexagesimal system rather than the base ten system or decimal system. In this system numbers above the units (from 1 to 59) are arranged according to power of 60. At that time 60 itself was called "sussu" or "soss" and 60^2 was called "sar". The Babylonians divided the day into twenty four hours, each hours into sixty minutes and each minute to sixty second. They used only two symbols ∇ and < to represent all the numbers from 1 to 59. By grouping these two symbols together, they created symbols for all 59 numbers.

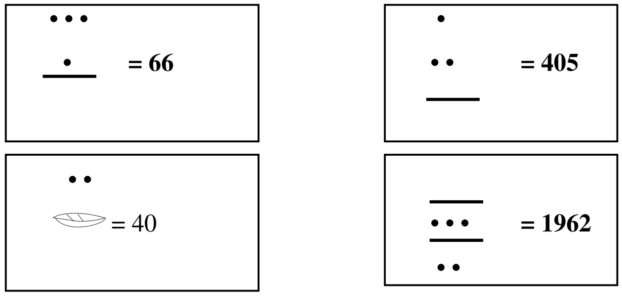
Some of the Babylonian numbers in sexagesimal system are given below.

Place value				Sexagesimal Value
60^3	60^2	60	1	

		∇	∇	∇∇ = 61
	< <	< ∇∇	∇∇∇	<< <∇∇ ∇∇∇ = 72, 37, 723
∇ ∇	< ∇	< < <	< < < ∇ ∇	∇∇< ∇ <<<<<<∇∇ = 4, 37, 792
	∇ ∇	< ∇∇	∇∇∇	∇∇ <∇∇ ∇ ∇∇ = 7, 923
	∇	∇ ∇	< <	∇∇∇ < < = 3, 722

2.4 The Mayans used a vigesimal system, which had a base 20. This system is believed to have been used because 20 was the total number of fingers and toes. The Mayans wrote their numbers vertically with the lowest denomination on the bottom. Their system was set up so that the first five place values were based on the multiples of 20. The Mayan numbers are read from bottom to top. The Mayans departed from a pure base 20 system by letting the symbols in the third position (from the bottom) represent the number of 360=18×20 instead of $400=20^2$. According to some scholars the reason may be that their year consisted of 360 days. In the table below are represented some Mayan numbers.

Place value	$18\times20^2 = 7200$						•••
	$18\times20 = 360$			•	•	••	• ≡
	20	•	••	•	—	•• —	𝋠
	Unit	—	••••	• ≡	••• —	••• =	•••• =
	Vigesimal Value	• — = 25	•• •••• = 44	• • • ≡ **= 396**	• — ••• — **= 468**	•• •• — ••• = **= 873**	••• • ≡ 𝋠 •••• = **= 27,374**



3. POSITION OF ZERO

In Egyptian number system there was no symbol for zero. In Egyptian number system the symbol of zero was not necessary to write a number. In modern number system we cannot write 306 or 2010 without the zero.

But the Egyptian wrote 306 = 

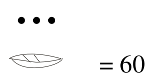
2010 = ♣ ♣ ∩

Babylonians also did not have a symbol for zero. But they used the idea of zero. When it was necessary to express zero, they just left a blank space in the number they were writing or used a wedge mark. When they wrote "60", they would put a single wedge mark in the second place of the numeral.

$\nabla^{/} = 60, \quad \nabla\nabla^{/} = 120, \quad < \nabla^{/} = 660,$

The Mayans were symbolized the concept of nothing or zero. The most common symbol of zero was

 → a head.



4. A COMPARATIVE STUDY OF SEXAGESIMAL, DECIMAL AND VIGESIMAL SYSTEM

4.1 Place Value

Numerals 1 – 2 – 3 – 2 in different place value notation

Place Value				
60^3	60^2	60	1	Sexagesimal
1	2	3	2	$1\times60^3+2\times60^2+3\times60+2\times1$ $=2,23,382$
10^3	10^2	10	1	Decimal Value
1	2	3	2	$1\times10^3+2\times10^2+3\times10+2\times1$ $=1232$
20^3	20^2	20	1	Vigesimal Value
1	2	3	2	$1\times20^3+2\times20^2+3\times20+2\times1$ $=8862$

Value “Three thousand seven hundred twenty one” in Base 60, Base 10 and Base 20.

Place Value				
60^3	60^2	60	1	Sexagesimal
	1	2	1	Value = 3721
10^3	10^2	10	1	Decimal Value
3	7	2	1	Value = 3721
20^3	20^2	20	1	Vigesimal
	9	6	1	Value = 3721

Numerals 2 – 1 – 6 – 3 in different place value notation.

Place Value				
60^3	60^2	60	1	Sexagesimal
2	1	6	3	$2\times60^3+1\times60^2+6\times60+3\times1$ $=4,35,963$
10^3	10^2	10	1	Decimal
2	1	6	3	Value $(200+100+60+3)$ $=2163$
20^3	20^2	20	1	Vigesimal
2	1	6	3	Value $2\times20^3+1\times20^2+6\times20+3\times1$ $=16,523$

Value “Four thousand three hundred twenty one” in Base 60, Base 10 and Base 20.

Place Value				
60^3	60^2	60	1	Sexagesimal
	1	12	1	Value = 4321

10^3	10^2	10	1	Decimal
4	3	2	1	Value = 4321
20^3	20^2	20	1	Vigesimal
	10	16	1	Value = 4321

From above example, we have observed that the sexagesimal and vigesimal number system is more time consuming then decimal system. In decimal system it is easy to find out the place value of any digit in a number but in sexagesimal and vigesimal system it is quite difficult. Due to such type of drawbacks of sexagesimal and vigesimal system modern world give more preference to decimal system then sexagesimal and vigesimal system.

4.2 ADDITION IN DECIMAL, SEXAGESIMAL AND VIGESIMAL SYSTEM

Let us take two numbers 7866 and 3727

In our decimal system if we add these two numbers, we will get

$$\begin{array}{r} 7866 \\ +\,3727 \\ \hline 11593 \end{array}$$

If we add these two numbers in Babylonian number system then we will get

	60^2	60	1	Number in English form
	∇ ∇	< ∇	∇∇∇ ∇∇∇	$3600\times2+60\times11+1\times6=7866$
+	∇	∇ ∇	∇∇∇ ∇∇∇∇	$60^2\times1+60\times2+1\times7=3727$
	∇∇∇	< ∇ ∇∇	< ∇ ∇∇	

$$=3\times60^2+13\times60+13\times1$$

$$=10800+780+13$$

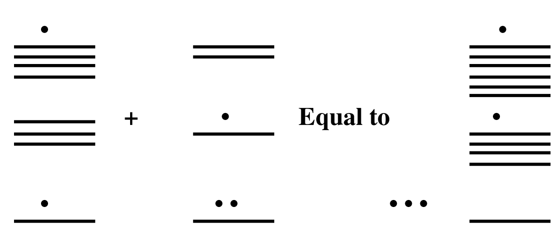
$$=11593$$

Now we will add these two numbers in Mayan number system,

60

20

1



=31×360=11,160

+

=21×20=420

+

=13×1=13

Number in English form

$360\times21+20\times15$
$+1\times6=7866$

$360\times10+20\times6$
$+1\times7=3727$

Total 11593

The result (11593) is same as result of decimal system and sexagesimal system.

Now let us consider three numbers 8930,887, and 532.

In decimal system, if we add them, we will get

$$\begin{array}{rr} & 8930 \\ & 887 \\ + & 532 \\ \hline & 10{,}349 \end{array}$$

Now their place value notation in Babylonian number system is

	60^2	60	1	Number in English form
	∇ ∇	<< ∇∇∇∇ ∇∇∇∇	<< <<<	$60^2\times2+60\times28+1\times50=8930$
+		< ∇∇ ∇∇	<< << ∇∇∇ ∇∇∇∇	$60\times14+1\times47=887$
		∇∇∇∇ ∇∇∇∇	<< <<< ∇∇	$60\times8+1\times52=532$
	∇∇	<<<<<∇ ∇	<<∇ ∇∇∇ ∇∇∇∇∇	

$$=2\times60^2+52\times60+29\times1$$

$$=7200+3120+29$$

$$=10349$$

Now we will add these three numbers in Mayan number system,

7200	•						•	= 7200
360	••••		••		•		••• ——	= 2880
20	•••• —— ——	+	••• ——	+	••• ——	Equal to	••• —— ——	= 260
1	—— ——		•• ——		•• —— ——		•••• ——	= 9
Number in English form	8930		887		532		Total = 10349	

[In vigesimal system we cannot write the numbers greater than 19 in unit place so, in case of 29 we have to distribute 29 as 9 in unit place and 20 in 20th place. Since the system is vigesimal therefore 20 becomes 1 in 20th place]

Therefore from above examples we have observed that in case of addition the results are same in decimal, sexagesimal and vigesimal system.

4.3 Subtraction in Decimal, sexagesimal and Vigesimal system

Let us take two numbers 7866 and 3727. In our decimal system if we subtract 3727 from 7866, we will get

$$\begin{array}{r} 3866 \\ -\ 3727 \\ \hline 4139 \end{array}$$

If we subtract 3727 from 7866, in Babylonian number system, then we will get

	$\underline{60^2}$	$\underline{60}$	$\underline{1}$	number in English
	∇∇	< $\boxed{∇}$	$\boxed{<<<<<<}$ ∇∇∇ ∇∇∇	7866
—	∇	∇ ∇	∇∇∇ ∇∇∇∇	3727
	∇	∇∇∇∇ ∇∇∇∇	<<<<< ∇∇∇∇ ∇∇∇∇∇	

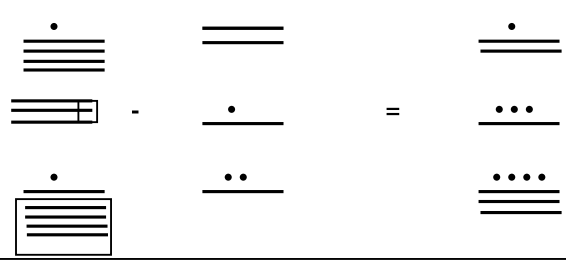
$= 60^2\times 1+60\times 8+1\times 59$

$=3600+480+59$

$=4139$ same as the result of decimal system.

[We cannot subtract from 6 to 7, therefore we carry over 1 from 60th place. Since the system is sexagesimal, therefore in unit place 1 becomes 60]

Now we will subtract 3727 from 7866 in Mayan number system



360 $=360\times 11=3960$

20 $=20\times 8=160$

1 $=1\times 19=19$

Number in English form: 7866 - 3727 = Total = 4139

Therefore in Mayan number system when we subtract 3727 from 7866, the result is 4139. The result is same as decimal system and sexagesimal system. (We cannot subtract from 6 to 7, therefore we carry over 1 from 20th place. Since the system is vigesimal, therefore in unit place 1 becomes 20). Now, let us take another example

Suppose we need to subtract 887 from 8930. In decimal system

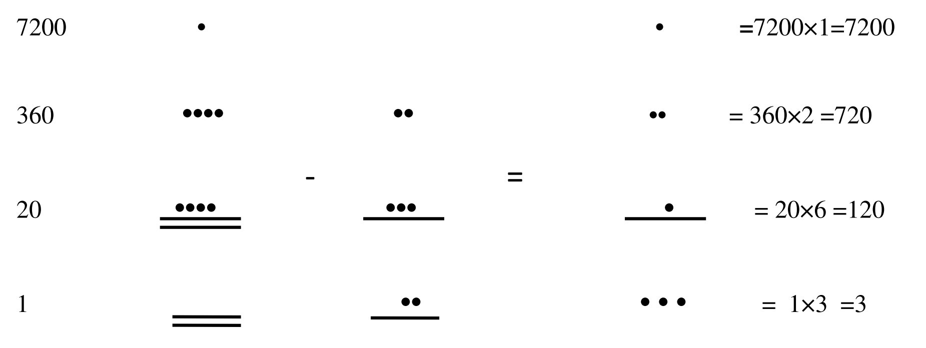
$$\begin{array}{r} 8930 \\ -\ \ 887 \\ \hline 8043 \end{array}$$

Now we will subtract 887 from 8930 in Babylonian number system or sexagesimal system

60^2	60	1	number in English form
∇∇	<< ∇∇∇∇ ∇∇∇∇	<< <<<	8930
—	< ∇∇ ∇∇	<<<< ∇∇∇ ∇∇∇∇	887
∇∇	< ∇∇∇∇	∇∇∇	

$$= 60^2\times 2+60\times 14+1\times 3$$
$$=7200+840+3$$
$$=8043$$

The result is same as decimal system. Now we will subtract 887 from 8930 in Mayan number system or vigesimal system.



Total = 8043

Thus from the above examples we have observed that in case of subtraction the results are same in decimal, sexagesimal and vigesimal system.

The Multiplication and division in sexagesimal and vigesimal system are left for the readers.

Comment:

The following observations are being made out during my studies.

(i) In Egyptian number system a number is expressed without using any symbol representing zero. Also in this system the same number may be expressed from right to left as well from left to right, which is not normally seen in other number systems. The symbols used here to represent numbers are very laborious to write.

(ii) In Babylonian and Mayan number systems, any number is expressed by using only two symbols (as in case of binary system?).One may take the opportunity for comparison of this system with that of well known binary system.

(iii) The process of expressing any number in all the above three systems is time consuming.

(iv) In decimal system we can easily find out the place value of any digit from a number, but in sexagesimal and vigesimal system it is quite difficult. This is the reason that the modern number system follows the decimal system.

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