International Journal of Mathematical Archive-8(11), 2017, 188-195
Available online through www.ijma.info ISSN 2229 – 5046

SOME NEW CONTRA-CONTINUOUS FUNCTIONS

S. S. BENCHALLI[1], P. G. PATIL[1*], J. B. TORANAGATTI[2] AND S. R. VIGHNESHI[3]

[1]Department of Mathematics,
Karnatak University, Dharwad-580 003, Karnataka, India.

[2]Department of Mathematics,
Karnatak University's Karnatak College, Dharwad-580 001, Karnataka, India.

[3]Department of Mathematics,
R. L. S College, Dharwad-580 008, Karnataka, India.

(Received On: 23-06-17; Revised & Accepted On: 19-10-17)

ABSTRACT

In this paper, the notion of δgβ-closed sets in topological spaces is applied to study new class of functions called contra δgβ -continuous and almost contra δgβ-continuous functions as a new generalization of contra continuity and obtain their characterizations and properties.

***AMS Mathematics Subject Classification**: 54C08, 54C10.*

***Keywords**: δgβ - open, δgβ - closed, δgβ - connected, δgβ – continuous, contra δgβ - continuous.*

1. INTRODUCTION

In 1996, J. Dontchev [8] introduced the concept of contra continuous functions. Later, M. Caldas and S. Jafari [9] introduced and studied contra β-continuous functions and K. Amutha *et al.* [4] introduced and obtained the properties of contra gβ-continuous functions.

2. PRELIMINARIES

The following definitions, which are useful in the sequel are recalled.

Definition 2.1: A subset A of a topological space X is called

(i) β-closed [1] if int(cl(int(A)))⊆A.
(ii) b-closed [2] if cl(int(A))∩int(cl(A))⊆A.
(iii) regular-closed [18] if A=cl(int(A)).
(iv) α-closed [13] if cl(int(cl(A)))⊆A.
(v) semi-closed [11] if int(cl(A))⊆A.
(vi) δ-closed[20] if $A=cl_\delta(A)$ where $cl_\delta(A)=\{x\in X:int(cl(U))\cap A\neq\Phi, U\in\tau \text{ and } x\in U\}$.
(vii) δgβ-closed [7] if βcl(A)⊆G whenever A⊆G and G is δ-open in X.

The complements of the above mentioned closed sets are their respective open sets.

The β-closure of a subset A of X is the intersection of all β-closed sets containing A and is denoted by βcl(A) .

Definition 2.2: A function f: X→Y from a topological space X into a topological space Y is called contra continuous [8] (resp, contra β-continuous [9], contra gβ-continuous[4], contra gδs-continuous[6], contra δgb-continuous [5] and δgβ-continuous[7])if $f^{-1}(G)$ is closed (resp, β-closed, gβ-closed, gδs-closed, δgb-closed and δgβ -open) in X for every open set G of Y.

Corresponding Author: P. G. Patil[1]*
[1]Department of Mathematics, Karnatak University, Dharwad-580 003, Karnataka, India.

Definition 2.3[7] A topological space X is said to be
(1) $T_{\delta g\beta}$ -space if every δgβ-closed subset of X is closed.
(2) $\delta g\beta T_{1/2}$-space if every δgβ-closed subset of X is β-closed.

3. CONTRA δgβ-CONTINUOUS FUNCTIONS

Definition 3.1: A function f: X→Y is called contra δgβ-continuous if the inverse image of open set in Y is δgβ-closed in X.

Theorem 3.2: A function f: X→Y is contra δgβ-continuous if and only if $f^{-1}(G)$ is δgβ-open in X for every closed set G in Y.

Theorem 3.3:
(i) Every contra β-continuous function is contra δgβ-continuous function.
(ii) Every contra gβ-continuous function is contra δgβ-continuous function.
(iii) Every contra gδs-continuous function is contra δgβ-continuous function.
(iv) Every contra δgb-continuous function is contra δgβ-continuous function.

Proof: Follows from definitions. However, converse does not hold.

Example 3.4: Let X={a, b, c, d} and Y={a, b, c}. Let τ = {X, Φ, {a}, {b}, {a, b}, {a, c}, {a, b, c}} and σ ={X, Φ,{a},{b},{a, b}} be topologies on X and Y respectively. Let f : X→Y be a function defined by f(a) = a, f(b) = c and f(c) = d, f(d) = b. Then f is contra δgβ-continuous but not contra gβ-continuous.

Example 3.5: Let X = {a, b, c, d, e} and Y={a, b, c, d}. Let τ = {X, Φ, {a, b}, {c, d}, {a, b, c, d}, {a, b, c}} and σ ={X, Φ,{a},{b},{a, b},{a, b, c}} be topologies on X and Y respectively. Let f: X→Y be a function defined by f(a) = a = f(d),f(b) = b = f(e) and f(c) = c. Then f is contra δgβ-continuous but not contra gδs-continuous.

Example 3.6: Let X=Y={a, b, c}. Let τ = {X, Φ, {a}, {b},{a, b}} and σ = {X, Φ,{a}} be topologies on X and Y respectively. Let f: X→Y be a function defined by f(a) = a = f(b) and f(c) = c. Then f is contra δgβ-continuous but not contra δgb-continuous function.

Theorem 3.7: If f: X→Y is contra δgβ-continuous with X as $T_{\delta g\beta}$-space, then f is contra continuous.

Proof: Suppose X is $T_{\delta g\beta}$ -space and f is contra δgβ-continuous. Let V be an open set in Y, by hypothesis $f^{-1}(V)$ is δgβ-closed in X and hence $f^{-1}(V)$ is closed in X since X is $T_{\delta g\beta}$-space. Therefore, f is contra continuous.

Converse is obvious.

Theorem 3.8 If f: X→Y is contra δgβ-continuous with X as $\delta g\beta T_{1/2}$-space then f is contra β-continuous.

Proof: Suppose X is $\delta g\beta T_{1/2}$-space and f is contra δgβ-continuous. Let G be an open set in Y by hypothesis $f^{-1}(G)$ is δgβ-closed in X and hence $f^{-1}(G)$ is β-closed in X because X is $\delta g\beta T_{1/2}$-space. Therefore, f is contra β-continuous.

Converse is obvious.

Theorem 3.9: If f: X→Y is contra δgβ-continuous with X is semi-regular space, then f is contra gβ-continuous.

Proof: Follows from the fact that every open set is δ-open in semi-regular space.

Definition 3.10[10]: A space X is submaximal and extremally disconnected if every β-open set is open.

Theorem 3.11: If f: X→Y is contra δgβ-continuous with X is submaximal and extremely disconnected space, then f is contra gδs-continuous.

Theorem 3.12: If f: X→Y is contra δgβ-continuous with X is submaximal and extremely disconnected space, then f is contra δgb-continuous.

Definition 3.13: A space X is called locally δgβ-indiscrete if every δgβ-open set is closed in X.

Theorem 3.14: If f: X→Y is a contra δgβ-continuous and X is locally δgβ-indiscrete space, then f is continuous.

Proof: Let G be a closed set in Y. Since f is contra δgβ-continuous and X is locally δgβ-indiscrete space, then $f^{-1}(G)$ is a closed set in X. Hence f is continuous.

Definition 3.15[14]: A space X is called locally indiscrete if every open set is closed in X.

Theorem 3.16: If f: X→Y is a contra δgβ-continuous preclosed surjection and X is $T_{\delta g\beta}$-space then Y is locally indiscrete.

Proof: Let V be an open set in Y. Since f is contra δgβ-continuous and X is $T_{\delta g\beta}$-space then $f^{-1}(G)$ is closed in X. Since f is preclosed, then V is preclosed in Y. We have cl(V) = cl(int(V)) ⊆V and hence Y is indiscrete.

Theorem 3.17: If f is contra δgβ-continuous, then for each x∈X and each closed set F of Y containing f(x), there exists an δgβ-open set G in X containing x such that f(G)⊆F.

Proof: Let F be a closed set in Y containing f (x) then x∈ $f^{-1}(F)$. By hypothesis, $f^{-1}(F)$ is δgβ-open set in X containing x. Let G = $f^{-1(}F)$, then f(G) = $f(f^{-1}(F))$ ⊆ F.

Theorem 3.18: Suppose that δgβC(X) is closed under arbitrary intersections. Then the following are equivalent for a function f: X→Y:

(i) f is contra δgβ-continuous.

(ii) For each x∈X and each closed set B of Y containing f(x), there exists an δgβ-open set A in X containing x such that f(A) ⊆ B.

(iii) For each x∈X and each open set G of Y not containing f(x), there exists an δgβ-closed set H in X not containing x such that $f^{-1}(G)$⊆H.

Proof:

(i) → (ii): Let B be a closed set in Y containing f (x), then x∈ $f^{-1}(B)$. By (i), $f^{-1}(B)$ is δgβ-open set in X containing x. Let A = $f^{-1}(F)$, then f (A) = $f(f^{-1}(B))$ ⊆ B.

(ii) → (i): Let F be a closed set in Y containing f (x), then x∈ $f^{-1}(F)$. From (ii), there exists δgβ-open set G_x in X containing x such that $f(G_x)$⊆F which implies $G_x \subseteq f^{-1}(F)$. Thus $f^{-1}(F)=\cup\{G_x : x\in f^{-1}(F)\}$, which is δgβ -open. Hence $f^{-1}(F)$ is δgβ-open set in X.

(ii) → (iii): Obvious.

Theorem 3.19: If A⊆X is regular open, then it is β-closed.

Theorem 3.20[7]: If A⊆X is both δ-open and δgβ-closed then it is β-closed.

Theorem 3.21: A⊆X is semi-open if and only if cl(int(A)) = cl(A).

Lemma 3.22[12]: For a subset A of a space X, the following are equivalent:

(i) A is regular open.
(ii) A is α-open and β-closed.
(iii) A is open and semi-closed.
(iv) A is open and β-closed.
(v) A is pre-open and semi-closed.

Lemma 3.23: For a subset A of a space X, the following are equivalent:

(i) A is regular open.
(ii) A is δ-open and semi-closed.
(iii) A is δ-open and β-closed.

Lemma 3.24[4]: For a subset A of a space X, the following are equivalent:

(i) A is open and gβ-closed.
(ii) A is regular open.

Definition 3.25[3]: A function f: X→Y said to be completely-continuous if $f^{-1}(G)$ is regular-open in X for every open set G of Y.

Lemma 3.26: For a subset A of a space X, the following are equivalent:
- (i) A is regular open.
- (ii) A is open and gβ-closed.
- (iii) A is δ-open and β-closed.
- (iv) A is δ-open and gδs-closed.
- (v) A is δ-open and δgβ-closed.

Proof: Follows from Lemma 3.22, Lemma 3.23 and Lemma 3.24.

As a consequence of the above lemma 3.26, we have the following result:

Theorem 3.27: For a function f: X→Y, the following statements are equivalent:
- (i) f is completely continuous.
- (ii) f is contra β-continuous and α-continuous.
- (iii) f is contra gβ-continuous and continuous.
- (iv) f is contra δgβ-continuous and super-continuous.
- (v) f is contra gδs-continuous and super-continuous

Definition 3.28[19]: A set A⊆X is said to be Q-set if int(cl(A)) = cl(int(A)).

Definition 3.29 [19]: A function f : X→Y is Q-continuous if $f^{-1}(V)$ is Q-set in X for every open set V of Y.

Theorem 3.30: For a subset A of a space X, the following are equivalent:
- (i) A is clopen.
- (ii) A is α-open, Q-set and β-closed.
- (iii) A is open, Q-set and gβ-closed.
- (iv) A is δ-open, Q-set and δgβ-closed.

Theorem 3.31: The following statements are equivalent for a function f: X→Y:
- (i) f is perfectly continuous.
- (ii) f is δ-continuous, Q-continuous and contra δgβ-continuous.
- (iii) f is continuous, Q-continuous and contra β-continuous.

Recall that for a function f: X→Y, the subset {(x, f(x)):x∈X}⊆XxY is called the graph of f and is denoted by G(f).

Definition 3.32: The graph G(f) of a function f: X→Y is said to be contra δgβ-closed if for each (x, y) ∈(XxY)-G(f), there exists U∈ δgβO (X, x) and V∈C (Y, y) such that (UxV)∩G(f)=Φ.

Theorem 3.33: Let f: X→Y be a function and g: X→XxY the graph function of f, defined by g(x) = (x, f(x)) for each x∈X. If g is contra δgβ-continuous, then f is contra δgβ-continuous.

Proof: Let U be an open set in Y, then XxU is an open set in XxY. Since g is contra δgβ-continuous. It follows that $f^{-1}(U) = g^{-1}(XxU)$ is δgβ-closed in X. Hence f is contra δgβ-continuous.

Theorem 3.34: If A and B are δgβ-closed sets in submaximal and extremally disconnected space X, then A∪B is δgβ-closed in X.

Proof: Let A∪B⊆G where G is δ-open in X. Since A⊆G, B⊆G and A and B are δgβ-closed sets, then βcl(A)⊆G and βcl(B)⊆G. As X is submaximal and extremally disconnected, βcl(M)=cl(M) for any M⊆X.
Therefore, βcl(A∪B) = βcl(A)∪βcl(B)⊆G and hence A∪B is δgβ-closed.

Corollary 3.35: If A and B are δgβ-open sets in submaximal and extremally disconnected space X, then A∩B is δgβ-open in X.

Theorem 3.36 [7]: Let A be a subset of a space X. Then x∈ δgβcl(A) if and only if G∩A≠ Φ for every δgβ-open set G containing x.

Theorem 3.37: Suppose that δGβO(X) is a topology on X. If f: X→Y and g:X→Y are contra δgβ -continuous and Y is Urysohn, then K={x∈X: f(x)=g(x)} δgβ-closed in X.

Proof: Let x∈X-K. Then f(x)≠g(x). Since Y is Urysohn, there exist open sets U and V such that f(x) ∈U, g(x) ∈V and cl(U)∩cl(V)=Φ. Also f and g are contra δgβ-continuous, f^{-1}(cl(U)) and g^{-1}(cl(V)) are δgβ-open sets in X. Let C=f^{-1}(cl(U)) and D=g^{-1}(cl(V)). Then C and D are δgβ-open sets containing x. Set E=C∩D, then E is δgβ-open set in X. Hence f(E)∩g(E) = f(C∩D) ∩g(C∩D) ⊆f(C)∩g(D) = cl(U)∩cl(V)=Φ. Therefore, E∩K = Φ. By Theorem 3.36, x ∉ δgβcl(K). Hence K is δgβ-closed in X.

Definition 3.38: A space X is called δgβ-connected provided that X is not the union of two disjoint nonempty δgβ-open sets.

Theorem 3.39: If f is a contra δgβ-continuous function from a δgβ- connected space X onto any space Y, then Y is not a discrete space.

Proof: Since f is contra δgβ-continuous and X is δgβ-connected space. Suppose Y is a discrete space. Let V be a proper non empty open and closed subset of Y. Then f^{-1}(V) is proper nonempty δgβ-open and δgβ-closed subset of X, which contradicts the fact that X is δgβ-connected space. Hence Y is not a discrete space.

Theorem 3.40: If a surjective function f: X→Y is contra δgβ-continuous with X is δgβ-connected space, then Y is connected.

Proof: Suppose Y is a not connected space. Then there exist disjoint open sets U and V in Y such that Y = U∪V. Therefore, U and V are clopen in Y. Since f is contra δgβ-continuous, f^{-1}(U) and f^{-1}(V) are δgβ-open sets in X. Further f is surjective implies f^{-1}(U) and f^{-1}(V) are nonempty disjoint and X = f^{-1}(U)∪f^{-1}(V). This contradicts the fact that X is δgβ-connected space. Therefore, Y is connected.

Theorem 3.41: If f: X→Y is contra δgβ-continuous, X is δgβ-connected and Y is T_1 -space, then f is constant.

Proof: Since Y is T_1-space, U ={f^{-1}(y): y∈Y} is a disjoint δgβ-open partition of X. If |U|≥2, then X is the union of two nonempty δgβ-open sets. This is contradiction to the fact that X is δgβ-connected. Therefore |U|=1 and hence f is constant.

Definition 3.42: A topological space X is said to be δgβ-T_2 space if for any pair of distinct points x and y, there exist disjoint δgβ-open sets G and H such that x∈G and y∈H.

Theorem 3.43: Let f:X→Y be contra δgβ-continuous injective function from a space X into Urysohn space Y, then X is δgβ-T_2.

Proof: Let x and y be any distinct points in X, then f(x)≠f(y), there exist open sets V and W in Y containing f(x) and f(y) respectively, such that cl(V)∩cl(W) = Φ. Since f is contra δgβ-continuous, then there exist δgβ-open sets M and N in X such that f(M)⊆cl(V) and f(N)⊆cl(W) we have M∩N=Φ. Hence X is δgβ-T_2

Remark 3.44: The composition of two contra δgβ-continuous functions need not be contra δgβ- continuous as seen from the following examples.

Example 3.45: Let X = Y = Z ={a, b, c}, τ={X, Φ,{a},{b},{a, b}}, σ={Y,Φ, {a}} and η={Z,Φ,{b, c}} be topologies on X,Y and Z respectively. Define a function f: X→Y as f(a) = a, f(b) = b and f(c) = c and a function g: Y→Z as g(a) = b, g(b) = c and g(c) = a. Then f and g are contra δgβ-continuous but g∘f: X→Z is not contra δgβ-continuous, since there exists a open set {b, c} in Z such that $(g\circ f)^{-1}$[{b, c}]={a, b} is not δgβ-closed in X.

Theorem 3.46: Let f: X→Y and g: Y→Z be any two functions.

(i) If f is contra δgβ-continuous and g is continuous then g∘f is contra δgβ-continuous.
(ii) If f is contra δgβ-continuous and g is contra continuous then g∘f is δgβ-continuous.
(iii) If f is δgβ-continuous and g is contra continuous then g∘f is contra δgβ-continuous.
(iv) If f is δgβ-irresolute and g is contra δgβ-continuous then g∘f is contra δgβ-continuous.

Proof: (i) Let h = g∘f and V be an open set in Z.

Since g is continuous, $g^{-1}(V)$ is open in Y. Therefore $f^{-1}[g^{-1}(V)] = h^{-1}(V)$ is δgβ-closed in X because f is contra δgβ-continuous .Hence g∘f is contra δgβ-continuous.

The proofs of (ii), (iii) and (iv) are analogous to (i) with the obvious changes.

Theorem 3.47: Let f: X→Y be contra δgβ-continuous and g: Y→Z be δgβ-continuous. If Y is $T_{\delta g\beta}$-space, then g∘f: X→Z is contra δgβ-continuous.

Proof: Let V be any open set in Z. Since g is δgβ-continuous, $g^{-1}(V)$ is δgβ-open in Y and since Y is $T_{\delta g\beta}$-space, $g^{-1}(V)$ open in Y. Since f is contra δgβ-continuous $f^{-1}[g^{-1}(V)] = (g\circ f)^{-1}(V)$ is δgβ-closed set in X. Therefore, g∘f is contra δgβ-continuous.

4. ALMOST CONTRA δgβ-CONTINUOUS FUNCTIONS

In this section, almost contra delta generalized β-continuous functions are introduced and studied.

Definition 4.1: A function f: X→Y is called almost contra delta generalized β-continuous if $f^{-1}(G)$ is δgβ-closed in X for every regular open set G in Y

Theorem 4.2: A function f: X→Y is almost contra δgβ-continuous if and only if for every regular closed set F of Y, $f^{-1}(V)$ is δgβ-open set of X.

Theorem 4.3: Every contra δgβ-continuous function is almost contra δgβ-continuous.

Proof: Follows from the fact that every regular-open set is open.

The converse of the Theorem 4.3 need to be true in general as seen from the following example.

Example 4.4: Let X = Y = {a, b, c} with τ = {X, Φ, {a}} and σ = {X,Φ,{a},{b},{a, b}}be topologies on X and Y respectively. Let f: X→Y be a function defined by f(a) = a, f(b) = b and f(c) = c. Then f is almost contra δgβ-continuous function but not contra δgβ-continuous, because for the open set {b} in Y and $f^{-1}(\{b\})$ ={a} is not δgβ-closed in X.

Theorem 4.5: The following are equivalent for a function f: X→Y:

(i) f is almost contra δgβ-continuous.
(ii) $f^{-1}(cl(G))$ is δgβ-open set in X for every β-open subset G of Y.
(iii) $f^{-1}(cl(G))$ is δgβ-open set in X for every semi-open subset G of Y.
(iv) $f^{-1}(int(cl(G)))$ is δgβ-closed set in X for every pre-open subset G of Y.

Proof:
(i) → (ii): Let G be β-open set of Y. It follows from Theorem 2.4 of [3] that cl(G) is regular closed. Then $f^{-1}(cl(G))$ is δgβ-open set in X.

(ii) → (iii): Obvious.

(iii)→ (iv): Let G be a pre-open set of Y. Then Y-int(cl(G)) is regular closed and hence it is semi-open. Then, we have $f^{-1}(cl(Y\text{-}int(cl(G))) = f^{-1}(Y\text{-}int(cl(G)) = X\text{-}f^{-1}(int(Cl(G)))$ is δgβ-open set in X .
Hence $f^{-1}(int(cl(G)))$ is δgβ-closed set in X.

(iv)→ (v): Let G be regular-open set of Y. Then G is pre-open in X and hence $f^{-1}(G) = f^{-1}(int(Cl(G)))$ is δgβ-closed set in X.

Theorem 4.6[16]: For a subset A of a space X, the following properties hold:

(i) αcl(A) = cl(A) for every β-open subset A of X.
(ii) pcl(A) = cl(A) for every semi-open subset A of X.
(iii) scl(A) = int(cl(A)) for every pre-open subset A of X.

Theorem 4.7: The following are equivalent for a function f: X→Y:

(i) f is almost contra δgβ-continuous.
(ii) for every β-open subset G of Y, $f^{-1}(\alpha cl(G))$ is δgβ-open set in X.
(iii) for every semi-open subset G of Y, $f^{-1}(pcl(G))$ is δgβ-open set in X.
(iv) for every pre-open subset G of Y, $f^{-1}(scl(G)))$ is δgβ-closed set in X.

Definition 4.8 [16]: A function f: X→Y is said to be R-map if $f^{-1}(V)$ is regular open in X for each regular open set V of Y.

Definition 4.9[15]: A function f: X→Y is said to be perfectly continuous if $f^{-1}(V)$ is clopen in X for each regular open set V of Y.

Theorem 4.10: For two functions f: X→Y and g: Y→Z, let g∘f: Y→Z is composition function. Then the following properties hold:

(i) If f is almost contra δgβ-continuous and g is an R-map, then g∘f is almost contra δgβ-continuous.
(ii) If f is almost contra δgβ-continuous and g is perfectly continuous, then g∘f is contra δgβ-continuous.
(iii) If f is contra δgβ-continuous and g is almost continuous, then g∘f is almost contra δgβ-continuous.

Proof: (i) Let V be any regular open set in Z. Since g is an R-map, $g^{-1}(V)$ is regular open in Y. Since f is almost contra δgβ-continuous, $f^{-1}[g^{-1}(V)] = (g\circ f)^{-1}(V)$ is δgβ-closed set in X. Therefore, g∘f is almost contra δgβ-continuous.

Proofs of (ii) and (iii) are similar to (i).

Theorem 4.11: Let f: X→Y be a contra δgβ-continuous and g: Y→Z be δgβ-continuous. If Y is $T_{\delta g\beta}$-space, then g∘f: X→Z is almost contra δgβ-continuous.

Proof: Let V be any regular open and hence open set in Z. Since g is δgβ-continuous $g^{-1}(V)$ is δgβ-open in Y. Since f is contra δgβ-continuous, $f^{-1}[g^{-1}(V)] = (g\circ f)^{-1}(V)$ is δgβ-closed set in X. Therefore, g∘f is almost contra δgβ-continuous.

Definition 4.12: A space X is called locally δgβ-indiscrete if every δgβ-open set is closed in X.

Theorem 4.13: If f: X→Y is almost contra δgβ-continuous and X is locally δgβ-indiscrete space then f is almost continuous.

Proof: Let U be any regular open set of Y. Since f is almost contra δgβ-continuous $f^{-1}(U)$ is δgβ-closed set in X. As X is locally δgβ-indiscrete space, $f^{-1}(U)$ is an open set in X. Therefore, f is almost continuous.

Theorem 4.14[7]: The intersection of a δgβ-closed set and a δ-closed set of X is always δgβ-closed.

Theorem 4.15: If f: X→Y is almost contra δgβ-continuous, and A is δ-closed in X then the restriction (f/A): A→Y X is almost contra δgβ- continuous.

Proof: Let V be any regular open set of Y. Then $f^{-1}(V)$ is δgβ-closed set in X. By Theorem 4.14, $(f/A)^{-1}(V)=A\cap f^{-1}(V)$ is δgβ-closed it follows that (f/A) is almost contra δgβ-continuous.

ACKNOWLEDGEMENT

The authors are grateful to the University Grants Commission, New Delhi, India for financial support under UGC SAP DRS-III, F-510/3/DRS-III/2016(SAP-I) dated 29th Feb 2016 to the Department of Mathematics, Karnatak University, Dharwad, India.

REFERENCES

1. Abd. El-Monsef M.E., El-Deeb S.N. and Mahmoud. R.A., β-open sets and β-continuous mappings, Bull. Fac. Sci. Assiut Univ., 12(1983)77-90.
2. D. Andrijivic, On b-open sets, Mat. Vesnik, 48(1996), 59-64.
3. S.P.Arya and R. Gupta, On strongly continuous mappings, Kyungpook Math.Jl., 14(1974), 131-143.
4. K. Amutha, K. M. Dharmalingam and O. Ravi, On contra gβ-continuous functions, Int. Jl. Curr. Res. Sci. and Tech., 3(2017), 1-11.
5. S. S. Benchalli, P.G. Patil, J.B. Toranagatti and S.R. Vighneshi, Contra δgb-continuous functions in topological spaces, Eur. Jl. Pure Appl. Math, 10(2017), 312-322.
6. S.S. Benchalli, Umadevi Neeli and G. P. Siddapur, Contra gδs-continuous functions in topological spaces, Int. Jl. Appl. Math, 25(2012), 457-471.
7. S.S. Benchalli, P.G. Patil, J.B. Toranagatti and S.R. Vighneshi, A new class of generalized closed sets in topological spaces, Global Jl. of Pure and Applied Mathematics, 13(2017),331-345
8. J. Dontchev, Contra continuous functions and strongly S-closed mappings, Int. Jl. Math. Sci., 19(1996), 303-310.
9. M. Caldas and S. Jafari, Some properties of contra-β-continuous functions, Mem. Fac. Sci. Kochi Univ., 22(2001)19-28.

10. M. Ganster and D. Andrijevic, On some questions concerning semi-preopen sets, Jl. Inst. Math. Compu. Sci. Math., 1(1988), 65-75.
11. N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.
12. Mohammad S. Sarsak, On b-open sets and associated generalized open sets, Que. and Ans. in General Topology, 27(2009), 157-173.
13. O. Njastad, On some classes of nearly open sets, Pacific. Jl. Math., 15(1965), 961-970.
14. T. Nieminen, On ultra-pseudo compact and related spaces, Ann. Acad. Sci. Fenn. Ser. A.I. Math. 3(1977), 185-205.
15. T. Noiri, Super-continuity and some strong forms of continuity, Indian Jl. Pure Appl. Math., 15(1984), 241-250.
16. T. Noiri, Almost continuous functions, Indian. Jl. Pure Appl. Math., 20(1989), 571-576.
17. R. Staum, The algebra of bounded continuous functions into a non-Archimedean field, Pacific Jl. Math., 50(1974), 169-185.
18. M. Stone, Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc., 41(1937), 371-381.
19. K. V. Tamilselvi, P. Thangaraj and O. Ravi, On contra gγ-continuous functions, Int. Jl. on Recent and Innovation Trends in Computing and Communication, 4(2016),260-269.
20. N.V. Velicko, H-closed topological spaces, Amer.Math.Soc.Transl., 78(1968), 103-118.

Source of support: University Grants Commission, New Delhi, India.
Conflict of interest: None Declared.