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RADIO ODD MEAN AND EVEN MEAN LABELING OF SOME GRAPHS

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ABSTRACT

A Radio Mean labeling of a connected graph G is a one to one map h from the vertex set V(G) to the set of natural numbers N such that for any two distinct vertices x and y of G, $d(x, y) + \left\lceil \frac{h(x)+h(y)}{2} \right\rceil \geq 1 + diam(G)$*. The radio mean number of h, rmn(h), is the maximum number assigned to any vertex of G. The radio mean number of G, rmn(G), is the minimum value of rmn(h) taken over all radio mean labelings h of G. In this paper we find the radio odd mean and even mean number of some graphs such as Umbrella graph, a Rooted tree and a* $K_1 + C_n$ *graph.*

Keywords: *Radio odd mean labeling, Radio even mean labeling, Distance, Eccentricity, Diameter, Umbrella graph, a Rooted tree and a* $K_1 + C_n$ *graph.*

1. INTRODUCTION AND DEFINITION

Throughout this paper we consider finite, simple, undirected and connected graphs. V(G) and E(G) respectively denote the vertex set and edge set of G. Radio labeling, or multilevel distance labeling, is motivated by the channel assignment problem for radio transmitters [1]. Ponraj *et al.* [10] introduced the notion of radio mean labeling of graphs and investigated radio mean number of some graphs [3, 11]. C. Davidraj, A. Subramanian and K.Sunitha investigated radio mean number for some graphs [7]. D.S.T.Ramesh, A. Subramanian and K. Sunitha investigated radio labeling of some graphs [9] and introduced the radio mean square labeling of some graphs [8]. N. Revathi [5] introduced the notion of Vertex Odd Mean and Even Mean labeling of Some Graphs. The span of a labeling h is the maximum integer that h maps to a vertex of G. The radio mean number of G, rmn(G) is the lowest span taken over all radio mean labelings of the graph G. For standard terminology and notations we follow Harary [4] and Gallian [6]. The distance between two vertices x and y of G is denoted by d(x, y) and diam(G) indicate the diameter of G.

Definition 1.1[2]: The distance d(u, v) from a vertex u to a vertex v in a connected graph G is the minimum of the lengths of the u-v paths in G.

Definition 1.2[2]: The eccentricity e(v) of a vertex v in a connected graph G is the distance between v and a vertex farthest from v in G.

Definition 1.3[2]: The diameter diam(G) of G is the greatest eccentricity among the vertices of G.

Definition 1.4: A radio odd mean labeling is a one to one mapping h: V(G)→{1, 3, 5, …, N} satisfying the condition $d(x, y) + \left\lceil \frac{h(x)+h(y)+1}{2} \right\rceil \geq 1 + diam(G)$ for every x, y∈V(G). The radio odd mean number of G is denoted by romn(G).

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Definition 1.5: A radio even mean labeling is a one to one mapping h: V(G) $\rightarrow$ {2, 4, 6,…, N} satisfying the condition $d(x, y) + \left\lceil \frac{h(x)+h(y)}{2} \right\rceil \geq 1 + diam(G)$ for every x, y $\in$ V(G). The radio even mean number of G is denoted by remn(G).

Definition 1.6[5]: For any integer n > 2, the umbrella graph U(n, n-1) is obtained by joining a path P_n with the central vertex of a fan f_n .

Definition 1.7 [2]: A tree in which one vertex is distinguished from all the others is called a rooted tree and the vertex is called the root of the tree.

Definition 1.8: The join of graphs K_1 and C_n, $K_1 + C_n$, is obtained by joining a vertex of K_1 with every vertex of C_n with an edge.

2. MAIN RESULTS

Theorem 2.1: romn (U(n, n-1)) = 4n - 1, n $\geq$ 3.

Proof: Let $x_1, x_2, ..., x_n$ be the vertices of the path P_n which are joined to the central vertex y_{n-1} of the fan f_n. The resultant graph is U(n, n-1) whose vertex set is V(U(n,n-1)) = $\{x_1, x_2, …, x_n, y_1, y_2, …, y_{n-1}\}$ and edge set is E(U(n,n-1)) = $\{x_i x_{i+1}: 1 \leq i \leq n-1\} \cup \{y_i y_{i+1}: 1 \leq i \leq n-2\} \cup \{x_i y_{n-1}: 1 \leq i \leq n\}$.

Clearly, diam(U(n, n-1)) = n – 1. Also the graph U(n, n-1) has 2n - 1 vertices and 3n - 3 edges. Define a function h: V(U(n, n-1)) $\rightarrow$ {1, 3, 5, ..., 4n – 1} by

$h(x_i) = 4i - 1, 1 \leq i \leq n$

$h(y_i) = 4i - 3, 1 \leq i \leq n-1$

Now we check the radio mean condition for h.

Case-a: Consider the pair (x_i, x_j), $i \neq j$, $1 \leq i, j \leq n$

$$d(x_i, x_j) + \left\lceil \frac{h(x_i) + h(x_j) + 1}{2} \right\rceil \geq 1 + \left\lceil \frac{4i+4j-1}{2} \right\rceil \geq n = 1 + diam (U (n, n-1))$$

Case-b: Consider the pair (y_i, y_j), $i \neq j$, $1 \leq i, j \leq n - 1$

$$d(y_i, y_j) + \left\lceil \frac{h(y_i) + h(y_j) + 1}{2} \right\rceil \geq 1 + \left\lceil \frac{4i+4j-5}{2} \right\rceil \geq n$$

Case-c: Consider the pair (y_{n-1}, x_i), $1 \leq i \leq n$

$$d(y_{n-1}, x_i) + \left\lceil \frac{h(y_{n-1}) + h(x_i) + 1}{2} \right\rceil = 1 + \left\lceil \frac{4(n-1)+4i-3}{2} \right\rceil \geq n$$

Case-d: Consider the pair (x_i, y_j), $1 \leq i \leq n$, $1 \leq j \leq n - 1$

$$d(x_i, y_j) + \left\lceil \frac{h(x_i) + h(y_j) + 1}{2} \right\rceil \geq 1 + \left\lceil \frac{4i+4j-3}{2} \right\rceil \geq n$$

Thus, the radio odd mean condition is satisfied for all pairs of vertices. Hence h is a valid radio odd mean labeling of U(n, n-1).Therefore romn(U(n, n-1)) $\leq$ romn(h) = 4n – 1.

Since h is injective, romn(U(n, n-1)) $\geq$ 4n – 1 for all radio mean labelings h and hence romn(U(n, n-1)) = 4n - 1, n $\geq$ 3.

Example 2.1: For the graph U(5,4) in Figure 1, romn(U(5,4)) = 19

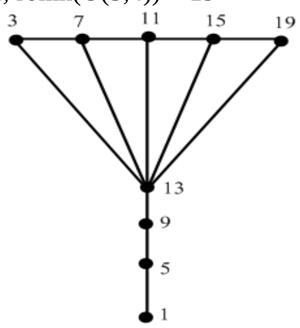

Figure-1

Theorem 2.2: remn(U(n, n - 1)) = 4 n, $n \geq 3$

Proof: Let $x_1, x_2, ..., x_n$ be the vertices of the path P_n which are joined to the central vertex y_{n-1} of the fan f_n. The resultant graph is U(n, n-1) whose vertex set is $V(U(n,n-1)) = \{x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_{n-1}\}$ and edge set is $E(U(n, n-1)) = \{x_i x_{i+1}: 1 \leq i \leq n-1\} \cup \{y_i y_{i+1}: 1 \leq i \leq n-2\} \cup \{x_i y_{n-1}: 1 \leq i \leq n\}$.

Clearly, diam(U(n, n-1)) = n – 1. Also the graph U(n, n-1) has 2n - 1 vertices and 3n - 3 edges. Define a function h: $V(U(n, n-1)) \rightarrow \{2, 4, 6, ..., 4n\}$ by

$h(x_i) = 4i,\ 1 \leq i \leq n;$

$h(y_i) = 4i - 2,\ 1 \leq i \leq n-1$

Now we check the radio mean condition for h.

Case-a: Consider the pair (x_i, x_j), $i \neq j$, $1 \leq i, j \leq n$

$$d(x_i, x_j) + \left\lceil \frac{h(x_i) + h(x_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{4i+4j}{2} \right\rceil \geq n = 1 + \text{diam}(U(n, n-1))$$

Case-b: Examine the pair (y_i, y_j), $i \neq j$, $1 \leq i, j \leq n-1$

$$d(y_i, y_j) + \left\lceil \frac{h(y_i) + h(y_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{4i+4j-4}{2} \right\rceil \geq n$$

Case-c: Examine the pair (y_{n-1}, x_i), $1 \leq i \leq n$

$$d(y_{n-1}, x_i) + \left\lceil \frac{h(y_{n-1}) + h(x_i)}{2} \right\rceil \geq 1 + \left\lceil \frac{4(n-1)+4i-2}{2} \right\rceil \geq n$$

Case-d: Consider the pair (x_i, y_j), $1 \leq i \leq n$, $1 \leq j \leq n-1$

$$d(x_i, y_j) + \left\lceil \frac{h(x_i) + h(y_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{4i+4j-2}{2} \right\rceil \geq n$$

Thus, the radio even mean condition is satisfied for all pairs of vertices. Hence h is a valid radio even mean labeling of U(n, n-1).Therefore $\text{remn}(U(n, n-1)) \leq \text{remn}(h) = 4n$.

Since h is injective, $\text{remn}(U(n, n-1)) \geq 4n$ for all radio even mean labelings h and hence $\text{remn}(U(n, n-1)) = 4n$, $n \geq 3$.

Example 2.2: For the graph U(5,4) in Figure 2, remn(U(5,4)) = 20

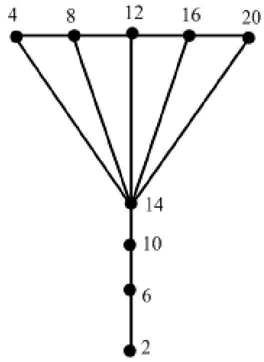

Figure-2

Theorem 2.3: $\text{romn}(RT_{n,n}) = 4n + 1$, $n \geq 2$

Proof: Let u be the root of the tree and let $x_1, x_2, ..., x_n$ be the vertices which are joined to the vertex u of the tree. Let $y_1, y_2, ..., y_n$ be the vertices which are joined to the vertex x_i, $1 \leq i \leq n$. The resultant graph is $RT_{n,n}$ whose edge set is $E = \{x_i y_i / 1 \leq i \leq n\} \cup \{u x_i / 1 \leq i \leq n\}$ and $\text{diam}(RT_{n,n}) = 4$.

Define the radio odd mean labeling h: $V(RT_{n,n}) \rightarrow \{1, 3, 5, \ldots, 4n+1\}$ by

$h(u) = 4n + 1;$

$h(x_i) = 2i - 1,\ 1 \leq i \leq n$

$h(y_i) = 2n + 2i - 1,\ 1 \leq i \leq n.$

Next we check the radio mean condition for h.

Case-a: Consider the pair (u, x_i), $1 \le i \le n$

$$d(u,x_i) + \left\lceil \frac{h(u)+h(x_i)+1}{2} \right\rceil = 1 + \left\lceil \frac{4n+2i+1}{2} \right\rceil \ge 5 = 1 + diam(RT_{n,n})$$

Case-b: Consider the pair (u, y_i), $1 \le i \le n$

$$d(u,y_i) + \left\lceil \frac{h(u)+h(y_i)+1}{2} \right\rceil = 2 + \left\lceil \frac{6n+2i+1}{2} \right\rceil \ge 5$$

Case-c: Consider the pair (x_i, x_j), $i \ne j$, $1 \le i, j \le n$

$$d(x_i, x_j) + \left\lceil \frac{h(x_i)+h(x_j)+1}{2} \right\rceil = 2 + \left\lceil \frac{2i+2j-1}{2} \right\rceil \ge 5$$

Case-d: Consider the pair (y_i, y_j), $i \ne j$, $1 \le i, j \le n$

$$d(y_i, y_j) + \left\lceil \frac{h(y_i)+h(y_j)+1}{2} \right\rceil = 4 + \left\lceil \frac{4n+2i+2j-1}{2} \right\rceil \ge 5$$

Case-e: Consider the pair (x_i, y_j), $1 \le i, j \le n$

$$d(x_i, y_j) + \left\lceil \frac{h(x_i)+h(y_j)+1}{2} \right\rceil \ge 1 + \left\lceil \frac{2n+2i+2j-1}{2} \right\rceil \ge 5$$

Thus, the radio odd mean condition is satisfied for all pairs of vertices. Hence h is a valid radio odd mean labeling of $RT_{n,n}$. Therefore $romn(RT_{n,n}) \le romn(h) = 4n + 1$.

Since h is injective, $romn(RT_{n,n}) \ge 4n+1$ for all radio odd mean labelings h and hence $romn(RT_{n,n}) = 4n + 1$.

Example 2.3: For the graph $RT_{5,5}$ in Figure 3, $romn(RT_{5,5}) = 21$

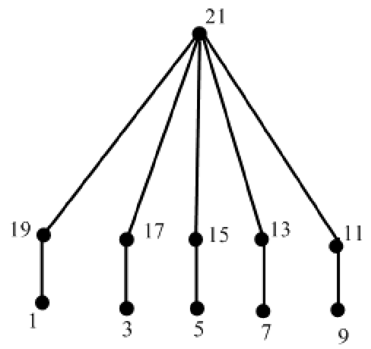

Figure-3

Theorem 2.4: $remn(RT_{n,n}) = 4n + 2$, $n \ge 2$.

Proof: Let u be the root of the tree and let $x_1, x_2, ..., x_n$ be the vertices which are joined to the vertex u of the tree. Let $y_1, y_2, ..., y_n$ be the vertices which are joined to the vertex x_i, $1 \le i \le n$. The resultant graph is $RT_{n,n}$ whose edge set is $E = \{x_i y_i / 1 \le i \le n\} \cup \{u x_i / 1 \le i \le n\}$ and $diam(RT_{n,n}) = 4$.

Define $h: V(RT_{n,n}) \to \{2, 4, 6, .., 4n + 2\}$ by

$h(u) = 4n+2$;
$h(x_i) = 2i$, $1 \le i \le n$;
$h(y_i) = 2n+2i$, $1 \le i \le n$.

Next we check the radio mean condition for h.

Case-a: Consider the pair (u, x_i), $1 \le i \le n$

$$d(u, x_i) + \left\lceil \frac{h(u)+h(x_i)}{2} \right\rceil = 1 + \left\lceil \frac{4n+2i+2}{2} \right\rceil \ge 5 = 1+diam(RT_{n,n})$$

Case-b: Consider the pair (u, y_i), $1 \le i \le n$

$$d(u, y_i) + \left\lceil \frac{h(u) + h(y_i)}{2} \right\rceil = 2 + \left\lceil \frac{6n + 2i + 2}{2} \right\rceil \ge 5$$

Case-c: Consider the pair (x_i, x_j), $i \ne j$, $1 \le i, j \le n$

$$d(x_i, x_j) + \left\lceil \frac{h(x_i) + h(x_j)}{2} \right\rceil = 2 + \left\lceil \frac{2n + 2i + 2j}{2} \right\rceil \ge 5$$

Case-d: Consider the pair (y_i, y_j), $i \ne j$, $1 \le i, j \le n$

$$d(y_i, y_j) + \left\lceil \frac{h(y_i) + h(y_j)}{2} \right\rceil = 4 + \left\lceil \frac{4n + 2i + 2j}{2} \right\rceil \ge 5$$

Case-e: Consider the pair (x_i, y_j), $1 \le i, j \le n$

$$d(x_i, y_j) + \left\lceil \frac{h(x_i) + h(y_j)}{2} \right\rceil \ge 1 + \left\lceil \frac{2n + 2i + 2j}{2} \right\rceil \ge 5$$

Thus, the radio even mean condition is satisfied for all pairs of vertices. Hence h is a valid radio even mean labeling of $RT_{n,n}$. Therefore $remn(RT_{n,n}) \le remn(h) = 4n + 2$.

Since h is injective, $remn(RT_{n,n}) \ge 4n+2$ for all radio even mean labelings h and hence $remn(RT_{n,n}) = 4n + 2$.

Example 2.4: For the graph $RT_{4,4}$ in Figure 4, $remn(RT_{4,4}) = 18$

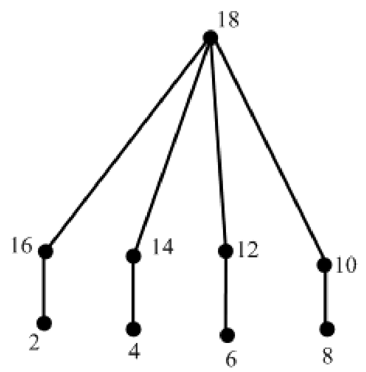

Figure-4

Theorem 2.5: $romn(K_1 + C_n) = 4n + 1$, $n > 2$.

Proof: Let $x_1, x_2, \ldots, x_n$ be the vertices of the cycle C_n and let u be the vertex of K_1 which are joined to the vertex x_i of the cycle C_n, $1 \le i \le n$. The resultant graph is $K_1 + C_n$ whose edge set is $E = \{x_i x_{i+1}, x_n x_1 / 1 \le i \le n-1\} \cup \{ux_i / 1 \le i \le n\}$ and $diam(K_1 + C_n) = 2$.

Define a radio odd mean labeling $h: V(K_1 + C_n) \to \{1, 3, 5, \ldots, 4n + 1\}$ by $h(u) = 3$ and $h(x_i) = 4i + 1$, $1 \le i \le n$.

Now we check the radio mean condition for h.

Case-a: Consider the pair (x_i, x_j), $i \ne j$, $1 \le i, j \le n$

$$d(x_i, x_j) + \left\lceil \frac{h(x_i) + h(x_j) + 1}{2} \right\rceil \ge 1 + \left\lceil \frac{4i + 4j + 2}{2} \right\rceil \ge 3 = 1 + diam(K_1 + C_n)$$

Case-b: Examine the pair (u, x_i), $1 \le i \le n$

$$d(u, x_i) + \left\lceil \frac{h(u) + h(x_i) + 1}{2} \right\rceil = 1 + \left\lceil \frac{4i + 5}{2} \right\rceil \ge 3$$

Thus, the radio odd mean condition is satisfied for all pairs of vertices. Hence h is a valid radio odd mean labeling of $K_1 + C_n$. Therefore $romn(K_1 + C_n) \le romn(h) = 4n + 1$.

Since h is injective, $romn(K_1 + C_n) \ge 4n + 1$ for all radio odd mean labelings h and hence $romn(K_1 + C_n) = 4n + 1$.

Example 2.5: For the graph $K_1 + C_5$ in Figure 5, romn $(K_1 + C_5) = 21$

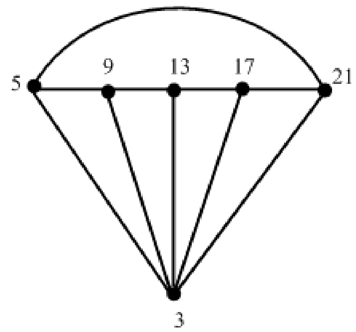

Figure-5

Theorem 2.6: remn $(K_1 + C_n) = 4n + 2$, $n > 2$

Proof: Let $x_1, x_2, ..., x_n$ be the vertices of the cycle C_n and let u be the vertex of K_1 which are joined to the vertex x_i of the cycle C_n, $1 \le i \le n$. The resultant graph is $K_1 + C_n$ whose edge set is $E = \{x_i x_{i+1}, x_n x_1 / 1 \le i \le n-1\} \cup \{ux_i / 1 \le i \le n\}$ and diam$(K_1 + C_n) = 2$.

Define the radio even mean labeling h: $V(K_1 + C_n) \rightarrow \{2, 4, 6, \ldots, 4n+2\}$ by
$h(u) = 4$ and $h(x_i) = 4i + 2$, $1 \le i \le n$

Next we check the radio mean condition for h.

Case-a: Consider the pair (x_i, x_j), $i \ne j$, $1 \le i, j \le n$

$$d(x_i, x_j) + \left\lceil \frac{h(x_i) + h(x_j)}{2} \right\rceil \ge 1 + \left\lceil \frac{4i + 4j + 4}{2} \right\rceil \ge 3 = 1 + \text{diam}(K_1 + C_n)$$

Case-b: Examine the pair (u, x_i), $1 \le i \le n$

$$d(u, x_i) + \left\lceil \frac{h(u) + h(x_i)}{2} \right\rceil = 1 + \left\lceil \frac{4i + 6}{2} \right\rceil \ge 3$$

Thus, the radio even mean condition is satisfied for all pairs of vertices. Hence h is a valid radio even mean labeling of $K_1 + C_n$. Therefore remn $(K_1 + C_n) \le$ remn(h) $= 4n + 2$.

Since h is injective, remn$(K_1 + C_n) \ge 4n + 2$ for all radio even mean labelings h and hence remn$(K_1+C_n) = 4n+2$.

Example 2.6: For the graph $K_1 + C_5$ in Figure 6, remn $(K_1 + C_5) = 22$

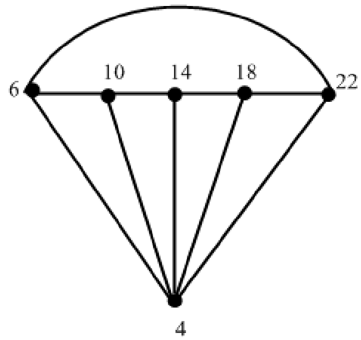

Figure-6

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