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NEW DERIVED POWER SERIES DISTRIBUTION AND ITS PROPERTIES

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ABSTRACT

Jayasree and Swamy (2006) derived new series of distributions called SJS derived power series distributions. In this paper an attempt is made to derive a new discrete probability distribution using restriction on one of the two parameters in the distribution given by Kulasekera and Tonkyn (1992). Properties of the derived distribution are presented with suitable examples.

Keywords: *Derived Power Series Distribution (DPSD), SJS Distribution.*

1. INTRODUCTION

Suppose that $\underline{a} = (a_0, a_1, a_2, \ldots)$ is a sequence of nonnegative real numbers. The partial sum of order n ∈ N is $g_n(\theta) = \Sigma\, a_k\theta^k$, k=0, 1 … n for all θ ∈ R. The power series is then defined by $g(\theta) = \lim_{n\to\infty} g_n(\theta)$ for θ ∈ R for which the limit exists, and is denoted $g_n(\theta) = \Sigma a_k\theta^k$, k = 0,1 … ∀ θ∈R. A random variable n with values in N has the power series distribution associated with the function g (or equivalently with the sequence *a*), with parameter θ ∈ [0, r), if N has discrete probability density function $f_\theta(n) = a_n\theta^n / g(\theta)$, n ∈ N.

Let $P_1(s)$ and $P_2(s)$; | s | ≤1 are the probability generating functions of two power series distributions. Jayasree and Swamy (2006) defined a family of new power series distributions with the convolution of $P_1(s)$ and $[P_2(s)]^{-1}$ called Derived Power Series Distributions (DPSD). Consider a DPSD for which the probability mass function is given by

$$P[X = x] = \left\{\frac{a_0\, g(\theta_2)}{b_0 f(\theta_1)}\right\} d_x ; \qquad x = 1,2, \ldots \ldots \tag{1.1}$$

where $d_x = a'_x - \sum_{j=1}^{x} b'_j\, d_{x-j}$; $x = 1, 2, \ldots$ with $a'_x = \frac{a_x}{a_0}(\theta_1)^x$ and $b'_j = \frac{b_j}{b_0}(\theta_2)^j$; $\theta_1, \theta_2 > 0$

The mean and variance of derived power series distributions are $\mu = \mu_1 - \mu_2$ and $\sigma^2 = \sigma_1^2 - \sigma_2^2$ where μ_1, σ_1^2 are the mean and variance of $P_1(s)$ and μ_2, σ_2^2 are the mean and variance of $P_2(s)$ power series distributions. Some of the power series distributions defined by Kulasekera and Tonkyn (1992) and Jayasree and Swamy (2006) are presented below.

Definition 1.1: A random variable X is said be a derived power series probability distribution if it satisfies the probability law $P[X = x] = x^\alpha q^x / (\Sigma\, x^\alpha q^x)$; $x = 1, 2, \ldots$; $-\infty < \alpha < \infty$; $0 < q < 1$; p+q =1. For fixed value of α, the distribution belongs to the family of power series distribution. The mean and variance of the distribution are $(1+q)\, p^{-1}$ and $2qp^{-2}$.

Definition 1.2: A random variable X is said to be a Geometrico-Poisson Distribution if it satisfies the probability law $P[X = x] = e^\delta (\delta\theta^{-1})^x (1-\delta\theta^{-1})\, S(x,\theta)$ where $x = 1, 2, \ldots$; $0 < \delta < \theta < 1$, where $S(x,\theta) = \sum_{x=0}^{\infty} (-1)^x \theta^x / x!$; θ = λ/q; δ=θ(1-p). The mean and variance of Geometrico-Poisson Distribution are $\mu = \delta[\theta^{-1}(1-\delta\theta^{-1})^{-1}-1]$ and $\sigma^2 = \delta\theta^{-1}[(1-\delta\theta^{-1})^{-2}-\theta]$.

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Definition 1.3: A random variable X is said to be a N-Bino-Geometric Distribution if it satisfies the probability law $P[X = x] = {}^{r+x-2}C_x\, q^x p^{r-1}$; $x = 1,2,\ldots$; $r > 2$; $0 < q < 1$. The mean and variance of NBGD are $\mu = (r-1)\, qp^{-1}$ and $\sigma^2 = (r-1)\, qp^{-2}$

Definition 1.4: A random variable X is said to be Log-Geometric Distribution if it satisfies the probability law

$$P_n = \theta\, p^{-1}\alpha \left\{ \frac{\theta^{n-2}}{(n-1)} - \frac{q\,\theta^{n-3}}{(n-2)} \right\}; \quad n = 2, 3, 4, \ldots \tag{1.2}$$

with $P_1 = \theta\ p^{-1}\alpha\ d_0$; where, $\alpha = -\{\log(1-\theta)\}^{-1}$. The mean and variance of LGD are $\mu = \alpha\theta(1-\theta)^{-1}-p^{-1}$ and $\sigma^2 = [\alpha\theta(1-\theta)^{-1}]\,[(1-\theta)^{-1} - \alpha\theta] - p^{-2}$.

2. NEW DERIVED POWER SERIES DISTRIBUTION

Let X be the random variable follows a power series probability distributions of Kulasekera and Tonkyn (1992), and Y be the random variable follows Geometric distribution. Let $P_1(s)$ and $P_2(s)$ are the probability generating functions of the two power series distributions in s and convergent for $|s| \le 1$,

$$P_1(s) = b^{-1}\sum_{x=1}^{\infty} x q_1^{\,x} s^x \quad \text{and} \quad P_2(s) = p_2 \sum_{y=1}^{\infty} q_2^{\,y-1} s^y \ ; \quad |s| \le 1 \tag{2.1}$$

Where $P[X = x] = x^{\alpha} q_1^{\,x} / (\sum_{x=1}^{\infty} x^{\alpha} q_1^{\,x})$; and $[Y = y] = p_2\, q_2^{\,y-1}$; $y=1, 2, \ldots$

The convolution of $P_1(s)$ and $[P_2(s)]^{-1}$ is

$$\begin{aligned}
P(s) = [P_1(s)].[P_2(s)]^{-1} &= \left[b^{-1}\sum_{x=1}^{\infty} k q_1^{\,x} s^x \right] q_2 \left[p_2 \sum_{y=1}^{\infty} q_2^{\,y} s^y \right]^{-1} \\
&= (bp_2)^{-1} q_2 \ \left[(q_1 s) + 2(q_1 s)^2 + 3(q_1 s)^3 + \ldots\right] \left[(q_2 s) + (q_2 s)^2 + (q_2 s)^3 + \ldots\right]^{-1} \\
&= q_1 (bp_2)^{-1}\left[1 + 2(q_1 s) + 3(q_1 s)^2 + \ldots\right] \left[1 + (q_2 s) + (q_2 s)^2 + \ldots\right]^{-1} \\
&= q_1 (bp_2)^{-1}\left[1 + \sum_{k=1}^{\infty}(k+1)(q_1 s)^k\right] \left[1 + \sum_{k=1}^{\infty}(q_2 s)^k\right]^{-1} \\
&= \sum_{x=1}^{\infty} p_x s^x \quad \text{where } p_x = q_1 (bp_2)^{-1}\left[(x+1)\ q_1^{\,x} - x\, q_1^{\,x-1} q_2\right]
\end{aligned} \tag{2.2}$$

Therefore, P(s) is a power series in 's' which converges for $|s| \le 1$. Under appropriate conditions for the coefficients of the power series P(s) to be non-negative, one can consider this as the probability generating function of a random variable, where from, the p.m.f. can be obtained by identifying the coefficients in P(s).

Theorem 3.1: The vector $P = (p_0, p_1, p_2, \ldots \ldots)$ where p_x is given by (2.2) defines a proper probability distribution, for $0 < p_1 < p_2 < 1$.

Proof: The probabilities specified in equation (2.2) is

$$P(s) = \sum_{x=1}^{\infty} p_x s^x \quad \text{where } p_x = q_1 (bp_2)^{-1}\left[(x+1)\ q_1^{\,x} - x\, q_1^{\,x-1} q_2\right]$$

Let $P = [\,p_0\ \ p_1\ \ p_2 \ldots \ldots\,]$ be the vector of probabilities generated from Probability distribution, then we must have, $p_x \ge 0$ and $\sum_{x=0}^{\infty} p_x = 1$.

Let $d_x = [(x+1)q_1^{\,x} - xq_1^{\,x-1}q_2] = [xq_1^{\,x} + q_1^{\,x} - xq_1^{\,x-1}q_2] \Rightarrow d_x \ge 0$, $x = 1, 2 \ldots$ Hence p_x, $x = 1, 2, \ldots$ defined in (2.2) are positive.

$$\sum_{x=0}^{\infty} p_x = \sum_{x=0}^{\infty}\ q_1 b^{-1} p_2^{-1}\left[(x+1)\ q_1^{\,x} - x\, q_1^{\,x-1} q_2\right]$$

$$= p_1^{\ 2} p_2^{\ -1} \sum_{x=0}^{\infty} [\{(x+1)q_1^{\ x}\} - \{xq_1^{\ x-1} q_2\}]$$
$$= p_1^{\ 2} p_2^{\ -1} [\sum_{x=0}^{\infty} \{(x+1)q_1^{\ x}\} - \{q_2 \sum_{x=0}^{\infty} xq_1^{\ x-1}\}]$$
$$= p_1^{\ 2} p_2^{\ -1} [p_1^{\ -2} - q_2 p_1^{\ -2}]$$
$$= 1.$$

Theorem 3.2: The mean of the derived power series distribution is difference between the means of numerator and the denominator power series distributions $= (1+q_1)p_1^{\ -1} - p_2^{\ -1}$

Proof: $\mu = E[X] \ = p_1^{\ 2} p_2^{\ -1} \sum_{x=1}^{\infty} xd_x$

$$= p_1^{\ 2} p_2^{\ -1} \sum_{x=1}^{\infty} [x\{(x+1)q_1^{\ x}\} - \{x^2 q_1^{\ x-1} q_2\}]$$
$$= p_1^{\ 2} p_2^{\ -1} [\sum_{x=1}^{\infty} [x^2 q_1^{\ x} \{(q_1 - q_2)q_1^{\ -1}\}] - \{\sum_{x=1}^{\infty} xq_1^{\ x}\}]$$
$$= [p_1^{\ 2} (p_2 q_1)^{-1} (q_1 - q_2) \sum_{x=1}^{\infty} x^2 q_1^{\ x}] + [p_1^{\ -2} q_1 p_1^{\ 2} p_2^{\ -1}]$$
$$= [p_1^{\ 2} (p_2 q_1)^{-1} (q_1 - q_2)(2q_1^{\ 2} + p_1 q_1)/p_1^{\ 3}] + q_1 p_2^{\ -1}$$
$$= (1+q_1)p_1^{\ -1} - p_2^{\ -1}.$$

Theorem 3.2: The variance of the derived power series distribution is the difference between the variances of the distributions whose probability generating functions were considered in the numerator and the denominator respectively

Proof: $\sigma^2 = E[X^2] - \{E[X]\}^2 \ = p_1^{\ 2} p_2^{\ -1} \sum_{x=1}^{\infty} x^2 d_x - \mu^2$

Consider $\sum_{x=1}^{\infty} x^2 d_x = \sum_{x=1}^{\infty} \{x(x-1) + x\} d_x$

$$= \sum_{x=1}^{\infty} [x(x-1)\{(x+1)q_1^{\ x}\} - \{xq_1^{\ x-1} q_2\}] + \sum_{x=1}^{\infty} xd_x$$
$$= \sum_{x=1}^{\infty} [x(x-1)(x+1)q_1^{\ x}] - q_2 q_1^{\ -1} \sum_{x=1}^{\infty} x^2 (x-1) q_1^{\ x} + \mu$$
$$= \sum_{x=1}^{\infty} [x(x-1)(x+1)q_1^{\ x}] - q_2 q_1^{\ -1} [\sum_{x=1}^{\infty} x(x-1)^2 q_1^{\ x} + \sum_{x=1}^{\infty} x(x-1) q_1^{\ x}] + \mu$$
$$= 6q_1^{\ 2} p_1^{\ -4} - 2q_1 q_2 p_1^{\ -3} - 6q_1^{\ 2} q_2 p_1^{\ -4} - 2q_1 q_2 p_1^{\ -3} + \mu$$

From (3.2) and (3.3)

$$\sigma^2 = p_1^{\ 2} p_2^{\ -1} [6q_1^{\ 2} p_1^{\ -4} - 2q_1 q_2 p_1^{\ -3} - 6q_1^{\ 2} q_2 p_1^{\ -4} - 2q_1 q_2 p_1^{\ -3}] + \mu p_1^{\ 2} p_2^{\ -1} - \mu^2$$

On simplification,

$$\sigma^2 = 2q_1 p_1^{\ -2} - q_2 p_2^{\ -2} \ .$$

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Applications:

1. In statistical quality control the control limits for the shewhart control chart based on probability can be set, satisfying $P\left[\sum_{i=1}^{n} x_i > UCL/H_0\right] = \alpha_1$ and $P\left[\sum_{i=1}^{n} x_i < LCL/H_0\right] = \alpha_2$, where, α_1 and α_2 are specified, in such a way that $\alpha_1 + \alpha_2$ is the probability of false alarm. Since $T_n = \sum_{i=1}^{n} x_i$ has Negative Binomial distribution with parameters (n,q) , the UCL and the LCL can be obtained from the tails of that distribution. Further, the OC-function $P(q_1, q_2)$ can be obtained from the SJS-4 distribution for $q_1 \neq q_2$ .So also the ARL function $ARL(q_1, q_2)$ can be obtained in the usual manner as $ARL(q_1,q_2) = \{1 - P(q)\}^{-1}$.
2. In testing of hypotheses $H_0 : q_1 = q_2 = q$ Vs. $H_1 : q_1 \neq q_2$ at l.o.s. α**,** under H_0, from remark (3.5) one can observe that the SJS-4 distribution in (3.6) collapses to the geometric distribution, in which, the sufficient statistics for q is given by $\sum_{i=1}^{n} x_i$ and the distribution of $\sum_{i=1}^{n} x_i$ is Negative Binomial (n,q) where n is the size of the random sample drawn from the SJS-4 distribution. Hence the cut-off points of the test are obtained corresponding to a given level α of significance and thereby, the power of the test can be obtained..

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