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SEMI FUZZY GRAPH

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ABSTRACT

In this paper, we introduce the new concept of semi fuzzy graph and some basic definitions on semi fuzzy graphs.

Keywords: *Semi fuzzy graph, semi fuzzy subgraph, domination of semi fuzzy graph.*

1. INTRODUCTION

Graph theory is a very important tool to represent many real world problems. For example, a social network may be represented as a graph where vertices represent accounts(persons, institutions, etc.) and edges represent the relation between the accounts. If the relations among the accounts are to be measured as good or bad according to the frequency of contacts among the accounts, fuzzyness should be added to representation. This and many other problems motivated to define fuzzy graphs. Rosenfeld (1975) introduced the notion of fuzzy graphs and several analogs of graph theoretic concepts such as path, cycle, connectedness etc. After that, fuzzy graph theory becomes a vast research area. Zadeh (1987) introduced the concept of fuzzy relations. The concept of a complete fuzzy graphs was investigated by Sunitha and Vijayakumar (2002). The concept of domination in fuzzy graphs was introduced by Somasundaram(1998). In this paper we introduce the new concept semi fuzzy graphs

2. SEMI FUZZY GRAPHS

2.1 Definition: A semi fuzzy graph G is a pair (V, μ) where V is a non empty set and μ is a function from VxV $\rightarrow$[0, 1]. The elements of V are called vertices and the elements VxV are called edges. The elements of VxV are denoted either by (x, y) or xy.

If $\mu(e=xy)=1$, then the nodes x and y are said to be perfectly adjacent.

If $\mu(e=xy\,)=0$, then the nodes x and y are said to be perfectly non -adjacent.

If $0<\mu(e=xy)<1$, then the nodes x and y are said to be partially adjacent and $\mu(xy)$ is called the level of adjacency of x and y.

2.2 Notation: Given a semi fuzzy graph G: (V, μ), the order of G is defined and denoted as

p = |V| and q = $\sum_{x,y\in V}\mu(xy)$.

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2.3 Example:

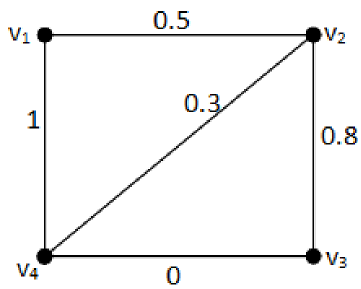

Figure-2.1: G(V, μ)

Here, G(V, μ) is a semi fuzzy graph. $\mu(v_1v_2) = 0.5$ v_1 and v_2 are partially adjacent. Similarly, $\mu(v_1v_4) = 1$ implies v_1 and v_4 are perfectly adjacent; $\mu(v_2v_3) = 0.8$ implies v_2 and v_3 are partially adjacent; $\mu(v_2v_4) = 0.3$ implies v_2 and v_4 are partially adjacent; $\mu(v_3v_4) = 0$ implies v_3 and v_4 are non – adjacent.

2.4 Definition: A semi fuzzy graph H = (U, σ) is called a semi fuzzy subgraph of G = (V, μ) if $U \subseteq V$ and σ (e) $\leq$ μ (e) for every edge $e \in H$.

2.5 Example:

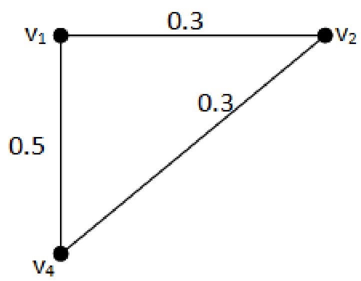

Figure-2.2: H (U, σ)

Comparing the figure 2.1 and 2.2, U is a subset of V and $\sigma(v_1v_2) < \mu(v_1v_2)$; $\sigma(v_1v_4) < \mu(v_1v_4)$; $\sigma(v_2v_4) = \mu(v_2v_4)$. Therefore, H(U, σ) is a semi fuzzy subgraph of G(V, μ)

2.6 Definition: For any subset U of V, the semi fuzzy subgraph of (V, μ) induced by U is the semi fuzzy graph (U, μ).

2.7 Definition: A semi fuzzy graph H = (U, σ) is called semi fuzzy spanning subgraph of G = (V, μ) if U = V.

2.8 Definition: Let G be a semi fuzzy graph. The degree of a vertex u is defined as
d(u) = $\sum_{u \neq v} \mu(uv)$. δ (G) = min{d(u)/ u$\in$ V(G)}; Δ (G) = max{d(u)/ u$\in$ V(G)}.

2.9 Example:

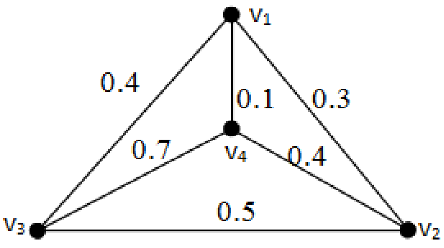

Figure-2.3

In figure 2.3, $d(v_1) = 0.8$, $d(v_2) = 1.2$, $d(v_3) = 1.6$, $d(v_4) = 1.2$. Therefore, $\delta(G) = 0.8$, $\Delta(G) = 1.6$.

2.10 Definition: A semi fuzzy graph is said to be regular if every vertex is of same degree.

2.11 Definition: A semi fuzzy graph G = (V, μ) is said to be complete if $\mu(xy) > 0.5$ for all x, y ∈ V.

2.12 Example:

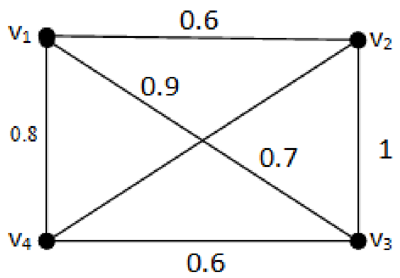

Figure-2.4

2.13 Definition: Complement of a semi fuzzy graph G= (V, μ) is defined as $\overline{G} = (V, \overline{\mu})$ where $\overline{\mu}(xy) = 1 - \mu(xy)$.

2.14 Example:

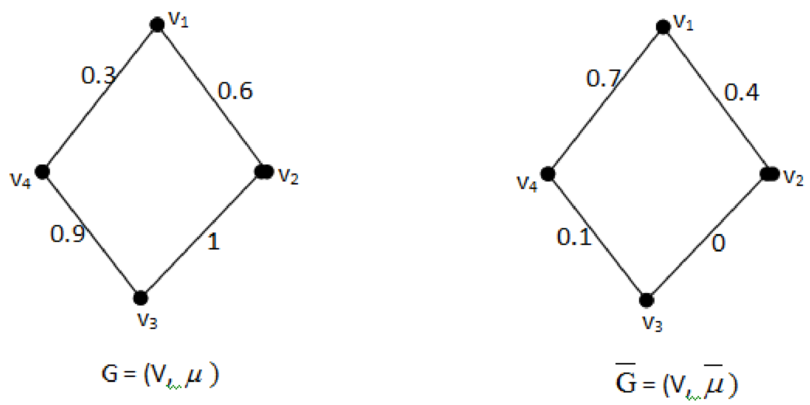

Figure-2.5

2.15 Definition: A path in a semi fuzzy graph G(V, μ) is a sequence of distinct nodes $v_0, v_1, v_2, v_3, \ldots, v_n$ such that μ ($v_{i-1}v_i$) > 0, for all i=1 to n. $\sum_{i=1}^{n} \frac{1}{\mu(v_{i-1}v_i)}$ is called the length of the path and the strength of the path is defined as min{ μ (v_iv_{i+1})/i=1 to n}.

2.16 Example:

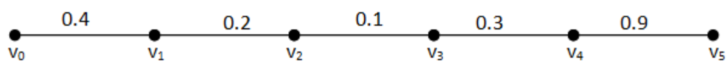

Figure-2.6

Here, the length of path is 21.9, and the strength of the path is 0.1.

2.17 Definition: A semi fuzzy graph G = (V, μ) is connected if any two vertices are joined by a path. Otherwise, it is called a disconnected graph.

2.18 Definition: Let G = (V, μ) be a semi fuzzy graph. The strength of connectedness between two nodes x and y is defined as the maximum of strength of all paths between x and y and it is denoted by $\mu^{\infty}(xy)$.

2.19 Definition: Let G = (V, μ) be a semi fuzzy graph. A xy path P is called a strongest xy path if its strength equals $\mu^{\infty}(xy)$.

2.20 Definition: The μ -semi fuzzy distance d(u, v) is the smallest length of any u-v path.

2.21 Definition: Let G = (v, μ) be a connected semi fuzzy graph. The μ-semi fuzzy detour distance D(u,v) between the vertices u and v is defined to be the maximum length of any u-v path. A u-v path of length D(u, v) is called a μ-semi fuzzy u-v detour.

2.22 Example:

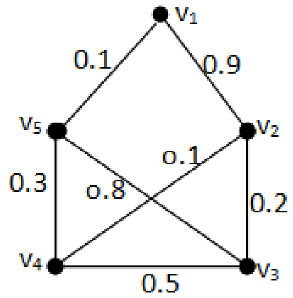

Figure-2.7

Let $P_1(v_1v_5)$:v_1,v_2,v_4,v_3,v_5; $P_2(v_1v_5)$:v_1,v_2,v_3,v_4,v_5; $P_3(v_1v_5)$:v_1,v_2,v_3,v_5; $P_4(v_1v_5)$:v_1,v_2,v_4,v_5; $P_5(v_1v_5)$: v_1v_5. L (P_1)=1.1+10+2+1.25=14.35; l(P_2)=1.1+5+2+3.3=11.4; l(P_3)=1.1+5+1.25=7.35; l(P_4) = 1.1 + 10 + 3.3 = 14.4; l(P_5) = 10. Therefore, D(v_1v_5) is 14.4 and d(v_1v_5) is 7.35.

2.23 Definition: Let G = (V, μ) be a semi fuzzy graph. A subset D of V is said to be a dominating set of G if every vertex v in V – D is joined to a vertex u in D with $\mu(\mathrm{uv}) > 0$. The minimum cardinality among all dominating sets of G is called the domination number of G and is denoted by γ (G).

2.24 Example:

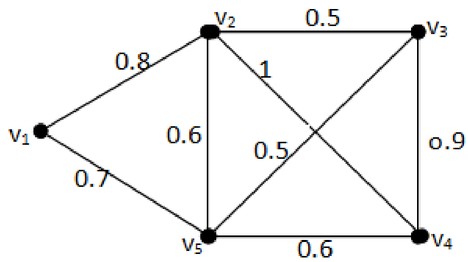

Figure-2.8

In Figure 2.8, $\{v_2\}$ minimum is a dominating set of G. Hence, $\gamma(\mathrm{G}) = 1$.

2.25 Definition: Let G = (V, μ) be a semi fuzzy graph. A subset D of V is said to be a weak independent set if $\mu(\mathrm{uv}) > 0.5$ for all u, v ∈ D.

The maximum cardinality taken over all weak independent sets of G is called the weak independence number. It is denoted by i_w(G).

2.26 Definition: Let G = (V, μ) be a semi fuzzy graph. A subset D of V is said to be a strong independent set if $\mu(\mathrm{uv}) \le 0.5$ for all u, v ∈ D.

The maximum cardinality taken over all strong independent sets of G is called the strong independence number. It is denoted by i_s(G).

2.27 Remark: If $\mu(\mathrm{uv}) = 0$, for all u, v ∈ D then D is called an independent set of G and the maximum cardinality taken over all independent sets of G is called the independence number of G.

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