SOME NEW CONTRA-CONTINUOUS FUNCTIONS

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ABSTRACT

In this paper, the notion of δgβ-closed sets in topological spaces is applied to study new class of functions called contra δgβ-continuous and almost contra δgβ-continuous functions as a new generalization of contra continuity and obtain their characterizations and properties.

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1. INTRODUCTION


2. PRELIMINARIES

The following definitions, which are useful in the sequel are recalled.

Definition 2.1: A subset A of a topological space X is called
(i) β-closed [1] if int(cl(int(A)))⊆A.
(ii) b-closed [2] if cl(int(A))∩int(cl(A))⊆A.
(iii) regular-closed [18] if A=cl(int(A)).
(iv) α-closed [13] if cl(int(cl(A)))⊆A.
(v) semi-closed [11] if int(cl(A))⊆A.
(vi) δ-closed [20] if A=cl(A) where cl(A)={x∈X:int(cl(U))∩A≠Ø, U∈τ and x∈U}.
(vii) δgβ-closed [7] if βcl(A)⊆G whenever A⊆G and G is δ-open in X.

The complements of the above mentioned closed sets are their respective open sets.

The β-closure of a subset A of X is the intersection of all β-closed sets containing A and is denoted by βcl(A).

Definition 2.2: A function f: X→Y from a topological space X into a topological space Y is called contra continuous [8] (resp, contra β-continuous [9], contra gβ-continuous[4], contra gδs-continuous[6], contra δgb-continuous [5] and δgb-continuous[7]) if f⁻¹(G) is closed (resp, β-closed, gβ-closed, gδs-closed, δgb-closed and δgb-open) in X for every open set G of Y.

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Definition 2.3[7] A topological space $X$ is said to be

(1) $T_{\delta g^\beta}$-space if every $\delta g^\beta$-closed subset of $X$ is closed.

(2) $\delta g^\beta T_{1/2}$-space if every $\delta g^\beta$-closed subset of $X$ is $\beta$-closed.

3. CONTRA $\delta g^\beta$-CONTINUOUS FUNCTIONS

Definition 3.1: A function $f: X \rightarrow Y$ is called contra $\delta g^\beta$-continuous if the inverse image of open set in $Y$ is $\delta g^\beta$-closed in $X$.

Theorem 3.2: A function $f: X \rightarrow Y$ is contra $\delta g^\beta$-continuous if and only if $f^{-1}(G)$ is $\delta g^\beta$-open in $X$ for every closed set $G$ in $Y$.

Theorem 3.3:
(i) Every contra $\beta$-continuous function is contra $\delta g^\beta$-continuous function.
(ii) Every contra $g^\beta$-continuous function is contra $\delta g^\beta$-continuous function.
(iii) Every contra $g^\delta s$-continuous function is contra $\delta g^\beta$-continuous function.
(iv) Every contra $g^\delta b$-continuous function is contra $\delta g^\beta$-continuous function.

Proof: Follows from definitions. However, converse does not hold.

Example 3.4: Let $X = \{a, b, c, d\}$ and $Y = \{a, b, c\}$. Let $\tau = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$ and $\sigma = \{X, \Phi, \{a\}, \{b\}, \{a, b, c\}\}$ be topologies on $X$ and $Y$ respectively. Let $f: X \rightarrow Y$ be a function defined by $f(a) = a$, $f(b) = c$ and $f(c) = d$, $f(d) = b$. Then $f$ is contra $\delta g^\beta$-continuous but not contra $g^\beta$-continuous.

Example 3.5: Let $X = \{a, b, c, d, e\}$ and $Y = \{a, b, c, d\}$. Let $\tau = \{X, \Phi, \{a\}, \{b\}, \{a, b, c\}\}$ and $\sigma = \{X, \Phi, \{a\}, \{b\}, \{a, b, c\}\}$ be topologies on $X$ and $Y$ respectively. Let $f: X \rightarrow Y$ be a function defined by $f(a) = a = f(d), f(b) = b = f(c)$. Then $f$ is contra $\delta g^\beta$-continuous but not contra $g^s$-continuous.

Example 3.6: Let $X = Y = \{a, b, c\}$. Let $\tau = \{X, \Phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{X, \Phi, \{a\}\}$ be topologies on $X$ and $Y$ respectively. Let $f: X \rightarrow Y$ be a function defined by $f(a) = a = f(b)$ and $f(c) = c$. Then $f$ is contra $\delta g^\beta$-continuous but not contra $g^\delta b$-continuous.

Theorem 3.7: If $f: X \rightarrow Y$ is contra $\delta g^\beta$-continuous with $X$ as $T_{\delta g^\beta}$-space, then $f$ is contra continuous.

Proof: Suppose $X$ is $T_{\delta g^\beta}$-space and $f$ is contra $\delta g^\beta$-continuous. Let $V$ be an open set in $Y$, by hypothesis $f^{-1}(V)$ is $\delta g^\beta$-closed in $X$ and hence $f^{-1}(V)$ is closed in $X$ since $X$ is $T_{\delta g^\beta}$-space. Therefore, $f$ is contra continuous.

Converse is obvious.

Theorem 3.8: If $f: X \rightarrow Y$ is contra $\delta g^\beta$-continuous with $X$ as $\delta g^\beta T_{1/2}$-space then $f$ is contra $\beta$-continuous.

Proof: Suppose $X$ is $\delta g^\beta T_{1/2}$-space and $f$ is contra $\delta g^\beta$-continuous. Let $G$ be an open set in $Y$ by hypothesis $f^{-1}(G)$ is $\delta g^\beta$-closed in $X$ and hence $f^{-1}(G)$ is $\beta$-closed in $X$ because $X$ is $\delta g^\beta T_{1/2}$-space. Therefore, $f$ is contra $\beta$-continuous.

Converse is obvious.

Theorem 3.9: If $f: X \rightarrow Y$ is contra $\delta g^\beta$-continuous with $X$ as semi-regular space, then $f$ is contra $g^\beta$-continuous.

Proof: Follows from the fact that every open set is $\delta$-open in semi-regular space.

Definition 3.10[10]: A space $X$ is submaximal and extremally disconnected if every $\beta$-open set is open.

Theorem 3.11: If $f: X \rightarrow Y$ is contra $\delta g^\beta$-continuous with $X$ as submaximal and extremely disconnected space, then $f$ is contra $g^\delta s$-continuous.

Theorem 3.12: If $f: X \rightarrow Y$ is contra $\delta g^\beta$-continuous with $X$ as submaximal and extremely disconnected space, then $f$ is contra $g^\delta b$-continuous.

Definition 3.13: A space is called locally $\delta g^\beta$-indiscrete if every $\delta g^\beta$-open set is closed in $X$.

Theorem 3.14: If $f: X \rightarrow Y$ is a contra $\delta g^\beta$-continuous and $X$ is locally $\delta g^\beta$-indiscrete space, then $f$ is continuous.
Proof: Let G be a closed set in Y. Since f is contra $\delta\beta$-continuous and X is locally $\delta\beta$-indiscrete space, then $f^{-1}(G)$ is a closed set in X. Hence f is continuous.

Definition 3.15\cite{14}: A space X is called locally indiscrete if every open set is closed in X.

Theorem 3.16: If f: X→Y is a contra $\delta\beta$-continuous preclosed surjection and X is $T_{\delta\beta}$-space then Y is locally indiscrete.

Proof: Let V be an open set in Y. Since f is contra $\delta\beta$-continuous and X is $T_{\delta\beta}$-space then $f^{-1}(G)$ is closed in X. Since f is preclosed, then V is preclosed in Y. We have cl(V) = cl(int(V)) $\subseteq$ V and hence Y is indiscrete.

Theorem 3.17: If f is contra $\delta\beta$-continuous, then for each $x \in X$ and each closed set F of Y containing f(x), there exists an $\delta\beta$-open set G in X containing x such that $f(G) \subseteq F$.

Proof: Let F be a closed set in Y containing f(x) then $x \in f^{-1}(F)$. By hypothesis, $f^{-1}(F)$ is $\delta\beta$-open set in X containing x. Let $G = f^{-1}(F)$, then $f(G) = f(f^{-1}(F)) \subseteq F$.

Theorem 3.18: Suppose that $\delta\beta\gamma(C(X))$ is closed under arbitrary intersections. Then the following are equivalent for a function f: X→Y:

(i) f is contra $\delta\beta$-continuous.

(ii) For each $x \in X$ and each closed set B of Y containing f(x), there exists an $\delta\beta$-open set A in X containing x such that $f(A) \subseteq B$.

(iii) For each $x \in X$ and each open set G of Y not containing f(x), there exists an $\delta\beta$-closed set H in X not containing x such that $f^{-1}(G) \subseteq H$.

Proof:

(i) → (ii): Let B be a closed set in Y containing f(x), then $x \in f^{-1}(B)$. By (i), $f^{-1}(B)$ is $\delta\beta$-open set in X containing x. Let $A = f^{-1}(B)$, then $f(A) = f(f^{-1}(B)) \subseteq B$.

(ii) → (i): Let F be a closed set in Y containing f(x), then $x \in f^{-1}(F)$. From (ii), there exists $\delta\beta$-open set $G_x$ in X containing x such that $f(G_x) \subseteq F$ which implies $G_x \subseteq f^{-1}(F)$. Thus $f^{-1}(F) = \cup \{G_x: x \in f^{-1}(F)\}$, which is $\delta\beta$-open. Hence $f^{-1}(F)$ is $\delta\beta$-open set in X.

(ii) → (iii): Obvious.

Theorem 3.19: If $A \subseteq X$ is regular open, then it is $\beta$-closed.

Theorem 3.20\cite{7}: If $A \subseteq X$ is both $\delta$-open and $\delta\beta$-closed then it is $\beta$-closed.

Theorem 3.21: $A \subseteq X$ is semi-open if and only if cl(int(A)) = cl(A).

Lemma 3.22\cite{12}: For a subset A of a space X, the following are equivalent:

(i) A is regular open.

(ii) A is $\alpha$-open and $\beta$-closed.

(iii) A is open and semi-closed.

(iv) A is open and $\beta$-closed.

(v) A is pre-open and semi-closed.

Lemma 3.23: For a subset A of a space X, the following are equivalent:

(i) A is regular open.

(ii) A is $\delta$-open and semi-closed.

(iii) A is $\delta$-open and $\beta$-closed.

Lemma 3.24\cite{4}: For a subset A of a space X, the following are equivalent:

(i) A is open and $g\beta$-closed.

(ii) A is regular open.
Definition 3.25[3]: A function $f: X \rightarrow Y$ said to be completely-continuous if $f^{-1}(G)$ is regular-open in $X$ for every open set $G$ of $Y$.

Lemma 3.26: For a subset $A$ of a space $X$, the following are equivalent:
   (i) $A$ is regular open.
   (ii) $A$ is open and $g\beta$-closed.  
   (iii) $A$ is $\delta$-open and $\beta$-closed.  
   (iv) $A$ is $\delta$-open and $g\delta\beta$-closed.  
   (v) $A$ is $\delta$-open and $\delta g\beta$-closed.


As a consequence of the above lemma 3.26, we have the following result:

Theorem 3.27: For a function $f: X \rightarrow Y$, the following statements are equivalent:
   (i) $f$ is completely continuous.
   (ii) $f$ is contra $\beta$-continuous and $\alpha$-continuous.
   (iii) $f$ is contra $g\beta$-continuous and continuous.
   (iv) $f$ is contra $\delta g\beta$-continuous and super-continuous.
   (v) $f$ is contra $g\delta\beta$-continuous and super-continuous.

Definition 3.28[19]: A set $A \subseteq X$ is said to be $Q$-set if $\text{int}(\text{cl}(A)) = \text{cl}(\text{int}(A))$.

Definition 3.29 [19]: A function $f : X \rightarrow Y$ is $Q$-continuous if $f^{-1}(V)$ is $Q$-set in $X$ for every open set $V$ of $Y$.

Theorem 3.30: For a subset $A$ of a space $X$, the following are equivalent:
   (i) $A$ is clopen.
   (ii) $A$ is $\alpha$-open, $Q$-set and $\beta$-closed.
   (iii) $A$ is open, $Q$-set and $g\beta$-closed.
   (iv) $A$ is $\delta$-open, $Q$-set and $\delta g\beta$-closed.

Theorem 3.31: The following statements are equivalent for a function $f: X \rightarrow Y$:
   (i) $f$ is perfectly continuous.
   (ii) $f$ is $\delta$-continuous, $Q$-continuous and contra $\delta g\beta$-continuous.
   (iii) $f$ is continuous, $Q$-continuous and contra $\beta$-continuous.

Recall that for a function $f: X \rightarrow Y$, the subset $\{(x, f(x)): x \in X\} \subseteq X \times Y$ is called the graph of $f$ and is denoted by $G(f)$.

Definition 3.32: The graph $G(f)$ of a function $f: X \rightarrow Y$ is said to be contra $\delta g\beta$-closed if for each $(x, y) \in (X \times Y) - G(f)$, there exists $U \in \delta g\beta O(X, x)$ and $V \in C(Y, y)$ such that $(U \times V) \cap G(f) = \emptyset$.

Theorem 3.33: Let $f : X \rightarrow Y$ be a function and $g : X \rightarrow X \times Y$ the graph function of $f$, defined by $g(x) = (x, f(x))$ for each $x \in X$. If $g$ is contra $\delta g\beta$-continuous, then $f$ is contra $\delta g\beta$-continuous.

Proof: Let $U$ be an open set in $Y$, then $X \times U$ is an open set in $X \times Y$. Since $g$ is contra $\delta g\beta$-continuous. It follows that $f^{-1}(U) = g^{-1}(X \times U)$ is $\delta g\beta$-closed in $X$. Hence $f$ is contra $\delta g\beta$-continuous.

Theorem 3.34: If $A$ and $B$ are $\delta g\beta$-closed sets in submaximal and extremally disconnected space $X$, then $A \cup B$ is $\delta g\beta$-closed in $X$.

Proof: Let $A \cup B \subseteq G$ where $G$ is $\delta$-open in $X$. Since $A \subseteq G$, $B \subseteq G$ and $A$ and $B$ are $\delta g\beta$-closed sets, then $\beta \text{cl}(A) \subseteq G$ and $\beta \text{cl}(B) \subseteq G$. As $X$ is submaximal and extremally disconnected, $\beta \text{cl}(M) = \text{cl}(M)$ for any $M \subseteq X$. Therefore, $\beta \text{cl}(A \cup B) = \beta \text{cl}(A) \cup \beta \text{cl}(B) \subseteq G$ and hence $A \cup B$ is $\delta g\beta$-closed.

Corollary 3.35: If $A$ and $B$ are $\delta g\beta$-open sets in submaximal and extremally disconnected space $X$, then $A \cap B$ is $\delta g\beta$-open in $X$.

Theorem 3.36 [7]: Let $A$ be a subset of a space $X$. Then $x \in \delta g\beta \text{cl}(A)$ if and only if $G \cap A \neq \emptyset$ for every $\delta g\beta$-open set $G$ containing $x$. 

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Theorem 3.37: Suppose that \( \delta g|O(X) \) is a topology on \( X \). If \( f: X \to Y \) and \( g: X \to Y \) are contra \( \delta g \)-continuous and \( Y \) is Urysohn, then \( K = \{ x \in X : f(x) = g(x) \} \) is \( \delta g \)-closed in \( X \).

Proof: Let \( x \in X - K \). Then \( f(x) \neq g(x) \). Since \( Y \) is Urysohn, there exist open sets \( U \) and \( V \) such that \( f(x) \in U \), \( g(x) \in V \) and \( cl(U) \cap cl(V) = \emptyset \). Also \( f \) and \( g \) are contra \( \delta g \)-continuous, \( f'^{(cl(U))} \) and \( g'^{(cl(V))} \) are \( \delta g \)-open sets in \( X \). Let \( C = f'^{(cl(U))} \) and \( D = g'^{(cl(V))} \). Then \( C \) and \( D \) are \( \delta g \)-open sets containing \( x \). Set \( E = C \cap D \), then \( E \) is \( \delta g \)-open set in \( X \). Hence \( f(E) \cap g(E) = f(C \cap D) \cap g(C \cap D) \subseteq f(C) \cap g(D) = cl(U) \cap cl(V) = \emptyset \). Therefore, \( E \cap K = \emptyset \). By Theorem 3.36, \( x \notin \delta g|cl(K) \). Hence \( K \) is \( \delta g \)-closed in \( X \).

Definition 3.38: A space \( X \) is called \( \delta g \)-connected provided that \( X \) is not the union of two disjoint nonempty \( \delta g \)-open sets.

Theorem 3.39: If \( \delta f \) is a contra \( \delta g \)-continuous function from a \( \delta g \)-connected space \( X \) onto any space \( Y \), then \( Y \) is not a discrete space.

Proof: Since \( f \) is contra \( \delta g \)-continuous and \( X \) is \( \delta g \)-connected space. Suppose \( Y \) is a discrete space. Let \( V \) be a proper non empty open and closed subset of \( Y \). Then \( f'^{(V)} \) is nonempty \( \delta g \)-open and \( \delta g \)-closed subset of \( X \), which contradicts the fact that \( X \) is \( \delta g \)-connected space. Hence \( Y \) is not a discrete space.

Theorem 3.40: If a surjective function \( f: X \to Y \) is contra \( \delta g \)-continuous with \( X \) is \( \delta g \)-connected space, then \( Y \) is connected.

Proof: Suppose \( Y \) is a not connected space. Then there exist disjoint open sets \( U \) and \( V \) in \( Y \) such that \( Y = U \cup V \). Therefore, \( U \) and \( V \) are clopen in \( Y \). Since \( f \) is contra \( \delta g \)-continuous, \( f'^{(U)} \) and \( f'^{(V)} \) are \( \delta g \)-open sets in \( X \). Further \( f \) is surjective implies \( f'^{(U)} \) and \( f'^{(V)} \) are nonempty disjoint and \( X = f'^{(U)} \cup f'^{(V)} \). This contradicts the fact that \( X \) is \( \delta g \)-connected space. Therefore, \( Y \) is connected.

Theorem 3.41: If \( f: X \to Y \) is contra \( \delta g \)-continuous, \( X \) is \( \delta g \)-connected and \( Y \) is \( T_1 \)-space, then \( f \) is constant.

Proof: Since \( Y \) is \( T_1 \)-space, \( U = \{ f'(y) : y \in Y \} \) is a disjoint \( \delta g \)-open partition of \( X \). If \( |U| \geq 2 \), then \( X \) is the union of two nonempty \( \delta g \)-open sets. This is contradiction to the fact that \( X \) is \( \delta g \)-connected. Therefore \( |U| = 1 \) and hence \( f \) is constant.

Definition 3.42: A topological space \( X \) is said to be \( \delta g \)-T_2 space if for any pair of distinct points \( x \) and \( y \), there exist disjoint \( \delta g \)-open sets \( G \) and \( H \) such that \( x \in G \) and \( y \in H \).

Theorem 3.43: Let \( f:X \to Y \) be contra \( \delta g \)-continuous injective function from a space \( X \) into Urysohn space \( Y \), then \( X \) is \( \delta g \)-T_2.

Proof: Let \( x \) and \( y \) be any distinct points in \( X \), then \( f(x) \neq f(y) \), there exist open sets \( V \) and \( W \) in \( Y \) containing \( f(x) \) and \( f(y) \) respectively, such that \( cl(V) \cap cl(W) = \emptyset \). Since \( f \) is contra \( \delta g \)-continuous, then there exist \( \delta g \)-open sets \( M \) and \( N \) in \( X \) such that \( f(M) \subseteq cl(V) \) and \( f(N) \subseteq cl(W) \) we have \( M \cap N = \emptyset \). Hence \( X \) is \( \delta g \)-T_2.

Remark 3.44: The composition of two contra \( \delta g \)-continuous functions need not be contra \( \delta g \)-continuous as seen from the following examples.

Example 3.45: Let \( X = Y = Z = \{ a, b, c \} \) and \( \tau = \{ X, \emptyset, \{ a \}, \{ b \}, \{ a, b \} \} \) be topologies on \( X, Y \) and \( Z \) respectively. Define a function \( f: X \to Y \) as \( f(a) = a, f(b) = b \) and \( f(c) = c \) and a function \( g: Y \to Z \) as \( g(a) = b, g(b) = c \) and \( g(c) = a \). Then \( f \) and \( g \) are contra \( \delta g \)-continuous but \( g \circ f: X \to Z \) is not contra \( \delta g \)-continuous, since there exists a open set \( \{ b, c \} \) in \( Z \) such that \( (g \circ f)'(\{ b, c \}) \neq \{ a, b \} \) is not \( \delta g \)-closed in \( X \).

Theorem 3.46: Let \( f: X \to Y \) and \( g: Y \to Z \) be any two functions.

(i) If \( f \) is contra \( \delta g \)-continuous and \( g \) is continuous then \( g \circ f \) is contra \( \delta g \)-continuous.

(ii) If \( f \) is contra \( \delta g \)-continuous and \( g \) is contra continuous then \( g \circ f \) is \( \delta g \)-continuous.

(iii) If \( f \) is \( \delta g \)-continuous and \( g \) is contra continuous then \( g \circ f \) is contra \( \delta g \)-continuous.

(iv) If \( f \) is \( \delta g \)-irresolute and \( g \) is contra \( \delta g \)-continuous then \( g \circ f \) is contra \( \delta g \)-continuous.

Proof: (i) Let \( h = g \circ f \) and \( V \) be an open set in \( Z \).
Since \( g \) is continuous, \( g^{-1}(V) \) is open in \( Y \). Therefore \( f^{-1}[g^{-1}(V)] = h^{-1}(V) \) is \( \delta g \beta \)-closed in \( X \) because \( f \) is contra \( \delta g \beta \)-continuous. Hence \( g \circ f \) is contra \( \delta g \beta \)-continuous.

The proofs of (ii), (iii) and (iv) are analogous to (i) with the obvious changes.

**Theorem 3.47:** Let \( f: X \to Y \) be contra \( \delta g \beta \)-continuous and \( g: Y \to Z \) be \( \delta g \beta \)-continuous. If \( Y \) is \( T_{\delta g \beta} \)-space, then \( g \circ f \) is contra \( \delta g \beta \)-continuous.

**Proof:** Let \( V \) be any open set in \( Z \). Since \( g \) is \( \delta g \beta \)-continuous, \( g^{-1}(V) \) is \( \delta g \beta \)-open in \( Y \) and since \( Y \) is \( T_{\delta g \beta} \)-space, \( g^{-1}(V) \) open in \( Y \). Since \( f \) is contra \( \delta g \beta \)-continuous \( f^{-1}[g^{-1}(V)] = (g \circ f)^{-1}(V) \) is \( \delta g \beta \)-closed set in \( X \). Therefore, \( g \circ f \) is contra \( \delta g \beta \)-continuous.

### 4. ALMOST CONTRA \( \delta g \beta \)-CONTINUOUS FUNCTIONS

In this section, almost contra delta generalized \( \beta \)-continuous functions are introduced and studied.

**Definition 4.1:** A function \( f: X \to Y \) is called almost contra delta generalized \( \beta \)-continuous if \( f^{-1}(G) \) is \( \delta g \beta \)-closed in \( X \) for every regular open set \( G \) in \( Y \).

**Theorem 4.2:** A function \( f: X \to Y \) is almost contra \( \delta g \beta \)-continuous if and only if for every regular closed set \( F \) of \( Y \), \( f^{-1}(V) \) is \( \delta g \beta \)-open set of \( X \).

**Theorem 4.3:** Every contra \( \delta g \beta \)-continuous function is almost contra \( \delta g \beta \)-continuous.

**Proof:** Follows from the fact that every regular-open set is open.

The converse of the Theorem 4.3 need to be true in general as seen from the following example.

**Example 4.4:** Let \( X = Y = \{a, b, c\} \) with \( \tau = \{X, \Phi, \{a\}\} \) and \( \sigma = \{X, \Phi, \{a\}, \{b\}, \{a, b\}\} \) be topologies on \( X \) and \( Y \) respectively. Let \( f: X \to Y \) be a function defined by \( f(a) = a, f(b) = b \) and \( f(c) = c \). Then \( f \) is almost contra \( \delta g \beta \)-continuous function but not contra \( \delta g \beta \)-continuous, because for the open set \( \{b\} \) in \( Y \) and \( f^{-1}(\{b\}) = \{a\} \) is not \( \delta g \beta \)-closed in \( X \).

**Theorem 4.5:** The following are equivalent for a function \( f: X \to Y \):

(i) \( f \) is almost contra \( \delta g \beta \)-continuous.

(ii) \( f^{-1}(cl(G)) \) is \( \delta g \beta \)-open set in \( X \) for every \( \beta \)-open subset \( G \) of \( Y \).

(iii) \( f^{-1}(cl(G)) \) is \( \delta g \beta \)-open set in \( X \) for every semi-open subset \( G \) of \( Y \).

(iv) \( f^{-1}(int(cl(G))) \) is \( \delta g \beta \)-closed set in \( X \) for every pre-open subset \( G \) of \( Y \).

**Proof:**

(i) \( \to \) (ii): Let \( G \) be \( \beta \)-open set of \( Y \). It follows from Theorem 2.4 of [3] that \( cl(G) \) is regular closed. Then \( f^{-1}(cl(G)) \) is \( \delta g \beta \)-open set in \( X \).

(ii) \( \to \) (iii): Obvious.

(iii) \( \to \) (iv): Let \( G \) be a pre-open set of \( Y \). Then \( Y-int(cl(G)) \) is regular closed and hence it is semi-open. Then, we have \( f^{-1}(cl(Y-int(cl(G)))) = f^{-1}(Y-int(cl(G))) = X-f^{-1}(int(Cl(G))) \) is \( \delta g \beta \)-open set in \( X \). Hence \( f^{-1}(int(cl(G))) \) is \( \delta g \beta \)-closed set in \( X \).

(iv) \( \to \) (v): Let \( G \) be regular-open set of \( Y \). Then \( G \) is pre-open in \( X \) and hence \( f^{-1}(G) = f^{-1}(int(Cl(G))) \) is \( \delta g \beta \)-closed set in \( X \).

**Theorem 4.6[16]:** For a subset \( A \) of a space \( X \), the following properties hold:

(i) \( ucl(A) = cl(A) \) for every \( \beta \)-open subset \( A \) of \( X \).

(ii) \( pcl(A) = cl(A) \) for every semi-open subset \( A \) of \( X \).

(iii) \( scl(A) = int(cl(A)) \) for every pre-open subset \( A \) of \( X \).

**Theorem 4.7:** The following are equivalent for a function \( f: X \to Y \):

(i) \( f \) is almost contra \( \delta g \beta \)-continuous.

(ii) for every \( \beta \)-open subset \( G \) of \( Y \), \( f^{-1}(ucl(G)) \) is \( \delta g \beta \)-open set in \( X \).

(iii) for every semi-open subset \( G \) of \( Y \), \( f^{-1}(pcl(G)) \) is \( \delta g \beta \)-open set in \( X \).

(iv) for every pre-open subset \( G \) of \( Y \), \( f^{-1}(scl(G)) \) is \( \delta g \beta \)-closed set in \( X \).
Definition 4.8 [16]: A function $f: X \to Y$ is said to be R-map if $f^{-1}(V)$ is regular open in $X$ for each regular open set $V$ of $Y$.

Definition 4.9 [15]: A function $f: X \to Y$ is said to be perfectly continuous if $f^{-1}(V)$ is clopen in $X$ for each regular open set $V$ of $Y$.

Theorem 4.10: For two functions $f: X \to Y$ and $g: Y \to Z$, let $g \circ f: Y \to Z$ be composition function. Then the following properties hold:

(i) If $f$ is almost contra $\delta g \beta$-continuous and $g$ is an R-map, then $g \circ f$ is almost contra $\delta g \beta$-continuous.

(ii) If $f$ is almost contra $\delta g \beta$-continuous and $g$ is perfectly continuous, then $g \circ f$ is contra $\delta g \beta$-continuous.

(iii) If $f$ is contra $\delta g \beta$-continuous and $g$ is almost continuous, then $g \circ f$ is almost contra $\delta g \beta$-continuous.

Proof: (i) Let $V$ be any regular open set in $Z$. Since $g$ is an R-map, $g^{-1}(V)$ is regular open in $Y$. Since $f$ is almost contra $\delta g \beta$-continuous, $f^{-1}[g^{-1}(V)] = (g \circ f)^{-1}(V)$ is $\delta g \beta$-closed set in $X$. Therefore, $g \circ f$ is almost contra $\delta g \beta$-continuous.

Proofs of (ii) and (iii) are similar to (i).

Theorem 4.11: Let $f: X \to Y$ be a contra $\delta g \beta$-continuous and $g: Y \to Z$ be $\delta g \beta$-continuous. If $Y$ is $T_{\delta g \beta}$-space, then $g \circ f: X \to Z$ is almost contra $\delta g \beta$-continuous.

Proof: Let $V$ be any regular open and hence open set in $Z$. Since $g$ is $\delta g \beta$-continuous, $g^{-1}(V)$ is $\delta g \beta$-open in $Y$. Since $f$ is contra $\delta g \beta$-continuous, $f^{-1}[g^{-1}(V)] = (g \circ f)^{-1}(V)$ is $\delta g \beta$-closed set in $X$. Therefore, $g \circ f$ is almost contra $\delta g \beta$-continuous.

Definition 4.12: A space $X$ is called locally $\delta g \beta$-indiscrete if every $\delta g \beta$-open set is closed in $X$.

Theorem 4.13: If $f: X \to Y$ is almost contra $\delta g \beta$-continuous and $X$ is locally $\delta g \beta$-indiscrete space then $f$ is almost continuous.

Proof: Let $U$ be any regular open set of $Y$. Since $f$ is almost contra $\delta g \beta$-continuous, $f^{-1}(U)$ is $\delta g \beta$-closed set in $X$. As $X$ is locally $\delta g \beta$-indiscrete space, $f^{-1}(U)$ is an open set in $X$. Therefore, $f$ is almost continuous.

Theorem 4.14 [7]: The intersection of a $\delta g \beta$-closed set and a $\delta$-closed set of $X$ is always $\delta g \beta$-closed.

Theorem 4.15: If $f: X \to Y$ is almost contra $\delta g \beta$-continuous, and $A$ is $\delta$-closed in $X$ then the restriction $(f/A): A \to Y$ is almost contra $\delta g \beta$-continuous.

Proof: Let $V$ be any regular open set of $Y$. Then $f^{-1}(V)$ is $\delta g \beta$-closed set in $X$. By Theorem 4.14, $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is $\delta g \beta$-closed it follows that $(f/A)$ is almost contra $\delta g \beta$-continuous.

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