International Journal of Mathematical Archive-8(11), 2017, 179-187
Available online through www.ijma.info ISSN 2229 – 5046

PROPAGATION OF RAYLEIGH WAVES DUE TO TWO OPPOSITELY PLACED PARALLEL RIGID BARRIERS IN THE LIQUID LAYER

OMBIR DAHIYA*
Department of Mathematics,
B.P.S. Institute of Higher Learning,
BPSMV Khanpur-kalan, Sonipat-131305, India.

SIKANDER
Department of Mathematics, A.I.J.H.M. College, Rohtak-124001, India.

MEENA
Research Scholar, B.P.S. Institute of Teacher Training and Research,
BPSMV Khanpur-kalan, Sonipat-131305, India.

JAGAT SINGH
PGT Mathematics, GSSS Agwanpur, Sonipat – 131101, India.

(Received On: 15-10-17; Revised & Accepted On: 14-11-17)

ABSTRACT

In this paper the problem of Rayleigh wave propagation due to two oppositely placed, parallel rigid plane barriers has been discussed. The plane vertical barriers of small depth H (x = 0, a) are erected artificially in the surface of the deep ocean. Deep ocean is a liquid half space given by $z \geq 0, -\infty < x < \infty$. *The elastic medium is homogeneous, isotropic and slightly dissipative. The reflected, transmitted and scattered waves have been obtained by Fourier transform and Wiener-Hopf technique. The numerical computations for the amplitude of the scattered waves have been made versus the wave number. As the wave number goes on increasing, the amplitude of the reflected waves falls rapidly.*

Key Words: *Scattering, Fourier Transform, Rayleigh waves, Wiener-Hopf technique.*

I. INTRODUCTION

During an earthquake, seismic waves appear on the surface of the earth and loose their energy around the inhomogeneities and irregularities. Rayleigh waves are responsible for the damages to human beings and buildings on the surface of the earth. The problem of scattering of Rayleigh waves at the edges of rigid plane barriers requires investigation. The effect of a vertical barrier, fixed in an infinitely deep sea, on normally incident surface waves was first considered by Ursell [11] for a two dimensional case. The problem of diffraction of compressional waves due to a rigid barrier in the surface of a deep sea-water and in an ocean superimposed on a solid half space has been studied by Deshwal [3, 4] using the technique of Wiener and Hopf [9]. The attenuation of Rayleigh waves due to the presence of a surface impedence in the surface of a solid half space has been studied by Gregory [6]. Momoi [8] has considered the scattering of Rayleigh waves by semicircular and rectangular discontinuities in the surface of a solid half space using the technique of Fourier transformation. The problem of reflection and transmission of a plane SH-wave at a corrugated interface between a dry sandy half space and an anisotropic elastic half space has been studied by Tomar and Kaur [7]. They have used the Rayleigh's method of approximation for studying the effect of sandiness, the anisotropy, the frequency and the angle of incidence on the reflection and transmission coefficients. The reflection of shear waves in visco-elastic medium at parabolic irregularity has been studied by Chattopadhyay *et al.* [1]. They found that amplitude of reflected wave decreases with increasing length of notch and increases with increasing depth of irregularity.

Corresponding Author: Ombir Dahiya*
Department of Mathematics, B.P.S. Institute of Higher Learning,
BPSMV Khanpur-kalan, Sonipat-131305, India.

Here we discuss the problem of scattering of Rayleigh waves due to the presence of two oppositely placed parallel rigid barriers in the surface of a deep ocean. The barriers are given by x = 0, a and of equal length H. The barriers are rigid such that no displacement occurs across them. The problem is two dimensional in zx-plane.

II. FORMULATION OF THE PROBLEM

We take the x-axis along the free surface of the ocean and the z-axis pointing vertically downward. The oceanic water is assumed to be a homogeneous, isotropic and slightly dissipative liquid half space. The two rigid vertical plane barriers of small depth H are held parallel to z-axis at distance a in the free surface (Fig. 1). The two dimensional wave equation is

$$\frac{\partial^2 \overline{\phi}}{\partial x^2}+\frac{\partial^2 \overline{\phi}}{\partial z^2}=\frac{1}{c^2}\left(\frac{\partial^2 \overline{\phi}}{\partial t^2}+\varepsilon\frac{\partial \overline{\phi}}{\partial t}\right) \tag{1}$$

where c is the velocity of wave propagation and $\varepsilon > 0$ is a damping constant. A time harmonic two-dimensional Rayleigh wave incident on the barriers is given by

$$\phi_i(x,z)=A_0\exp\left(-ip_0x-\beta_0 z\right),\ \beta_0=\pm\left(p_0{}^2-k^2\right)^{1/2} \tag{2}$$

p_0 being the wave number for Rayleigh waves. If the potential for a time-harmonic wave be

$$\overline{\phi}\left(x,z,t\right)=\phi\left(x,z\right)e^{-iwt} \tag{3}$$

Then (1) reduces to

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial z^2}+k^2\phi=0,\ k=\sqrt{(w^2+i\varepsilon w)/c}=k_1+ik_2 \tag{4}$$

The imaginary part of k is assumed to be small and positive. Let the total potential be

$$\phi_t\left(x,z\right)=\phi_i\left(x,z\right)+\phi\left(x,z\right) \tag{5}$$

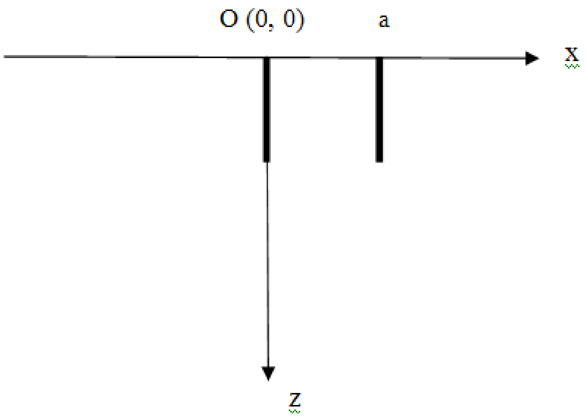

Figure-1: Geometry of the problem

Boundary Conditions:

The boundary conditions are

(i) $\phi\left(x,z\right)$ is bounded as $z\to\infty$. (6)

(ii) $\phi_t\left(x,z\right)=0,\ z=0$ for all x. (7)

(iii) $u=\dfrac{\partial \phi_t}{\partial x}=0,\ x=0\ and\ x=a, 0\le z\le H$ (8)

u is the displacement component at any point (x, z). It is assumed that for given z, $\phi\left(x,z\right)$ has the behavior of $e^{-d|x|}$ as $|x|\to\infty$, d > 0.

III. SOLUTION OF THE PROBLEM

We first define the Fourier transforms

$$\bar{\phi}(p,z) = \int_{-\infty}^{0} \phi(x,z) e^{ipx} dx + \int_{0}^{\infty} \phi(x,z) e^{ipx} dx$$

$$= \bar{\phi}_{-}(p,z) + \bar{\phi}_{+}(p,z), \; p = \xi + i\eta \tag{9}$$

Then $\bar{\phi}_{+}(p,z)$ and $\bar{\phi}_{-}(p,z)$ are analytic in the region $\eta > -d$ and $\eta < d$ respectively of the complex p-plane, ξ being the real part of the complex number p. Hence $\bar{\phi}(p,z)$ along with derivatives is analytic in the strip $-d < \eta < d$ of the complex p-plane.

Taking Fourier transformation of (4), we obtain

$$\left(\frac{d^2}{dz^2} - \beta^2\right)\bar{\phi}(p,z) = 0, \; \beta = \pm\sqrt{p^2 - k^2} \tag{10}$$

We choose that sign before the radical in (10) which makes the real part of $\beta \geq 0$ for all p. The solution of the equation (10) is

$$\bar{\phi}(p,z) = A(p) e^{\beta z} + B(p) e^{-\beta z} \tag{11}$$

Since $\bar{\phi}(p,z)$ is bounded as $z \to \infty$, therefore A(p) = 0 and from (11), we have

$$\bar{\phi}(p,z) = B(p) e^{-\beta z} \tag{12}$$

Differentiating (12) and eliminating B from (12) and resultant equation

$$\bar{\phi}(p) = -\frac{\bar{\phi}'(p)}{\beta} \tag{13}$$

where $\bar{\phi}(p), \bar{\phi}'(p)$ denote $\bar{\phi}(p,H)$ and $\bar{\phi}'(p,H)$ respectively. Similar notation are used for $\bar{\phi}_{+}(p,H)$ and $\bar{\phi}_{-}(p,H)$ and for teir deriaties. Deomposition of (13) by Wiener-Hopf technique and application of Liouville's theorem gives

$$\bar{\phi}_{+}(p) = h(p)\bar{\phi}'_{+}(k) - \frac{\bar{\phi}'_{+}(k)}{\beta} \tag{14}$$

$$\bar{\phi}_{-}(p) = -h(p)\bar{\phi}'_{+}(k) - \frac{\bar{\phi}'_{-}(k)}{\beta} \tag{15}$$

where $\bar{\phi}'_{+}(k) = \bar{\phi}'_{-}(-k)$ and

$$h(p) = \frac{1}{\sqrt{2k(p-k)}} - \frac{1}{\sqrt{2k(p+k)}} \tag{16}$$

Similarly,

$$\bar{\phi}_{+}(-p) = -h(p)\bar{\phi}'_{+}(k) - \frac{\bar{\phi}'_{+}(-p)}{\beta} \tag{17}$$

$$\bar{\phi}_{-}(-p) = h(p)\bar{\phi}'_{+}(k) - \frac{\bar{\phi}'_{-}(-p)}{\beta} \tag{18}$$

The Fourier transform of (4) from $-\infty$ to 0 and use of (8) leads to a differential equation whose complete solution is

$$\overline{\phi}_{-}(p,z)+\overline{\phi}_{-}(-p,z)=A_1(p)e^{\beta z}+A_2(p)e^{-\beta z}+\frac{2ip_0A_0e^{-\beta_0 z}}{p^2-p_o{}^2},0\le z\le H \quad (19)$$

The Fourier transform of (7) between $-\infty$ and 0 gives

$$\overline{\phi}_{-}(p,0)+\overline{\phi}_{-}(-p,0)=\frac{2ip_0A_0}{p^2-p_o{}^2},\, p\ne\pm p_0 \quad (20)$$

From (19) and (20) we get

$$\overline{\phi}_{-}(p,z)+\overline{\phi}_{-}(-p,z)=2A_1(p)\sinh p\ z+\frac{2ip_0A_0e^{-\beta_0 z}}{p^2-p_o{}^2} \quad (21)$$

Eliminating of $A_1(p)$ between (21) and its derivative with respect to z when z = H leads to

$$\overline{\phi}_{-}(p)+\overline{\phi}_{-}(-p)=\frac{\tanh\beta H}{\beta}\left[\overline{\phi}'_{-}(p)+\overline{\phi}'_{-}(-p)+\frac{2ip_0\beta_0A_0e^{-\beta_0 H}}{p^2-p_o{}^2}\right]+\frac{2ip_0A_0e^{-\beta_0 H}}{p^2-p_o{}^2} \quad (22)$$

Integrating (4) from x = 0 to x = a after multiplying it by e^{ipx} we get

$$\left(\frac{d^2}{dz^2}-\beta^2\right)\overline{\phi}_a(p,z)=-\left(\frac{\partial\phi}{\partial x}\right)_{x=a}e^{iap}+\left(\frac{\partial\phi}{\partial x}\right)_{x=0}+ip(\phi)_{x=a}e^{iap}-ip(\phi)_{x=0},0\le z\le H \quad (23)$$

where $\overline{\phi}_a(p,z)=\int_0^a\phi(x,z)e^{ipx}dx$

The right hand side of (22) is obtained by using the boundary conditions (7-8) and the result that $\phi_t=0$ on (0, H) and (a, H). Adding (23) to the new result obtained by changing p to –p in it, we get the differential equation whose solution is

$$\begin{aligned}\overline{\phi}_a(p,z)e^{-iap}+\overline{\phi}_a(-p,z)e^{iap}&=C_1(p)e^{\beta z}+C_2(p)e^{-\beta z}\\&-\frac{2iA_0e^{-\beta_0 z}}{p^2-p_0{}^2}\left[p_0\cos(ap)-ip\sin(ap)-p_0e^{-iap_0}\right]\end{aligned} \quad (24)$$

Similarly (7) is integrated on z = 0 to get

$$\overline{\phi}_a(p,0)e^{-iap}+\overline{\phi}_a(-p,0)e^{iap}=-\frac{2iA_0}{p^2-p_0{}^2}\left[p\cos(ap)-ip\sin(ap)-p_0e^{-iap_0}\right] \quad (25)$$

Using (24 and (25), we obtain

$$\overline{\phi}_a(p,z)e^{-iap}+\overline{\phi}_a(-p,z)e^{iap}=C_2(p)\sinh\beta z-\frac{2iA_0e^{-\beta_0 z}}{p^2-p_0{}^2}\left[p_0\cos(ap)-ip\sin(ap)-p_0e^{-ip_0a}\right] \quad (26)$$

Eliminating of $C_2(p)$ between (26) and its derivatives when z = H, gives

$$\begin{aligned}\overline{\phi}_a(p)e^{-iap}+\overline{\phi}_a(-p)e^{iap}&=\frac{\tanh\beta H}{\beta}\Bigg[\overline{\phi}_a{}'(p)e^{-iap}+\overline{\phi}_a{}'(-p)e^{iap}\\&-\frac{2iA_0\beta_0e^{-\beta_0 H}}{p^2-p_0{}^2}\left(p_o\cos(ap)-ip\sin(ap)-p_0e^{-iap_0}\right)\Bigg]\\&-\frac{2iA_0e^{-\beta_0 H}}{p^2-p_0{}^2}\left(p_o\cos(ap)-ip\sin(ap)-p_0e^{-iap_0}\right)\end{aligned} \quad (28)$$

Similarly, the Fourier transform of (4) between a and ∞ gives us

$$\overline{\phi}_{+a}(p,z)=\int_a^\infty\phi(x,z)e^{ipx}dx$$

Adding (27) and (28), we get

$$\bar{\phi}_+(p)e^{-iap}+\bar{\phi}_+(-p)e^{iap}=\frac{\tanh\beta H}{\beta}\Bigg[\bar{\phi}_+'(p)e^{-iap}+\bar{\phi}_+'(-p)e^{iap} -\frac{2iA_0\beta_0 e^{-\beta_0 H}}{p^2-p_0^2}\left(p_0\cos(ap)-ip\sin(ap)\right)\Bigg] -\frac{2iA_0 e^{-\beta_0 H}}{p^2-p_0^2}\left(p_0\cos(ap)-ip\sin(ap)\right)\Bigg] \quad (29)$$

Using (14) and (17) in (29), we get

$$\left(\bar{\phi}_+'(p)e^{-iap}+\bar{\phi}_+'(-p)e^{iap}\right)\frac{e^{\beta H}}{\beta\cosh\beta H}=\frac{2iA_0\beta_0 e^{-\beta_0 H}}{p^2-p_0^2}.\frac{e^{\beta H}}{\beta\cosh\beta H}$$
$$\times\left[p_0\cos(ap)-ip\sin(ap)\right]-2ih(p)\sin(ap)\bar{\phi}_+'(k)+\frac{2iA_0 e^{-\beta_0 H}}{p^2-p_0^2}\left[p_0\cos(ap)-ip\sin(ap)\right] \quad (30)$$

Similarly, from (15), (18) and (22), we get

$$\left[\bar{\phi}_-'(p)+\bar{\phi}_-'(-p)\right]\frac{e^{\beta H}}{\beta\cosh\beta H}=-\frac{2ip_0\beta_0 A_0 e^{-\beta_0 H}}{p^2-p_0^2}.\frac{e^{\beta H}}{\beta\cosh\beta H}-\frac{2ip_0 A_0 e^{-\beta_0 H}}{p^2-p_0^2} \quad (31)$$

IV. FACTORIZATION AND DECOMPOSITION

Let us now factorize $\cosh\beta H e^{-\beta H}$. We write

$$e^{-\beta H}=e^{\left[-G_+(p)-G_-(p)\right]} \quad (32)$$

Where

$$\cosh\beta H G_+(p)=\frac{\beta H\cos^{-1}(p/k)}{\pi}\sim\frac{ipH\log(2p/k)}{\pi}\text{ as }|p|\to\infty \quad (33)$$

And $\quad G_-(p)=G_+(-p) \quad (34)$

The factorization of $\cosh\beta H e^{-\beta H}/\beta H$ as an infinite product is

$$T(p)=T_+(p)T_-(p)=\frac{e^{-\beta H}\cosh\beta H}{\beta H}$$
$$=\frac{\exp\left[-G_+(p)-G_-(p)\right]}{H(p+k)^{1/2}(p-k)^{1/2}}\prod_{n=1}^{\infty}\left[1-k^2b_{n-1/2}^2+p^2b_{n-1/2}^2\right] \quad (35)$$

where

$$T_-(p)=\frac{\exp\left[X(p)-G_-(p)\right]}{\left[H(p-k)\right]^{1/2}}\prod_{n=1}^{\infty}\left[\left(1-k^2b_{n-1/2}^2\right)^{1/2}+ipb_{n-1/2}\right]\exp\left(\frac{-ipH}{\pi(n-1/2)}\right) \quad (36)$$

$b_{n-1/2}=H/(n-1/2)\pi$ and X(p) is an arbitrary function to give a suitable behaviour of $T_-(p)$ as $|p|\to\infty$. The behaviour of $T_-(p)$ as $|p|\to\infty$ is given by

$$T_-(p)=\frac{\exp\left[X(p)+ipH\pi^{-1}\log(-2p/k)\right]}{\left[H(p-k)\right]^{1/2}}\prod_{n=1}^{\infty}\left[1+\frac{ipH\pi^{-1}}{n-1/2}\right]\exp\left(-\frac{ipH\pi^{-1}}{n-1/2}\right) \quad (37)$$

The infinite product in (37) is approximated by the result

$$\prod_{n=1}^{\infty}\left[1+\frac{p}{n-1/2}\right]\exp\left(-\frac{p}{n-1/2}\right)\sim\exp\left(p+1/2-C_1p\right)2^{-2p} \quad (38)$$

where C_1= 0.5772 is Euler's constant. Therefore,

$$T_-(p) \sim \frac{\exp\left[X(p) + ipH\pi^{-1}\left(1 - C_1 + \log(\pi/2kH)\right) - pH/2\right]}{\left[H(p-k)\right]^{1/2}} \tag{39}$$

is asymptotic to $(p)^{-1/2}$ as $p \to \infty$, if

$$X(p) = -ipH\pi^{-1}\left(1 - C_1 + \log(\pi/2kH)\right) + pH/2 \tag{40}$$

Using (35) in (31) and decomposing the resulting equation, we obtain

$$\begin{aligned}&\frac{\bar{\phi}_-'(p)}{H(p-k)T_-(p)} + \frac{2ip_0\beta_0A_0e^{-\beta_0H}}{H(p+p_0)}\left[\frac{1}{(p-k)(p-p_0)T_-(p)} - \frac{1}{2(p_0+k)T_-(-p_0)}\right]\\ &+\frac{\bar{\phi}_-'(-p_m)}{H(p-k)T_-(p)} + \frac{iA_0e^{-\beta_0H}(p_0+k)T_+(p_0)}{p-p_0} = \frac{-1}{H(p-k)T_-(p)}\left[\bar{\phi}_-'(-p) - \bar{\phi}_-'(-p_m)\right]\\ &-\frac{i\beta_0A_0e^{-\beta_0H}}{H(p+p_0)(p_0+k)T_-(-p_0)} - \frac{2ip_0A_0e^{-\beta_0H}}{p-p_0}\left[\frac{T_+(p)(p+k)}{p+p_0} - \frac{T_+(p_0)(p_0+k)}{2p_0}\right]\end{aligned} \tag{41}$$

where $p_m = k, p_n$. p_n are zeros of $T_-(p) = 0$. By analytic continuation and Liouville,s theorem, each member is zero and we have

$$\begin{aligned}\bar{\phi}_-'(p) = &-\frac{2ip_0A_0e^{-\beta_0H}(p-k)T_-(p)}{p+p_0}\left[\frac{1}{(p-k)(p-p_0)T_-(p)} - \frac{1}{2(p_0+k)p_0T_-(-p_0)}\right]\\ &-\bar{\phi}_-'(-p_m) - \frac{iA_0e^{-\beta_0H}(p_0+k)T_+(p_0)H(p-k)T_-(p)}{p-p_0}\end{aligned} \tag{42}$$

Similarly decomposing (30), we get

$$\begin{aligned}\bar{\phi}_+'(p) = &\bar{\phi}_+'(p_m)e^{ia(p-p_m)} + \frac{iA_0\beta_0L_1(-p_0)(p-k)T_-(p)}{(p+p_0)(p_0+k)p_0T_-(-p_0)}e^{iap-\beta_0H}\\ &-2ih(p)\sin(ap)\bar{\phi}_+'(k)(p+k)(p-k)HT(p)\\ &+\frac{2iA_0e^{-\beta_0H}e^{iap}H(p-k)T_-(p)}{(p-p_0)}\left[\frac{L_1(p)(p+k)T(p)}{(p+p_0)} - \frac{L_1(p_0)(p_0+k)T(p_0)}{2p_0T_-(p_0)}\right]\end{aligned} \tag{43}$$

where

$$L_1(p) = p_0\cos(ap) - ip\sin(ap) \tag{44}$$

Adding (42) and (43), we obtain

$$\begin{aligned}\bar{\phi}'(p) = &\bar{\phi}_+'(p_m)e^{ia(p-p_m)} + \frac{iA_0\beta_0L_1(-p_0)(p-k)T_-(p)}{(p+p_0)(p_0+k)p_0T_-(-p_0)}e^{iap_0-\beta_0H}\\ &-2ih(p)\sin(ap)\bar{\phi}_+'(k)(p+k)(p-k)HT(p)\\ &+\frac{2iA_0e^{-\beta_0H}e^{iap}H(p-k)T_-(p)}{(p-p_0)}\left[\frac{L_1(p)(p+k)T(p)}{(p+p_0)T_-(p)} - \frac{L_1(p_0)(p_0+k)T(p_0)}{2p_0T_-(p_0)}\right]\\ &-\bar{\phi}_-'(-p_m) - \frac{iA_0e^{-\beta_0H}(p_0+k)T_+(p_0)H(p-k)T_-(p)}{(p-p_0)}\\ &-\frac{2iA_0p_0\beta_0e^{-\beta_0H}(p-k)T_-(p)}{(p+p_0)}\left[\frac{1}{(p-p_0)(p-k)T_-(p)} - \frac{1}{2(p_0+k)p_0T_-(-p_0)}\right]\end{aligned} \tag{45}$$

V. REFLECTED AND TRANSMITTED WAVES

The potential function $\phi(x,z)$ is obtained by the inverse Fourier transform

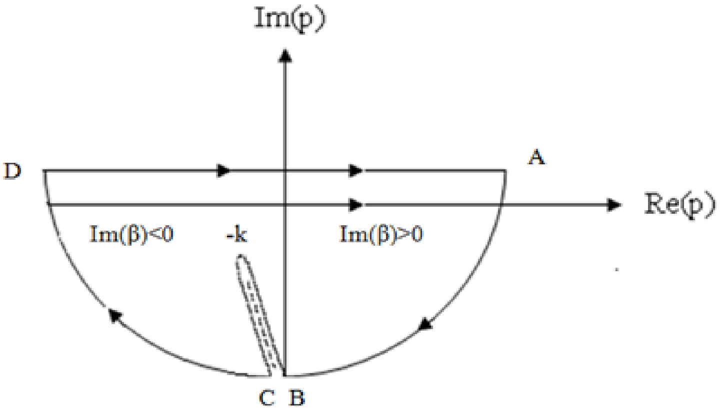

Figure-2: Contour of integration in the complex plane

$$\phi(x,z)=\frac{1}{2\pi}\int_{-\infty+i\eta}^{\infty+i\eta}\frac{\overline{\phi}'(p)e^{-\beta(z-H)e^{-ipx}}}{\beta}dp \tag{46}$$

To evaluate the integral (46), the contour is taken along the line $\eta=\mathrm{Im}(p_0)$ as shown in Fig. 2, avoiding the points $p=\pm p_0$. $p=\pm k$ are the branch points. The condition $\mathrm{Re}(\beta)=0$ on the -branch cut as discussed by Ewing and Press [5] gives the points of hyperbola to be used as branch cuts with $p=\pm k$ as branch points. The presence of the factor e^{-ipx} makes the integral vanish along the infinite circular arcs AB and CD. The contribution of identations are

$$\phi_1(x,z)=A_0e^{-ip_0x}e^{-\beta_0z}\left(1+e^{-\beta_0H}\cosh\beta_0H\right),\ p=p_0,\ x<0 \tag{47}$$

$$\phi_2(x,z)=A_0e^{ip_0(x-2a)}e^{-\beta_0z}\left(1+e^{-\beta_0H}\cosh\beta_0H\right),\ p=-p_0,\ x>0 \tag{48}$$

In (47), we have transmitted waves in the region x < 0 and in (48) we have reflected waves from the barriers in the region x > 0.

VI. SCATTERED WAVES

The scattered waves can be obtained by evaluating the integral (46) along the branch cut T_p. $T_-(p)$ being analytic in the lower half plane does not change its value on two sides of branch cut. $\mathrm{Im}(\beta)$ has different signs on the opposite sides of branch cut. The main contribution comes from the neighbourhood of the branch point p = -k, then p = -k –iu, u is small, since $\mathrm{Re}(\beta)$= 0 on the branch cut, therefore,

$$\beta=\pm\sqrt{p^2-k^2}=\pm\sqrt{(k+iu)^2-k^2}=\pm\sqrt{2i(k_1+ik_2)u-u^2}=\pm\sqrt{-\left(2k_2u+u^2\right)},\ k_1=0$$
$$=\pm i\beta_1,\ \beta_1=\sqrt{\left(2k_2u+u^2\right)} \tag{49}$$

Integrating (46) along two sides of the branch cut, we get

$$I(x,z)=\frac{ie^{-k_2x}}{2\pi}\int_0^{\infty}\left[\left(\frac{\overline{\phi}'(p)e^{-\beta(z-H)}}{\beta}\right)_{\beta=i\beta_1}-\left(\frac{\overline{\phi}'(p)e^{-\beta(z-H)}}{\beta}\right)_{\beta=-i\beta_1}\right]e^{-ux}du$$
$$=-\frac{e^{-k_2x}}{\pi}\int_0^{\infty}\left[\frac{H_1(u)\cos\beta_1(z-H)}{\beta_1}+2H_2(u)\cos\beta_1H\sin\beta_1z\right.$$
$$\left.-2H_3(u)\frac{\cos\beta_1H\cos\beta_1z}{\beta_1}\right]e^{-ux}du \tag{50}$$

Expanding $H_i(u)$ around u = 0,

$$H_i(u) = H_i(0) + uH_i'(0) + \frac{u^2}{2!}H_i''(0) + \dots \tag{51}$$

and retaining $H_i(0)$ only, we have

$$\begin{aligned} I(x,z) = \frac{e^{-k_2(x-a)}}{\pi}\int_0^\infty \Bigg[& \frac{H_1(0)\cos\sqrt{(2k_2u+u^2)}(z-H)}{\sqrt{(2k_2u+u^2)}} \\ & + H_2(0)(\sin\sqrt{(2k_2u+u^2)}(z+H) + \sin\sqrt{(2k_2u+u^2)}(z-H)) \\ & - \frac{H_3(0)(\cos\sqrt{(2k_2u+u^2)}(z+H) + \cos\sqrt{(2k_2u+u^2)}(z-H))}{\sqrt{(2k_2u+u^2)}} \Bigg] e^{-ux}du \end{aligned} \tag{52}$$

where

$$\begin{aligned} H_1(0) = & -\bar{\phi}_+'(p_m)e^{-iap_m} - \frac{2A_0\beta_0 L_1(-p_0)k_2T_-(-ik_2)e^{-\beta_0 H}}{(-ik_2+p_0)(p_0+ik_2)p_0T_-(-p_0)} \\ & + \frac{2A_0Hk_2L_1(-p_0)T_-(ik_2)(p_0+ik_2)T_+(p_0)e^{-\beta_0 H}}{(-ik_2-p_0)p_0} \\ & + \frac{\bar{\phi}_-'(-p_m)}{e^{ak_2}} + \frac{2A_0Hk_2T_-(-ik_2)(p_0+ik_2)T_+(p_0)e^{-\beta_0 H}}{(-ik_2-p_0)e^{ak_2}} \\ & + \frac{2A_0p_0\beta_0k_2T_-(-ik_2)e^{-\beta_0 H}}{(-ik_2-p_0)e^{ak_2}}\Bigg[\frac{1}{ik_2(ik_2+p_0)T_-(-ik_2)} \\ & - \frac{1}{p_0(ik_2+p_0)T_-(-p_0)}\Bigg] \end{aligned} \tag{53}$$

$$H_2(0) = -\frac{iA_0e^{-\beta_0 H}L_1(-ik_2)}{(k_2{}^2+p_0{}^2)} + \frac{\sinh(ak_2)\bar{\phi}_+'(ik_2)}{2k_2} \tag{54}$$

$$H_3(0) = \sinh(ak_2)\bar{\phi}_+'(ik_2) \tag{55}$$

To evaluate the integral in (52), we use results by Oberhettinger [10]

i.e. $$e^{-k_2(x-a)}\int_0^\infty \frac{\cos\left(\left(2k_2u+u^2\right)^{1/2}z\right)}{\left(2k_2u+u^2\right)^{1/2}}\mathrm{e}^{-ux}\,du = K_0(k_2r) \tag{56}$$

$$e^{-k_2(x-a)}\int_0^\infty \sin\left(\left(2k_2u+u^2\right)^{1/2}z\right)\mathrm{e}^{-ux}\,du = \frac{k_2z}{r^{1/2}}K_1(k_2r) \tag{57}$$

where $K_n(x)$ is the modified Hankel function of order n. Using (56) and (57) in (52). We get

$$\begin{aligned} I(x,z) = -\frac{1}{\pi}\Bigg[& H_1(0)K_0(k_2r_1) + H_2(0)\left(\frac{k_2(z+H)}{(r_2)^{1/2}}K_1(k_2r_2)\right. \\ & \left. + \frac{k_2(z-H)}{(r_1)^{1/2}}K_1(k_2r_1)\right) - H_3(0)\left(K_0(k_2r_2)+K_0(k_2r_1)\right)\Bigg] \end{aligned} \tag{58}$$

Where

$$r_1{}^2 = (x-a)^2 + (z-H)^2,\ r_2{}^2 = (x-a)^2 + (z+H)^2 \tag{59}$$

VII. NUMERICAL COMPUTATION AND CONCLUSIONS

The equation (47) represents the transmitted waves which are independent of the distance between the barriers but the reflected waves in (48) are found to depend on the distance. The scattered waves are obtained in (58). For small values of r, $K_0(k_2 r) \sim (\log z - \log k_2 r - C)$ and for large r, $K_0(k_2 r) \sim \exp(-k_2 r) / \sqrt{r}$. The scattered waves behave as a decaying cylindrical wave at distant points originating at the tips (a, H) of the barriers and at their images (a, -H) in the free surface. Close to the tips, when r_1 and r_2 are small, the scattered field possesses a logarithmic singularity implying very large amplitude close to the scatterer.

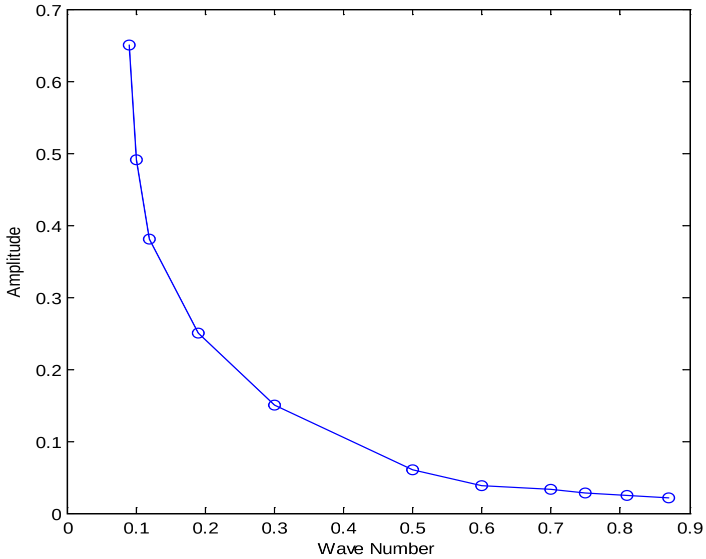

Figure-3: Variation of Amplitude of scattered waves vs the wave number

The numerical calculations for the amplitude oh the scattered waves have been obtained for a=0.01 km, r_1 = 0.1km, r_2 =12km, z=H and H= 6km. The graph of amplitude versus wave number of the scattered waves has been plotted in figure 3. The graph indicates that the amplitude decreases rapidly as the wave number increases very slowly.

REFERENCES

1. Chattopadhyay, A., Gupta, S., Sharma, V. K., Kumari, P., “Propagation of shear waves in visco-elastic medium at irregular boundaries”, Acta Geophysica, 58, 2009, pp. 195-214.
2. Copson, E. T., “Theory of functions of complex variables”, Oxford University Press, 1935.
3. Deshwal, P. S., “Diffraction of a compressional wave by a rigid barrier in a liquid half-space”, Pure Appl. Geophys., 85, 1971, pp. 107-204.
4. Deshwal, P. S., “Diffraction of a compressional wave by a rigid barrier in a liquid layer-Part II”, Pure Appl. Geophys., 91, 1971, pp. 14-33.
5. Ewing, W., Jardetzky, W.S. and Press, F., “Elastic Waves in Layered media”, McGraw-Hill Book Co., Inc., New York, 1957.
6. Gregory, R.D., “The attenuation of a Rayleigh wave in a half-space by surface impedance”, Proc. Camb. Phil. Soc., 62, 1966, pp. 811-827.
7. Kaur, J. and Tomar, S.K.,” SH-waves at a corrugated interface between a dry sandy half-space and anisotropic elastic half space”, Acta Mechanica, 190, 2007, 1-28.
8. Momoi, T., “Scattering of Rayleigh waves in an elastic quarter space”, J. Phys. Earth, 28, 1980, pp. 385-413.
9. Noble, B., “Methods based on the Wiener-Hopf technique”, 1958.
10. Oberhetttinger, F., Badii, L., “Tables of Laplace Transforms”, Springer-Verlang, New York, 1973.
11. Ursell, F., “The effect of a fixed vertical barrier on surface waves in deep water”, Proc. Camb. Phil. Soc., 43, 1947, pp. 374-382.

Source of support: Nil, Conflict of interest: None Declared.