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## INTERSECTION GRAPHS ON MINIMAL MAJORITY DOMINATING SETS OF A GRAPH

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#### Abstract

Let $G=(V, E)$ be a graph. The Minimal Majority Dominating Graph MMD(G) of a graph $G$ is the intersection graph defined on the family of all minimal majority dominating sets of vertices in $G$. The Common Minimal Majority Dominating Graph $\operatorname{CMMD}(G)$ of a graph $G$ is the graph having same vertex set as $G$ with two vertices adjacent in $C M M D(G)$ if and only if there exists a minimal majority dominating set in $G$ containing them.


## 1. INTRODUCTION

The graphs considered here are finite, undirected without loops or multiple edges. Any undefined term in this paper, may be found in Harary [1]. Suppose $G=(V, E)$ be a graph with p vertices and q edges. Let $S$ be a finite set and let $\mathrm{F}=\left\{\mathrm{S}_{1}, \mathrm{~S}_{2}, \ldots, \mathrm{~S}_{\mathrm{n}}\right\}$ be a partition of S . Then the intersection graph $\Omega(F)$ of F is the graph whose vertices are the subsets in F and in which two vertices $\mathrm{S}_{\mathrm{i}}$ and $\mathrm{S}_{\mathrm{j}}$ are adjacent if and only if $\mathrm{S}_{\mathrm{i}} \bigcap \mathrm{S}_{\mathrm{j}} \neq \phi$. Kulli and Janakiram [3] introduced a new class of intersection graphs in field of domination theory. The minimal dominating graph $\mathrm{MD}(\mathrm{G})$ of a graph $G$ is the intersection graph defined on the family of all minimal dominating sets of vertices in G . With the aim to extend this concept to majority dominating sets in a graph G, the Minimal Majority Dominating Graph MMD(G) is introduced in this article.

## 2. DEFINITION AND EXAMPLE

Definition 2.1: The Minimal Majority Dominating Graph $\operatorname{MMD}(\mathrm{G})$ of a graph $G$ is the intersection graph defined on the family of all minimal majority dominating sets of vertices in $G$. Each vertex of a $\operatorname{MMD}(\mathrm{G})$ is a minimal majority dominating set of G and two vertices of $\operatorname{MMD}(\mathrm{G})$ are adjacent if and only if there is a common element between the two minimal majority dominating sets of G.

Example 2.2: A graph $G$ and its $\operatorname{MMD}(\mathrm{G})$ are shown below.


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## 3. MAIN RESULTS ON MMD(G)

## Result 3.1:

1. For $\mathrm{G}=\mathrm{K}_{\mathrm{p}}, \mathrm{p} \geq 3$ and $\mathrm{G}=\mathrm{K}_{\mathrm{m}, \mathrm{n}}, \mathrm{m}=\mathrm{n}, \operatorname{MMD}(\mathrm{G})=\overline{K_{p}}$
2. If $G=W_{p}, p \geq 5, \operatorname{MMD}(G)$ is disconnected with atleast an isolate.
3. Suppose $\mathrm{G}=\mathrm{K}_{1, \mathrm{p}-1}, \mathrm{p} \geq 2$. $\mathrm{MMD}(\mathrm{G})$ is totally disconnected if $\mathrm{p} \leq 4$ and Disconnected with an isolate if $\mathrm{p}>4$.
4. Let $\mathrm{G}=\mathrm{D}_{\mathrm{r}, \mathrm{s}}$. Then $\operatorname{MMD}(\mathrm{G})$ is disconnected with isolates.
5. For $\mathrm{G}=\mathrm{P}_{\mathrm{p}}, \mathrm{p}>1 . \operatorname{MMD}(\mathrm{G})$ is totally disconnected if $\mathrm{p} \leq 6$ and connected if $\mathrm{p}>6$.

Theorem 3.2: For any graph $G$ with at least two vertices, $\operatorname{MMD}(G)$ is connected if and only if $\Delta(G)<\left\lceil\frac{p}{2}\right\rceil-1$.
Proof: Suppose MMD (G) is connected. Assume $\Delta(G) \geq\left\lceil\frac{p}{2}\right\rceil-1$. Let u be a vertex of degree $\left\lceil\frac{p}{2}\right\rceil-1$. Then $\{\mathrm{u}\}$ is a minimal majority dominating set of $G$ and $V-\{u\}$ also contains a minimal majority dominating set of $G$. This shows that $\operatorname{MMD}(\mathrm{G})$ has at least two components, a contradiction. Hence $\Delta(G)<\left\lceil\frac{p}{2}\right\rceil-1$.
Conversely, Let $\Delta(G)<\left\lceil\frac{p}{2}\right\rceil-1$. Then $\gamma_{M}(G) \geq 2$. Assume that $D_{1}$ and $D_{2}$ be any two disjoint minimal majority dominating sets of G. Then every minimal majority dominating set has at least two vertices of G. Suppose there exists two vertices $u \in D_{1}$ and $v \in D_{2}$ such that $u$ and $v$ are not adjacent. Then there exists a maximal independent set $D_{3}$ containing $u$ and $v$. Since $D_{3}$ is also a minimal majority dominating set, $D_{1}$ and $D_{2}$ are connected through $D_{3}$. Thus $\operatorname{MMD}(G)$ is connected. Suppose some vertex in $D_{1}$ is adjacent to some vertex in $D_{2}$. Let $u \in D_{1}$ and $v \in D_{2}$ such that $u$ and $v$ are adjacent. Then there exists a vertex $w \in D_{2}$ such that $u$ and $w$ are not adjacent. Then, there exists a maximal majority independent set $\mathrm{D}_{3}$ containing $\{\mathrm{u}, \mathrm{w}\}$. Since a maximal majority independent set is also a minimal majority dominating set, $D_{1}$ and $D_{2}$ are connected through the common vertices $u$ and $w$. Thus $\operatorname{MMD}(G)$ is connected. Suppose every vertex in $D_{1}$ is adjacent to every vertex in $D_{2}$. Since $\gamma_{M}(G) \geq 2$, Let $D_{1}=\left\{u_{1}, u_{2}, \ldots\right\}$ and $D_{2}=\left\{v_{1}, v_{2}, \ldots\right\}$ be such that every vertex in $D_{1}$ is adjacent to every vertex in $D_{2}$. Then there exists $w \in V-\left(D_{1} \cup D_{2}\right)$ such that $w$ is not adjacent to the vertices of $D_{1}$ and $D_{2}$. Choose two maximal majority independent sets $D_{3}$ and $D_{4}$ containing $\{u, w\}$ and $\{\mathrm{v}, \mathrm{w}\}$ respectively. Then $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$ are connected through the common vertex w . Thus the resulting graph MMD(G) is connected. Hence the proof.

Theorem 3.3: In a graph G, Every vertex is a majority dominating vertex if and only if $\operatorname{MMD}(\mathrm{G})$ is totally disconnected.

Proof: If every vertex $v$ in a graph $G$ is a majority dominating vertex then each vertex $v$ is a minimal majority dominating set of $G$. Let $F$ be the family of all minimal majority dominating sets of $G$. Then $|F|=\left|p_{M}\right|$. Since intersection of all vertices of $\operatorname{MMD}(\mathrm{G})$ is empty, there exists no edge among vertices of $\operatorname{MMD}(\mathrm{G})$. This implies that $\operatorname{MMD}(\mathrm{G})$ is totally disconnected. Conversely, Let $\operatorname{MMD}(\mathrm{G})$ be a totally disconnected graph. Therefore in MMD(G) each vertex is a minimal majority dominating set of $G$ and $d(v) \geq\left\lceil\frac{p}{2}\right\rceil-1 \forall \mathrm{v} \in \operatorname{MMD}(\mathrm{G})$.

Theorem 3.4: For any graph $G, \operatorname{MMD}(G)$ is disconnected with an isolate if and only if $G$ has a majority dominating vertex.

Proof: Suppose G has a majority dominating vertex v. Then $D=\{v\}$ is a minimal majority dominating set and there exists no minimal majority dominating set of G containing v . This shows that there exists no edge between D and any vertex of $\operatorname{MMD}(\mathrm{G})$. Therefore $\operatorname{MMD}(\mathrm{G})$ is disconnected with an isolate. Conversely, Suppose G has no majority dominating vertex $v$. Then $\gamma_{M}(G) \geq 2$ and By theorem [3-.1], $\operatorname{MMD}(G)$ is connected which is a contradiction. Therefore G has atleast one majority dominating vertex.

## 4. COMMON MINIMAL MAJORITY DOMINATING GRAPH OF A GRAPH

Definition 4.1: The Common Minimal Majority Dominating Graph CMMD(G) of a graph G is the graph having the same vertex set as $G$ with two vertices adjacent in $\operatorname{CMMD}(\mathrm{G})$ if and only if there exists a minimal majority dominating set in G containing them.

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Example 4.2: A graph $G$ and its $\mathrm{CMMD}(\mathrm{G})$ are given below.
G:

CMMD(G):


## Result 4.3:

1. For $G=K_{p}$, and $G=K_{m, n}, m=n, \operatorname{CMMD}(G)$ is totally disconnected.
2. If $\mathrm{G}=\mathrm{W}_{\mathrm{p}}, \mathrm{p} \geq 5, \mathrm{CMMD}(\mathrm{G})$ is disconnected with atleast an isolate.
3. Suppose $\mathrm{G}=\mathrm{K}_{1, \mathrm{p}-1}, \mathrm{p} \geq 2$. Then $\operatorname{CMMD}(\mathrm{G})=\mathrm{K}_{\mathrm{p}-1} \bigcup \mathrm{~K}_{1}$.
4. Let $\mathrm{G}=\mathrm{D}_{\mathrm{r}, \mathrm{s}}$. Then $\operatorname{CMMD}(\mathrm{G})=\mathrm{K}_{\mathrm{p}-2} \mathrm{U} 2 \mathrm{~K}_{1}$.

Theorem 4.4: For any graph $G$ with at least two vertices $\operatorname{CMMD}(G)$ is connected if and only if there is no majority dominating vertex in $G$.

Proof: Let CMMD(G) be connected. Suppose there exists atleast one majority dominating vertex vin G . Hence v is an isolate in $\mathrm{CMMD}(\mathrm{G})$, a contradiction. Therefore there is no majority dominating vertex. . Conversely suppose there is no majority dominating vertex. Then every minimal majority dominating set of $G$ contain atleast two vertices. Therefore there exists atleast one path between every pair of vertices of CMMD(G).

Proposition 4.5: If $\gamma_{M}(G)=1$ then $\operatorname{CMMD}(G)$ is disconnected with isolate. Proof: Suppose $\gamma_{M}(G)=1$. Then $\mathrm{D}=\{\mathrm{v}\}$ be a majority dominating set. Since $|\mathrm{D}|=1$, there exists no vertex in $\operatorname{CMMD}(\mathrm{G})$ adjacent to v . Therefore CMMD(G) is disconnected with isolate.

Proposition 4.6: If every vertex of $G$ is majority dominating vertex then $\mathrm{CMMD}(\mathrm{G})$ is totally disconnected.
Proof: Suppose every vertex of G is majority dominating vertex and CMMD $(G)$ is not totally disconnected. Then there exists a minimal majority dominating set containing two vertices which are a majority dominating set of G , a contradiction. Therefore CMMD(G) is totally disconnected.

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