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INTERSECTION GRAPHS ON MINIMAL MAJORITY DOMINATING SETS OF A GRAPH

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ABSTRACT

Let G = (V, E) be a graph. The Minimal Majority Dominating Graph MMD(G) of a graph G is the intersection graph defined on the family of all minimal majority dominating sets of vertices in G. The Common Minimal Majority Dominating Graph CMMD(G) of a graph G is the graph having same vertex set as G with two vertices adjacent in CMMD(G) if and only if there exists a minimal majority dominating set in G containing them.

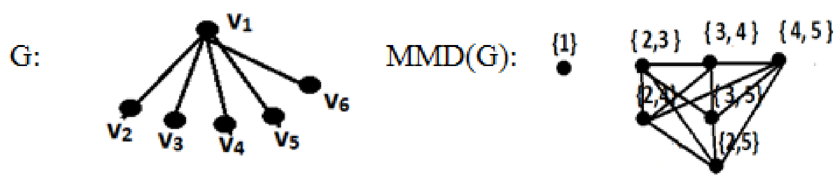
1. INTRODUCTION

The graphs considered here are finite, undirected without loops or multiple edges. Any undefined term in this paper, may be found in Harary [1]. Suppose G = (V, E) be a graph with p vertices and q edges. Let S be a finite set and let F = $\{S_1, S_2, \ldots, S_n\}$be a partition of S. Then the intersection graph $\Omega(F)$ of F is the graph whose vertices are the subsets in F and in which two vertices S_i and S_j are adjacent if and only if $S_i \cap S_j \neq \phi$. Kulli and Janakiram [3] introduced a new class of intersection graphs in field of domination theory. The minimal dominating graph MD(G) of a graph G is the intersection graph defined on the family of all minimal dominating sets of vertices in G. With the aim to extend this concept to majority dominating sets in a graph G, the Minimal Majority Dominating Graph MMD(G) is introduced in this article.

2. DEFINITION AND EXAMPLE

Definition 2.1: The Minimal Majority Dominating Graph MMD(G) of a graph G is the intersection graph defined on the family of all minimal majority dominating sets of vertices in G. Each vertex of a MMD(G) is a minimal majority dominating set of G and two vertices of MMD(G) are adjacent if and only if there is a common element between the two minimal majority dominating sets of G.

Example 2.2: A graph G and its MMD(G) are shown below.



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3. MAIN RESULTS ON MMD(G)

Result 3.1:

1. For G = K_p, p $\geq$ 3 and G = $K_{m,n}$, m = n, MMD(G) = $\overline{K_p}$
2. If G = W_p, p $\geq$5, MMD(G) is disconnected with atleast an isolate.
3. Suppose G = $K_{1,p-1}$, p$\geq$2. MMD(G) is totally disconnected if p $\leq$4 and Disconnected with an isolate if p > 4.
4. Let G = $D_{r,s}$. Then MMD(G) is disconnected with isolates.
5. For G = P_p, p > 1.MMD(G) is totally disconnected if p $\leq$ 6 and connected if p > 6.

Theorem 3.2: For any graph G with at least two vertices, MMD(G) is connected if and only if $\Delta(G) < \left\lceil \frac{p}{2} \right\rceil - 1.$

Proof: Suppose MMD (G) is connected. Assume $\Delta(G) \geq \left\lceil \frac{p}{2} \right\rceil - 1.$ Let u be a vertex of degree $\left\lceil \frac{p}{2} \right\rceil - 1.$ Then {u} is a minimal majority dominating set of G and V – {u} also contains a minimal majority dominating set of G. This shows that MMD(G) has at least two components, a contradiction. Hence $\Delta(G) < \left\lceil \frac{p}{2} \right\rceil - 1.$

Conversely, Let $\Delta(G) < \left\lceil \frac{p}{2} \right\rceil - 1.$ Then $\gamma_M(G) \geq 2$. Assume that D_1 and D_2 be any two disjoint minimal majority dominating sets of G. Then every minimal majority dominating set has at least two vertices of G. Suppose there exists two vertices u$\in D_1$ and v $\in D_2$ such that u and v are not adjacent. Then there exists a maximal independent set D_3 containing u and v. Since D_3 is also a minimal majority dominating set, D_1 and D_2 are connected through D_3. Thus MMD(G) is connected. Suppose some vertex in D_1 is adjacent to some vertex in D_2. Let u$\in D_1$ and v$\in D_2$ such that u and v are adjacent. Then there exists a vertex w$\in D_2$ such that u and w are not adjacent. Then, there exists a maximal majority independent set D_3 containing {u, w}. Since a maximal majority independent set is also a minimal majority dominating set, D_1 and D_2 are connected through the common vertices u and w. Thus MMD(G) is connected. Suppose every vertex in D_1 is adjacent to every vertex in D_2. Since $\gamma_M(G) \geq 2$, Let D_1 = {u_1, u_2, …} and D_2 = {v_1, v_2, …} be such that every vertex in D_1 is adjacent to every vertex in D_2. Then there exists w $\in$ V – ($D_1 \bigcup D_2$) such that w is not adjacent to the vertices of D_1 and D_2. Choose two maximal majority independent sets D_3 and D_4 containing {u, w}and {v, w} respectively. Then D_1 and D_2 are connected through the common vertex w. Thus the resulting graph MMD(G) is connected. Hence the proof.

Theorem 3.3: In a graph G, Every vertex is a majority dominating vertex if and only if MMD(G) is totally disconnected.

Proof: If every vertex v in a graph G is a majority dominating vertex then each vertex v is a minimal majority dominating set of G. Let F be the family of all minimal majority dominating sets of G. Then $|F| = |p_M|$. Since intersection of all vertices of MMD(G) is empty, there exists no edge among vertices of MMD(G). This implies that MMD(G) is totally disconnected. Conversely, Let MMD(G) be a totally disconnected graph. Therefore in MMD(G) each vertex is a minimal majority dominating set of G and d(v)$\geq \left\lceil \frac{p}{2} \right\rceil$-1 $\forall$ v$\in$MMD(G).

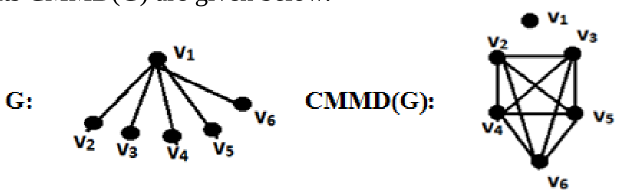
Theorem 3.4: For any graph G, MMD(G) is disconnected with an isolate if and only if G has a majority dominating vertex.

Proof: Suppose G has a majority dominating vertex v. Then D = {v} is a minimal majority dominating set and there exists no minimal majority dominating set of G containing v. This shows that there exists no edge between D and any vertex of MMD(G). Therefore MMD(G) is disconnected with an isolate. Conversely, Suppose G has no majority dominating vertex v. Then $\gamma_M(G) \geq 2$ and By theorem [3-.1], MMD(G) is connected which is a contradiction. Therefore G has atleast one majority dominating vertex.

4. COMMON MINIMAL MAJORITY DOMINATING GRAPH OF A GRAPH

Definition 4.1: The Common Minimal Majority Dominating Graph CMMD(G) of a graph G is the graph having the same vertex set as G with two vertices adjacent in CMMD(G) if and only if there exists a minimal majority dominating set in G containing them.

Example 4.2: A graph G and its CMMD(G) are given below.



Result 4.3:

1. For G = K_p, and G = $K_{m,n}$, m = n, CMMD(G) is totally disconnected.
2. If G = W_p, p ≥ 5, CMMD(G) is disconnected with atleast an isolate.
3. Suppose G = $K_{1,p-1}$, p $\geq$ 2. Then CMMD(G) = $K_{p-1} \cup K_1$.
4. Let G = $D_{r,s}$. Then CMMD(G) = K_{p-2} U $2K_1$.

Theorem 4.4: For any graph G with at least two vertices CMMD(G) is connected if and only if there is no majority dominating vertex in G.

Proof: Let CMMD(G) be connected. Suppose there exists atleast one majority dominating vertex v in G. Hence v is an isolate in CMMD(G), a contradiction. Therefore there is no majority dominating vertex. . Conversely suppose there is no majority dominating vertex. Then every minimal majority dominating set of G contain atleast two vertices. Therefore there exists atleast one path between every pair of vertices of CMMD(G).

Proposition 4.5: If $\gamma_M(G)$ = 1 then CMMD(G) is disconnected with isolate. **Proof:** Suppose $\gamma_M(G)$ =1. Then D = {v} be a majority dominating set. Since |D| = 1, there exists no vertex in CMMD(G) adjacent to v. Therefore CMMD(G) is disconnected with isolate.

Proposition 4.6: If every vertex of G is majority dominating vertex then CMMD(G) is totally disconnected.

Proof: Suppose every vertex of G is majority dominating vertex and CMMD(G) is not totally disconnected. Then there exists a minimal majority dominating set containing two vertices which are a majority dominating set of G, a contradiction. Therefore CMMD(G) is totally disconnected.

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