

More Functions Associated with Semi* δ -Open Sets

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ABSTRACT

The aim of this paper is to introduce various functions associated with semi* δ -open sets. Here strongly semi* δ -continuous, perfectly semi* δ -continuous, totally semi* δ -continuous and slightly semi* δ -continuous functions are defined. Characterizations and some of their properties are investigated.

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I. INTRODUCTION

In 1980, Jain [2] introduced the concept of totally continuous functions. Nour [6] defined totally semi-continuous and strongly semi-continuous functions. In 1995 T.M.Nour [7] introduced slightly semi-continuous functions. In 1997, Slightly continuity was introduced by Jain [3]. Quite recently, the authors [8, 9] introduced and studied some new concepts, namely semi* δ -open sets, semi* δ -closed sets. We have also defined semi* δ -continuous, semi* δ -irresolute functions and their contra versions [10]. In this paper we define the strongly semi* δ -continuous, perfectly semi* δ -continuous, totally semi* δ -continuous and slightly semi* δ -continuous functions and investigate their properties.

II. PRELIMINARIES

Throughout this paper (X, τ) , (Y, σ) and (Z, η) will always denote topological spaces on which no separation axioms are assumed, unless otherwise mentioned. When A is a subset of (X, τ) , $\text{cl}(A)$ and $\text{int}(A)$ denote the closure and the interior of A respectively. We recall some known definitions needed in this paper.

Definition 2.1: A subset A of a topological space (X, τ) is **semi* δ -open** [8] if $A \subseteq \text{Cl}^*(\delta\text{Int}(A))$.

Definition 2.2: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a **strongly continuous** [4] if $f^{-1}(V)$ is both open and closed in (X, τ) for each subset V in (Y, σ) .

Definition 2.3: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a **perfectly continuous** [5] if $f^{-1}(V)$ is both open and closed in (X, τ) for every open set V in (Y, σ) .

Definition 2.4: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called **totally continuous** [2] if $f^{-1}(V)$ is clopen set in X for each open set V of Y .

Definition 2.5: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a **slightly continuous** [3] if $f^{-1}(V)$ is open set in X for each clopen set V of Y .

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Definition 2.6: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a **contra continuous** [1] if $f^{-1}(V)$ is closed in (X, τ) for every open set V in (Y, σ) .

Definition 2.7: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be **semi* δ -continuous** [10] if $f^{-1}(V)$ is semi* δ -open in (X, τ) for every open set V in (Y, σ) .

Definition 2.8: A function $F: X \rightarrow Y$ is said to be **semi* δ -irresolute** [10] if $f^{-1}(V)$ is semi* δ -open in X for every semi* δ -open set V in Y .

Definition 2.9: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called **contra-semi* δ -continuous** [11] if $f^{-1}(V)$ is semi* δ -closed in X for every open set V in Y .

Definition 2.10: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be **contra-semi* δ -irresolute** [11] if $f^{-1}(V)$ is semi* δ -closed in (X, τ) for every semi* δ -open set V in (Y, σ) .

Definition 2.11: A function $f: X \rightarrow Y$ is said to be **pre-semi* δ -open** [11] if $f(V)$ is semi* δ -open in Y for every semi* δ -open set V in X .

Definition 2.12: A topological space (X, τ) is said to be **$T_{S^*\delta}$ -space** [11], if every semi* δ -open set of X is open in X .

Definition 2.13[12]: A space X is **locally indiscrete** if every open set in X is closed.

III. Strongly semi* δ -Continuous Function

Definition 3.1: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called **strongly semi* δ -continuous** if the inverse image of every semi* δ -open set in (Y, σ) is open in (X, τ) .

Example 3.2: Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = a$, $f(c) = d$, $f(d) = c$. The function f is strongly semi* δ -continuous.

Theorem 3.3: A map $f: X \rightarrow Y$ is strongly semi* δ -continuous if and only if the inverse image of every semi* δ -closed set in Y is closed in X .

Proof: Assume that f is strongly semi* δ -continuous. Let V be any semi* δ -closed set in Y . Then V^c is semi* δ -open in Y . Since f is strongly semi* δ -continuous, $f^{-1}(V^c)$ is open in X . But $f^{-1}(V^c) = X / f^{-1}(V)$ and so $f^{-1}(V)$ is closed in X . Conversely, assume that the inverse image of every semi* δ -closed set in Y is closed in X . Let V be any semi* δ -open set in Y . Hence V^c is semi* δ -closed in Y . By assumption, $f^{-1}(V^c)$ is closed in X , but $f^{-1}(V^c) = X / f^{-1}(V)$ and so $f^{-1}(V)$ is open in X . Therefore, f is strongly semi* δ -continuous.

Theorem 3.4: If a map $f: X \rightarrow Y$ is strongly semi* δ -continuous and a map $g: Y \rightarrow Z$ is semi* δ -continuous then $g \circ f: X \rightarrow Z$ is continuous.

Proof: Let V be any open set in Z . Since g is semi* δ -continuous, $g^{-1}(V)$ is semi* δ -open in Y . Since f is strongly semi* δ -continuous $f^{-1}(g^{-1}(V))$ is open in X . That is $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is open in X . Therefore, $g \circ f$ is continuous.

Theorem 3.5: If a map $f: X \rightarrow Y$ is strongly semi* δ -continuous and a map $g: Y \rightarrow Z$ is semi* δ -irresolute, then $g \circ f: X \rightarrow Z$ is strongly semi* δ -continuous.

Proof: Let V be any semi* δ -open set in Z . Since g is semi* δ -irresolute, $g^{-1}(V)$ is semi* δ -open in Y . Also since f is strongly semi* δ -continuous, $f^{-1}(g^{-1}(V))$ is open in X . Hence $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is open in X . Hence, $g \circ f: X \rightarrow Z$ is strongly semi* δ -continuous.

Theorem 3.6: If $f: X \rightarrow Y$ is strongly semi* δ -continuous and $g: Y \rightarrow Z$ is contra semi* δ -continuous, then their composition $g \circ f: X \rightarrow Z$ is contra continuous.

Proof: Let V be any open set in Z . Since, g is contra semi* δ -continuous, then $g^{-1}(V)$ is semi* δ -closed in Y and since f is strongly semi* δ -continuous, by theorem 3.2 $f^{-1}(g^{-1}(V))$ is closed in X . Therefore, $g \circ f$ is contra continuous.

Theorem 3.7: If a map $f: X \rightarrow Y$ is semi* δ -continuous and a map $g: Y \rightarrow Z$ is strongly semi* δ -continuous, then $g \circ f: X \rightarrow Z$ is semi* δ -irresolute.

Proof: Let V be any semi* δ -open set in Z . Since g is strongly semi* δ - continuous, $g^{-1}(V)$ is open in Y . Also since f is semi* δ -continuous, $f^{-1}(g^{-1}(V))$ is semi* δ -open in X . Hence $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semi* δ -open in X . Hence, $g \circ f: X \rightarrow Z$ is semi* δ - irresolute.

Theorem 3.8: If a map $f: X \rightarrow Y$ is contra semi* δ - continuous and a map $g: Y \rightarrow Z$ is strongly semi* δ -continuous, then $g \circ f: X \rightarrow Z$ is contra-semi* δ - irresolute.

Proof: Let V be any semi* δ -open set in Z . Since g is strongly semi* δ - continuous, $g^{-1}(V)$ is open in Y . Also since f is contra-semi* δ -continuous, $f^{-1}(g^{-1}(V))$ is semi* δ -closed in X . Hence $(gf)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semi* δ -closed in X . Hence, $g \circ f: X \rightarrow Z$ is contra-semi* δ - irresolute.

Theorem 3.9: If $f: X \rightarrow Y$ is continuous and $g: Y \rightarrow Z$ strongly semi* δ - continuous then their composition $g \circ f: X \rightarrow Z$ is strongly semi* δ - continuous.

Proof: Let V be any semi* δ - open set in Z . Since g is strongly semi* δ - continuous, $g^{-1}(V)$ is open in Y . Since f is continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is open in X . Hence, $(g \circ f)$ is strongly semi* δ - continuous.

Theorem 3.10: Let $f: X \rightarrow Y$ be a continuous function and Y be a $T_{S^*\delta}$ - space. Then f is strongly semi* δ - continuous.

Proof: Let V be any semi* δ - open set in Y . Since Y is $T_{S^*\delta}$ - space, V is open in Y . Also since f is continuous, $f^{-1}(V)$ is open in X . Hence, f is strongly semi* δ -continuous.

Theorem 3.11: Let $f: X \rightarrow Y$ be semi* δ -irresolute and Y be $T_{S^*\delta}$ - space. Then f is strongly semi* δ -continuous.

Proof: Let V be any semi* δ -open set in Y . Since f is semi* δ -irresolute, $f^{-1}(V)$ is semi* δ -open in X . Also since X is $T_{S^*\delta}$ - space, $f^{-1}(V)$ is open in X . Hence f is strongly semi* δ -continuous.

Theorem 3.12: If $g: Y \rightarrow Z$ is strongly semi* δ -continuous and injective, and $g \circ f: X \rightarrow Z$ is semi* δ -closed then $f: X \rightarrow Y$ is semi* δ -closed.

Proof: Let F be any closed set in X . Since, $g \circ f$ is semi* δ -closed which implies, $g \circ f(F)$ is semi* δ -closed in Z . Also since g is strongly semi* δ -continuous, $g^{-1}(g \circ f(F))$ is closed in Y . Since g is injective, $f(F)$ is closed in Y . Hence f is semi* δ -closed.

IV. Perfectly semi* δ -Continuous Function

Definition 4.1: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be **perfectly semi* δ - continuous** if the inverse image of every semi* δ -open set in (Y, σ) is both open and closed in (X, τ) .

Theorem 4.2: If a map $f: X \rightarrow Y$ is perfectly semi* δ - continuous then it is strongly semi* δ - continuous.

Proof: Assume that f is perfectly semi* δ -continuous. Let V be any semi* δ -open set in Y . Since f is perfectly semi* δ -continuous, $f^{-1}(V)$ is open in X . Therefore, f is strongly semi* δ - continuous.

Remark 4.3: The converse of the above theorem need not be true.

Example 4.4: Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = a$, $f(c) = d$, $f(d) = c$. Clearly, f is strongly semi* δ -continuous. But $f^{-1}(\{a\}) = \{b\}$ is open in X , but not closed in X . Therefore f is not perfectly semi* δ -continuous.

Theorem 4.5: A map $f: X \rightarrow Y$ is perfectly semi* δ - continuous if and only if $f^{-1}(V)$ is both open and closed in X for every semi* δ -closed set V in Y .

Proof: Let V be any semi* δ -closed set in Y . Then V^c is semi* δ -open in Y . Since, f is perfectly semi* δ -continuous, $f^{-1}(V^c)$ is both open and closed in X . But $f^{-1}(V^c) = X / f^{-1}(V)$ and so $f^{-1}(V)$ is both open and closed in (X, τ) . Conversely, assume that the inverse image of every semi* δ -closed set in Y is both open and closed in X . Let V be any semi* δ -open in Y . Then V^c is semi* δ -closed in Y . By assumption $f^{-1}(V^c)$ is both open and closed in X . But $f^{-1}(V^c) = X / f^{-1}(V)$ and so $f^{-1}(V)$ is both open and closed in X . Therefore, f is perfectly semi* δ - continuous.

Theorem 4.6: Let $f: X \rightarrow Y$ be a mapping from a discrete topological space X into any topological space Y , then the following statements are equivalent.

- 1) f is strongly semi* δ -continuous
- 2) f is perfectly semi* δ -continuous

Proof:

(1) \Rightarrow (2): Let V be any semi* δ -open set in Y . By hypothesis, $f^{-1}(V)$ is open in X . Since X is a discrete space, $f^{-1}(V)$ is closed in X . $f^{-1}(V)$ is both open and closed in X . Hence, f is perfectly semi* δ -continuous.

(2) \Rightarrow (1): Let V be any semi* δ -open set in Y . Then, $f^{-1}(V)$ is both open and closed in X . Hence, f is strongly semi* δ -continuous.

Theorem 4.7: If $f: X \rightarrow Y$ is continuous and $g: Y \rightarrow Z$ is perfectly semi* δ -continuous then their composition $g \circ f: X \rightarrow Z$ is strongly semi* δ -continuous.

Proof: Let V be any semi* δ -open set in Z . Since g is perfectly semi* δ -continuous, $g^{-1}(V)$ is open and closed in Y . Since f is continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is open in X . Hence, $g \circ f$ is strongly semi* δ -continuous.

V. Totally semi* δ -continuous functions

Definition 5.1: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called **totally semi* δ -continuous function** if the inverse image of every open set in (Y, σ) is both semi* δ -open and semi* δ -closed subset in (X, τ) .

Theorem 5.2: Every totally semi* δ -continuous function is semi* δ -continuous.

Proof: Let V be any open set in Y . Since, f is totally semi* δ -continuous function, $f^{-1}(V)$ is both semi* δ -open and semi* δ -closed in X . Therefore, f is semi* δ -continuous.

Remark 5.3: The converse of above theorem need not be true.

Example 5.4: Let $X=Y=\{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = d, f(d) = b$. $S^*\delta O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}, X\}$ and $S^*\delta C(X, \tau) = \{\emptyset, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. Clearly f is semi* δ -continuous. But, since $f^{-1}(\{a\}) = \{b\}$ is semi* δ -open in X but not semi* δ -closed in X , f is not totally semi* δ -continuous.

Theorem 5.5: If $f: X \rightarrow Y$ is semi* δ -irresolute and $g: Y \rightarrow Z$ is totally semi* δ -continuous then $g \circ f: X \rightarrow Z$ is totally semi* δ -continuous.

Proof: Let V be any open set in Z . Since g is totally semi* δ -continuous, $g^{-1}(V)$ is semi* δ -open and semi* δ -closed in Y . Since f is semi* δ -irresolute, $f^{-1}(g^{-1}(V))$ is semi* δ -open and semi* δ -closed in X . Since $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$, $g \circ f$ is totally semi* δ -continuous.

Theorem 5.6: If $f: X \rightarrow Y$ is totally semi* δ -continuous and $g: Y \rightarrow Z$ is continuous then $g \circ f: X \rightarrow Z$ is totally semi* δ -continuous.

Proof: Let V be any open set in Z . Since g is continuous, $g^{-1}(V)$ is open in Y . Since, f is totally semi* δ -continuous, $f^{-1}(g^{-1}(V))$ is semi* δ -clopen in X . Hence, $g \circ f$ is totally semi* δ -continuous.

Theorem 5.7: Every totally semi* δ -continuous function is totally semi-continuous.

Proof: Suppose $f: X \rightarrow Y$ is totally semi* δ -continuous. Let V be an open set in Y . Since f is totally semi* δ -continuous, $f^{-1}(V)$ is semi* δ -regular in X and hence semi-regular in X . Therefore f is totally semi-continuous.

Theorem 5.8: Every totally semi* δ -continuous function is totally semi*-continuous.

Proof: Suppose $f: X \rightarrow Y$ is totally semi* δ -continuous. Let V be an open set in Y . Since f is totally semi* δ -continuous, $f^{-1}(V)$ is semi* δ -regular in X and hence semi*-regular in X . Therefore f is totally semi*-continuous.

Theorem 5.9: Every totally semi* δ -continuous function is totally semi* α -continuous.

Proof: Suppose $f: X \rightarrow Y$ is totally semi* δ -continuous. Let V be an open set in Y . Since f is totally semi* δ -continuous, $f^{-1}(V)$ is semi* δ -regular in X and hence semi* α -regular in X . Therefore f is totally semi* α -continuous.

Theorem 5.10: A function $f: X \rightarrow Y$ is totally semi* δ -continuous if and only if f is both semi* δ -continuous and contra-semi* δ -continuous.

Proof: Let V be any open set in Y . Since f is totally semi* δ -continuous, $f^{-1}(V)$ is both semi* δ -open and semi* δ -closed in X . Hence f is both semi* δ -continuous and contra-semi* δ -continuous.

Conversely, Let V be any open set in Y . Since f is both semi* δ -continuous and contra-semi* δ -continuous, $f^{-1}(V)$ is both semi* δ -open and semi* δ -closed in X . Hence f is totally semi* δ -continuous.

VI. Slightly semi* δ -continuous functions

Definition 6.1: A function $(X, \tau) \rightarrow (Y, \sigma)$ is called **slightly semi* δ -continuous** at a point $x \in X$ if for each clopen subset V of Y containing $f(x)$, there exists a semi* δ -open subset U in X containing x such that $f(U) \subseteq V$. The function f is said to be slightly semi* δ -continuous if f is slightly semi* δ -continuous at each of its points.

Definition 6.2: A function $(X, \tau) \rightarrow (Y, \sigma)$ is said to be **slightly semi* δ -continuous** if the inverse image of every clopen set in Y is semi* δ -open in X .

Example 6.3: Let $X = Y = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b, c, d\}, Y\}$ and $S^*\delta O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. Let $g: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $g(a)=d$, $g(b)=c$, $g(c)=a$, $g(d)=b$. The function f is slightly semi* δ -continuous.

Theorem 6.4: The definition 6.1 and 6.2 are equivalent.

Proof: Suppose the definition 6.1 holds. Let V be any clopen set in Y and $x \in f^{-1}(V)$. Then $f(x) \in V$ and thus there exists a semi* δ -open set U_x such that $x \in U_x$ and $f(U_x) \subseteq V$. Now $x \in U_x \subseteq f^{-1}(V)$. And $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$. Since, arbitrary union of semi* δ -open set is semi* δ -open, $f^{-1}(V)$ is semi* δ -open in X and therefore, f is slightly semi* δ -continuous.

Suppose, the definition 6.2 holds. Let $f(x) \in V$ where, V is any clopen set in Y . Since, f is slightly semi* δ -continuous, $x \in f^{-1}(V)$ where $f^{-1}(V)$ is semi* δ -open in X . Let $U = f^{-1}(V)$. Then U is semi* δ -open in X , $x \in U$ and $f(U) \subseteq V$.

Theorem 6.5: For a function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent.

- (i) f is slightly semi* δ -continuous.
- (ii) The inverse image of every clopen set V of Y is semi* δ -open in X .
- (iii) The inverse image of every clopen set V of Y is semi* δ -closed in X .
- (iv) The inverse image of every clopen set V of Y is semi* δ -regular in X .

Proof:

(i) \Rightarrow (ii): Follows from the theorem 6.4

(ii) \Rightarrow (iii): Let V be a clopen set in Y which implies V^c is clopen in Y . By (ii), $f^{-1}(V^c) = (f^{-1}(V))^c$ is semi* δ -open in X . Therefore, $f^{-1}(V)$ is semi* δ -closed in X .

(iii) \Rightarrow (iv): By (ii) and (iii), $f^{-1}(V)$ is semi* δ -regular in X .

(iv) \Rightarrow (i): Let V be any clopen set in Y containing $f(x)$, by (iv) $f^{-1}(V)$ is semi* δ -regular in X . Take $U = f^{-1}(V)$, then $f(U) \subseteq V$. Hence, f is slightly semi* δ -continuous.

Theorem 6.6: Every semi* δ -continuous function is slightly semi* δ -continuous.

Proof: Let $f: X \rightarrow Y$ be a semi* δ -continuous function. Let V be any clopen set in Y . Since f is semi* δ -continuous, $f^{-1}(V)$ is semi* δ -open in X and semi* δ -closed in X . Hence by theorem 6.5, f is slightly semi* δ -continuous.

Remark 6.7: The converse of the above theorem need not be true as can be seen from the following example.

Example 6.8: Let $X=Y=\{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$, $\sigma = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$, and $\sigma^c = \{\phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, Y\}$, then $S^*\delta O(X, \tau) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a)= b$, $f(b)=c$ and $f(c)=a$. The function f is slightly semi* δ -continuous. But $f^{-1}(\{a\}) = \{c\}$ is not semi* δ -open in X . Hence f is not semi* δ -continuous.

Theorem 6.9: Every contra semi* δ -continuous function is slightly semi* δ -continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a contra semi* δ -continuous function. Let V be any clopen set in Y . Then by the definition of contra semi* δ -continuous and by theorem 3.11[11], $f^{-1}(V)$ is semi* δ -regular in X . Hence by theorem 6.5, f is slightly semi* δ -continuous.

Remark 6.10: The converse of the above theorem need not be true as can be seen from the following example.

Example 6.11: Let $X=Y=\{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$, $\sigma = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$, and $\sigma^c = \{\phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, Y\}$, then $S^*\delta O(X, \tau) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a)= c$, $f(b)= a$, $f(c)=b$. The function f is slightly semi* δ -continuous. But $f^{-1}(\{a, c\}) = \{a, b\}$ is not semi* δ -closed in X . Hence f is not contra semi* δ -continuous.

Theorem 6.12: If $f: X \rightarrow Y$ is semi* δ -irresolute and $g: Y \rightarrow Z$ is slightly semi* δ -continuous then $g \circ f : X \rightarrow Z$ is slightly semi* δ -continuous.

Proof: Let V be any clopen set in Z . Since g is slightly semi* δ -continuous, $g^{-1}(V)$ is semi* δ -open in Y . Since f is semi* δ -irresolute, $f^{-1}(g^{-1}(V))$ is semi* δ -open in X . Since $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$, $g \circ f$ is slightly semi* δ -continuous.

Theorem 6.13: If $f: X \rightarrow Y$ is semi* δ -irresolute and $g: Y \rightarrow Z$ is semi* δ -continuous then $g \circ f: X \rightarrow Z$ is slightly semi* δ -continuous.

Proof: Let V be any clopen set in Z . Since g is semi* δ -continuous, $g^{-1}(V)$ is semi* δ -open in Y . Since f is semi* δ -irresolute, $f^{-1}(g^{-1}(V))$ is semi* δ -open in X . Since $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$, $g \circ f$ is slightly semi* δ -continuous.

Theorem 6.14: If $f: X \rightarrow Y$ is semi* δ -continuous and $g: Y \rightarrow Z$ is slightly-continuous then $g \circ f: X \rightarrow Z$ is slightly semi* δ -continuous.

Proof: Let V be any clopen set in Z . Since, g is slightly-continuous, $g^{-1}(V)$ is open in Y . Since, f is semi* δ -continuous, $f^{-1}(g^{-1}(V))$ is semi* δ -open in X . Since $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$, $g \circ f$ is slightly semi* δ -continuous.

Theorem 6.15: If $f: X \rightarrow Y$ is strongly semi* δ -continuous and $g: Y \rightarrow Z$ is slightly semi* δ - continuous then $g \circ f: X \rightarrow Z$ is slightly-continuous.

Proof: Let V be any clopen set in Z . Since g is slightly semi* δ -continuous, $g^{-1}(V)$ is semi* δ -open in Y . Since f is strongly semi* δ -continuous, $f^{-1}(g^{-1}(V))$ is open in X . Therefore, $g \circ f$ is slightly continuous.

Theorem 6.16: If $f: X \rightarrow Y$ is slightly semi* δ -continuous and $g: Y \rightarrow Z$ is perfectly semi* δ - continuous then $g \circ f: X \rightarrow Z$ is semi* δ -irresolute.

Proof: Let V be any semi* δ -open set in Z . Since g is perfectly semi* δ -continuous, $g^{-1}(V)$ is both open and closed in Y . Since f is slightly semi* δ -continuous, $f^{-1}(g^{-1}(V))$ is semi* δ -open in X . Hence, $g \circ f$ is semi* δ -irresolute.

Theorem 6.17: If $f: X \rightarrow Y$ is slightly semi* δ -continuous and $g: Y \rightarrow Z$ is contra continuous then $g \circ f: X \rightarrow Z$ is slightly semi* δ - continuous.

Proof: Let V be any clopen set in Z . Since g is contra continuous, $g^{-1}(V)$ is open and closed in Y . Since, f is slightly semi* δ -continuous, $f^{-1}(g^{-1}(V))$ is semi* δ -open in X . Hence $g \circ f$ is slightly semi* δ -continuous.

Theorem 6.18: If $f: X \rightarrow Y$ is semi* δ - irresolute and $g: Y \rightarrow Z$ is contra semi* δ -continuous then $g \circ f: X \rightarrow Z$ is slightly semi* δ - continuous.

Proof: Let V be any clopen set in Z . Since g is contra semi* δ -continuous, $g^{-1}(V)$ is semi* δ -open and semi* δ -closed in Y . Since f is semi* δ -irresolute, $f^{-1}(g^{-1}(V))$ is semi* δ -open and semi* δ -closed in X . Hence $g \circ f$ is slightly semi* δ -continuous.

Theorem 6.19: If the function $f: X \rightarrow Y$ is slightly semi* δ -continuous and (X, τ) is $T_{S^*\delta}$ -space, then f is slightly continuous.

Proof: Let V be any clopen set in Y . Since g is slightly semi* δ -continuous, $f^{-1}(V)$ is semi* δ -open in X . Since X is $T_{S^*\delta}$ -space, $f^{-1}(V)$ is open in X . Hence f is slightly continuous.

Theorem 6.20: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions. If f is surjective and pre-semi* δ -open and $g \circ f: X \rightarrow Z$ is slightly semi* δ -continuous, then g is slightly semi* δ -continuous.

Proof: Let V be any clopen set in Z . Since $g \circ f: X \rightarrow Z$ is slightly semi* δ -continuous, $f^{-1}(g^{-1}(V))$ is semi* δ -open in X . Since f is surjective and pre-semi* δ -open $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is semi* δ -open in Y . Hence, g is slightly semi* δ -continuous.

Theorem 6.21: If $f: X \rightarrow Y$ is a slightly semi* δ -continuous and Y is a locally indiscrete space then f is semi* δ -continuous.

Proof: Let V be any open subset in Y . Since Y is a locally indiscrete space, V is closed in Y . Since f is slightly semi* δ -continuous, $f^{-1}(V)$ is semi* δ -open in X . Hence, f is semi* δ -continuous.

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7. REFERENCES

1. Dontchev, J., Contra-continuous functions and strongly s -closed spaces, *Internat. J. Math. & Math.Sci.*19 (2) (1996), 303-310.
2. Jain.R.C., The role of regularly open sets in general topology, Ph.D. thesis, Meerut University, Institute of advanced studies, Meerut-India, (1980).
3. Jain R.C. and Singal A.R., Slightly continuous mappings, *Indian Math. Soc.*, 64(1997), 195-203
4. Levine, N., Strong continuity in topological spaces, *Amer. Math.Monthly*, 67 (1960), 269.
5. Noiri, T., Strong form of continuity in topological spaces, *Rend. Circ. Math. Palermo*, (1986), 107-113.
6. Nour.T.M., Totally semi-continuous functions, *Indian J. Pure Appl. Math.*, 26(7) (1995), 675-678.
7. Nour. T.M., Slightly semi continuous functions *Bull.Cal.Math.Soc* 87, (1995) 187-190
8. Pious Missier .S and Reena.C, On Semi* δ -Open Sets in Topological Spaces, *IOSR Journal of Mathematics* (Sep – Oct.2016), 01-06.
9. Pious Missier .S and Reena.C, Between δ -Closed Sets and δ -Semi-Closed Sets, *International Journal of Mathematical Archive*-7(11), 2016, 60-67.
10. Pious Missier .S and Reena.C, Continuous and Irresolute Functions Via Star Generalised Closed Sets, *IJMER*-7(2), Feb 2017,
11. Contra Semi* δ -Continuous Functions in Topological Spaces. (communicated)
12. S. Willard, *General Topology*, Addison–Wesley Publishing Company, Inc, 1970.

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