International Journal of Mathematical Archive-8(11), 2017, 65-75
Available online through www.ijma.info ISSN 2229 – 5046

FORCED CONVECTIVE HEAT FLOW OF A VISCOUS LIQUID IN A POROUS MEDIUM OVER A FIXED HORIZONTAL IMPERMEABLE PLATE FOR DIFFERENT DEPTHS OF THE CHANNEL WITH A CONSTANT HEAT SOURCE DISTRIBUTED UNIFORMLY IN THE FLOW REGION

K. MOINUDDIN*

Faculty of Mathematics, Maulana Azad Nation Urdu University, Hyd, India.

(Received On: 13-10-17; Revised & Accepted On: 02-11-17)

ABSTRACT

In this paper the author deals with steady flow of a viscous fluid of finite depth in a porous medium over a fixed horizontal, impermeable bottom with a uniformly distributed constant heat source in the flow region. Momentum and Energy equations are solved analytically when the temperatures on the fixed bottom and on the free surface are prescribed. Velocity, Mean velocity, Temperature, Mean Temperature, Mean Mixed Temperature in the flow region and the heat transfer rate on the boundaries have been obtained. The cases of small depth (shallow fluid) and large depth (deep fluid) are also discussed. The results are illustrated graphically.

INTRODUCTION

Flows through porous media with heat transfer have been extensively investigated by mechanical engineers in the design of heat exchangers such as boilers, condensers and radiators of power plants and by chemical engineers working on absorption. Forced convective heat flow of a viscous liquid of finite depth in a porous medium over a fixed horizontal impermeable plate is studied by Moinuddin.K and Pattabhi Ramacharyulu N.Ch [1] in the year 2011. A problem of forced convective heat flow of a viscous liquid in a porous medium over a fixed horizontal impermeable plate for different depths of the channel is studied by Mohammed Ameenuddin [2] in the year 2016. A fluid flow and heat transfer problem in porous media with a uniformly distributed constant heat source in the flow region was studied by K.Moinuddin [3] in the year 2016.

In this paper the steady forced convective flow of a viscous liquid of viscosity μ and of finite depth H through a porous medium of porosity coefficient ‘k*’ over a fixed impermeable bottom with a uniformly distributed constant heat source is investigated. The flow is generated by a constant pressure gradient parallel to the fixed bottom plate. The momentum equation considered is the generalized Darcy’s law proposed by Yama Moto and Iwamura[4] which takes into account the convective acceleration and the Newtonian viscous stresses in addition to the classical Darcy force.

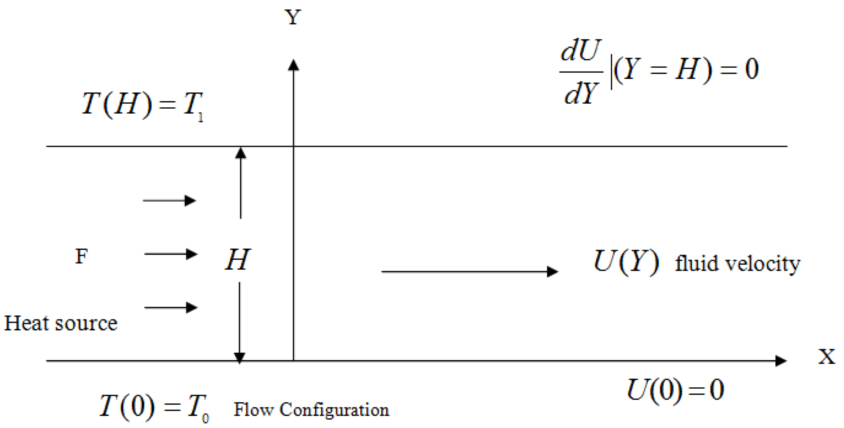
Momentum and Energy are solved analytically to give exact expressions for velocity and temperature distributions. Employing the flow rate, mean velocity, mean temperature, mean mixed temperature and the heat transfer rate at the fluid boundaries have been obtained and illustrated graphically.

The cases of 1. Large depths (Deep fluid) 2. Small depths (Shallow depths) are discussed.

MATHEMATICAL FORMULATION

Consider the steady forced convective flow of a Newtonian viscous fluid through a porous medium of viscosity coefficient μ and of finite depth (H) over a fixed horizontal impermeable bottom. The flow is generated by a constant pressure gradient parallel to the plate. Further the bottom is kept at a constant temperature T_0 and the free surface is exposed to atmospheric temperature T_1 with a uniformly distributed constant heat source F in the flow region. With reference to a rectangular Cartesian co-ordinates system with the origin ‘O’ on the bottom, X-axis in the flow direction. The Y-axis vertically upwards, the bottom is represented as Y=0 and the free surface as Y=H.

Corresponding Author: K. Moinuddin*
Faculty of Mathematics, Maulana Azad Nation Urdu University, Hyd, India.



Flow Configuration

Basic Equations:

Let the convective flow be characterized by the velocity field u = (U(Y), 0, 0) and the temperature T(Y). This choice of the velocity satisfies the continuity equation.

$$\nabla . u = 0 \tag{1}$$

The Momentum Equation is

$$-\frac{\partial P}{\partial X} + \mu \frac{d^2U}{dY^2} - \mu \frac{U}{k^*} = 0 \tag{2}$$

and the Energy Equation is

$$\rho c U \frac{\partial T}{\partial X} = K \frac{d^2T}{dY^2} + \mu \left(\frac{dU}{dY} \right)^2 + F \tag{3}$$

where F is a constant heat source distributed uniformly in the flow region. In the above equations ρ is the fluid density, k^* the coefficient of porosity of the medium, c is the specific heat, K the thermal conductivity of the fluid and P the fluid pressure.

Boundary Conditions:

Since the bottom is fixed U (0) =0 (4a)

the free surface shear stress $\mu \frac{dU}{dY} = 0$ at Y=H i.e $\frac{dU}{dY} = 0$ when Y=H. (4b)

Also T (0) = T_0 (5a)

and T (H) =T_1 (5b)

where T_0 is the temperature at the bottom and T_1 is the temperature of the atmosphere.

In terms of the non-dimensional variables defined as:

$$\text{Y=ay; X=ax; H=ah ; } U = \frac{\mu u}{\rho a^2};\ P = \frac{\mu^2 p}{\rho a^2};\ T = T_0 + (T_1 - T_0)\theta;\ \Pr = \frac{\mu c}{k};\ k^* = \frac{a^2}{\alpha^2};$$

$$E = \frac{\mu^3}{\rho^2 a^2 K (T_1 - T_0)};\ -\frac{\partial P}{\partial X} = \frac{\mu^2 c_1}{\rho a^3}\left(c_1 = -\frac{\partial p}{\partial x} \right) \text{ and } \frac{\partial T}{\partial X} = \frac{(T_1 - T_0)}{a} c_2 \text{ where } c_2 = \frac{\partial \theta}{\partial x}$$

$$f = \frac{a^2 F}{K (T_1 - T_0)} \tag{6}$$

where 'a' is some standard length, the basic field equations [2] , [3] can be rewritten as follows:

Momentum Equation:

$$\frac{d^2u}{dy^2} - \alpha^2 u = -c_1 \tag{7}$$

Energy Equation:

$$\frac{d^2\theta}{dy^2} = P_r c_2 u - E\left(\frac{du}{dy}\right)^2 - f \tag{8}$$

together with the boundary conditions

For velocity u (0) =0 and $\frac{du}{dy} = 0$ at y=h (9)

and for the temperature $\theta(0)$ =0 and $\theta(h) = 1$ (10)

The momentum equation together with the related boundary conditions [9] yields the velocity distribution:

$$u(y) = \frac{c_1}{\alpha^2}\left(1 - \frac{\cosh\alpha(h-y)}{\cosh\alpha h}\right) \tag{11}$$

The flow rate in the non-dimensional form is

$$q = \int_0^h u(y)\,dy = \frac{c_1}{\alpha^2}\left(h - \frac{\tanh\alpha h}{\alpha}\right) \tag{12}$$

The mean velocity in the non-dimensional form is

$$\frac{1}{h}\int_0^h u(y)\,dy = \frac{c_1}{h\alpha^2}\left(h - \frac{\tanh\alpha h}{\alpha}\right) \tag{13}$$

The energy equation with the boundary conditions [10] yields the temperature distribution:

$$\begin{aligned}\theta(y) = \frac{y}{h} + \frac{P_r c_1 c_2}{\alpha^2}\left[\frac{(h-y)}{h\alpha^2} - \frac{y(h-y)}{2} + \frac{1}{\alpha^2\cosh\alpha h}\left(\frac{y}{h} - \cosh\alpha(h-y)\right)\right] \\ + \frac{Ec_1^2}{2\alpha^2\cosh^2\alpha h}\left[\frac{(h-y)\cosh(2\alpha h)}{4\alpha^2 h} - \frac{y(h-y)}{2} + \frac{1}{4\alpha^2}\left(\frac{y}{h} - \cosh 2\alpha(h-y)\right)\right] + \frac{fy(h-y)}{2}\end{aligned} \tag{14}$$

Further the mean temperature in non-dimensional form is given by

$$\begin{aligned}\bar{\theta} &= \frac{1}{h}\int_0^h \theta\,dy \\ &= \frac{1}{2} + \frac{P_r c_1 c_2}{\alpha^2}\left(\frac{-h^2}{12} + \frac{1}{2\alpha^2} + \frac{1}{2\alpha^2\cosh\alpha h} - \frac{\tanh\alpha h}{h\alpha^3}\right) - \\ &\quad \frac{Ec_1^2}{2\alpha^2\cosh^2\alpha h}\left(\frac{-1}{8\alpha^2} + \frac{h^2}{12} - \frac{\cosh 2\alpha h}{8\alpha^2} + \frac{\sinh 2\alpha h}{8h\alpha^3}\right) + \frac{fh^2}{12}\end{aligned} \tag{15}$$

Also the mean mixed temperature in the dimensionless form is

$$\frac{\int_0^h \theta u dy}{\int_0^h u dy} = \frac{1}{\left(h - \frac{\tanh \alpha h}{\alpha}\right)} \left[\begin{array}{l} \left(\frac{h}{2} + \frac{1}{\alpha^2 h \cosh \alpha h} - \frac{1}{\alpha^2 h}\right) + \\ \frac{P_r c_1 c_2}{\alpha^2} \left\{ \begin{array}{l} \frac{-h^3}{12} + \frac{h}{\alpha^2} + \frac{1}{h\alpha^4} + \frac{h}{\alpha^2 \cosh \alpha h} \\ -\frac{5 \tanh \alpha h}{2\alpha^3} + \frac{h}{2\alpha^2 \cosh^2 \alpha h} - \frac{2}{h\alpha^4 \cosh \alpha h} + \frac{1}{h\alpha^4 \cosh^2 \alpha h} \end{array} \right\} + \\ \frac{Ec_1^2}{2\alpha^2 \cosh^2 \alpha h} \left\{ \begin{array}{l} \frac{5h}{8\alpha^2} - \frac{h^3}{12} - \frac{1}{4\alpha^4 h} - \frac{\sinh 2\alpha h}{8\alpha^3} + \frac{h \cosh 2\alpha h}{8\alpha^2} + \frac{h}{2\alpha^2 \cosh \alpha h} \\ -\frac{7 \tanh \alpha h}{8\alpha^3} + \frac{\sinh 3\alpha h}{24\alpha^3 \cosh \alpha h} - \frac{\sinh \alpha h \tanh \alpha h}{2\alpha^4 h} + \frac{\cosh 2\alpha h}{4\alpha^4 h} - \\ \frac{\cosh 2\alpha h \tanh \alpha h}{4\alpha^3} \end{array} \right\} \end{array} \right]$$

$$+ \frac{1}{(h - \frac{\tanh \alpha h}{\alpha})} f \left(\frac{h^3}{12} - \frac{h}{2\alpha^2} - \frac{h}{2\alpha^2 \cosh \alpha h} + \frac{\tanh \alpha h}{\alpha^3} \right) \quad (16)$$

Heat transfer coefficient (Nusselt Number):

On the bottom:

$$\frac{d\theta}{dy} \Big|_{y=0} = \frac{1}{h} + \frac{P_r c_1 c_2}{\alpha^2} \left(\frac{\tanh \alpha h}{\alpha} - \frac{h}{2} + \frac{1}{h\alpha^2 \cosh \alpha h} - \frac{1}{h\alpha^2} \right)$$
$$- \frac{Ec_1^2}{2\alpha^2 \cosh^2 \alpha h} \left(\frac{\sinh 2\alpha h}{-2\alpha} - \frac{1}{4h\alpha^2} + \frac{h}{2} + \frac{\cosh 2\alpha h}{4h\alpha^2} \right) + \frac{fh}{2} \quad (17)$$

On the free surface:

$$\frac{d\theta}{dy} \Big|_{y=h} = \frac{1}{h} + \frac{P_r c_1 c_2}{\alpha^2} \left(\frac{h}{2} + \frac{1}{h\alpha^2 \cosh \alpha h} - \frac{1}{h\alpha^2} \right) -$$
$$\frac{Ec_1^2}{2\alpha^2 \cosh^2 \alpha h} \left(-\frac{h}{2} - \frac{1}{4h\alpha^2} + \frac{\cosh 2\alpha h}{4h\alpha^2} \right) - \frac{fh}{2} \quad (18)$$

Case-(i): Flow for large depths that is for large H:

For large h $\sinh \alpha h \approx \frac{e^{\alpha h}}{2}$; $\cosh \alpha h \approx \frac{e^{\alpha h}}{2}$; $\tanh \alpha h \approx 1$ and neglecting terms of $O\frac{1}{h^3})$ we get

The velocity: $u(y) = \frac{c_1}{\alpha^2}(1 - e^{-\alpha y})$ (19)

Mean velocity: $u(y) = \frac{c_1}{h\alpha^2}\left(h - \frac{1}{\alpha}\right)$ (20)

Temperature:

$$\theta = \frac{y}{h} + \frac{P_r c_1 c_2}{\alpha^2}\left[\frac{1}{h\alpha^2}(h-y) - \frac{y}{2}(h-y) - \frac{e^{-\alpha y}}{\alpha^2}\right] + \frac{Ec_1^2}{\alpha^2}\left[\frac{h-y}{4\alpha^2 h} - \frac{e^{-2\alpha y}}{4\alpha^2}\right] + \frac{fy(h-y)}{2} \quad (21)$$

Mean temperature

$$\bar{\theta}=\frac{h}{2}+\frac{P_r c_2 c_2}{\alpha^2}\left\{\frac{1}{2\alpha^2}-\frac{h^2}{12}-\frac{1}{h\alpha^3}\right\}+\frac{Ec_1^2}{\alpha^2}\left\{\frac{1}{8\alpha^2}-\frac{1}{8h\alpha^3}\right\}+\frac{fh^2}{12} \tag{22}$$

Mean mixed temperature

$$\frac{\int_0^h \theta u dy}{\int_0^h u dy}=\frac{1}{\left(h-\frac{1}{\alpha}\right)}\left[\begin{array}{l}\left(\frac{h}{2}-\frac{1}{\alpha^2 h}\right)+\frac{P_r c_1 c_2}{\alpha^2}\left\{\frac{-h^3}{12}+\frac{h}{\alpha^2}+\frac{1}{h\alpha^4}-\frac{5}{2\alpha^3}\right\}+\\ \frac{Ec_1^2}{\alpha^2}\left\{\frac{h}{8\alpha^2}-\frac{7}{24\alpha^3}+\frac{1}{4\alpha^4 h}\right\}+f\left(\frac{h^3}{12}-\frac{h}{2\alpha}+\frac{1}{2\alpha^2}\right)\end{array}\right] \tag{23}$$

Nusselt Number:

On the bottom:

$$\frac{d\theta}{dy}\Big|_{y=0}=\frac{1}{h}+\frac{P_r c_1 c_2}{\alpha^2}\left[\frac{-1}{\alpha^2 h}-\frac{h}{2}+\frac{1}{\alpha}\right]+\frac{Ec_1^2}{\alpha^2}\left[\frac{-1}{4\alpha^2 h}+\frac{1}{2\alpha}\right]+\frac{fh}{2} \tag{24}$$

On the top

$$\frac{d\theta}{dy}\Big|_{y=h}=\frac{1}{h}+\frac{P_r c_1 c_2}{\alpha^2}\left[\frac{-1}{\alpha^2 h}+\frac{h}{2}\right]+\frac{Ec_1^2}{\alpha^2}\left[\frac{-1}{4\alpha^2 h}\right]-\frac{fh}{2} \tag{25}$$

Case-(ii): Flow for shallow fluids that is for small h:

Retaining terms up to the $o\left(h^2\right)$ and neglecting its higher powers we get

Velocity: $$u\left(y\right)=c_1\left[\frac{\left(2hy-y^2\right)}{2}-\frac{\alpha^2}{24}\left(-4hy^3+y^4\right)\right] \tag{26}$$

Mean velocity: $$\bar{u}=\frac{c_1 h^2}{3} \tag{27}$$

Temperature:

$$\begin{array}{l}\theta\left(y\right)=\frac{y}{h}+\frac{P_r c_1 c_2}{720}[(120hy^3-30y^4)-\alpha^2(-6hy^5+y^6)]+\\ \frac{Ec_1^2}{180}\left[(60hy^3-90h^2y^2-15y^4)-\alpha^2(15h^2y^4-12hy^5+2y^6)\right]+\frac{fy(h-y)}{2}\end{array} \tag{28}$$

Mean temperature:

$$\bar{\theta}=\frac{1}{h}\int_0^h \theta dy=\frac{1}{2}+\frac{fh^2}{12} \tag{29}$$

Mean mixed temperature:

$$\frac{\int_0^h \theta u dy}{\int_0^h u dy}=\frac{(150-31\alpha^2 h^2+21fh^2)}{240} \tag{30}$$

Nusselt Number:

On the bottom

$$\frac{d\theta}{dy}\Big|_{y=0} = \frac{1}{h} + \frac{fh}{2} \qquad (31)$$

On the top

$$\frac{d\theta}{dy}\Big|_{y=h} = \frac{1}{h} - \frac{fh}{2} \qquad (32)$$

CONCLUSIONS

1. It is noticed that the velocity profiles are more steep for large values of α that is the velocity of the fluid decreases with the increase in the value of α (fig 1). It can be observed from figure 2 thickness of the boundary layer decreases as the porosity parameter α increases for large h. In case of shallow fluids it can be observed from fig 3 that the velocity of the flow region increases with the increase in the porosity parameter α .
2. It is evident from the fig 4 that for the increasing values of the pressure gradient (c_1) the mean velocity increases and appears to be decreasing with the increase in the values of α . Figure 5 illustrates that the mean velocity decreases with the increase of the porosity parameter α in the case of large h. Figure 6 illustrates that the mean velocity of the flow region increases with the increasing values of the pressure gradient c_1 and depth of the channel 'h' for shallow fluids.
3. It is observed from fig 7 that the temperature of fluid flow slightly decreases with increasing porosity parameter α when the heat source f=10and from fig 8 it is clear that temperature remains unaltered for the small porosity parameters when f=100.Fig 9 clearly illustrates that the temperature of the fluid gradually increases with the increase in the porosity parameter α and reaches its maximum at half the distance of the channel. In the case of shallow fluids it has been observed that the temperature decreases with the increase in the values of the porosity parameter α Fig 10.
4. Fig 11 illustrates that the mean temperature decreases with the increasing values of the prandtl number 'p'. Fig 12 illustrates that the mean temperature decreases with the increase in the prandtal number for large values of 'h'. Mean temperature increases with the increasing values of the heat source 'f' in the case of shallow fluids Fig 13
5. It is evident from fig 14 mean mixed temperature decreases with the increase in the prandtl number 'p'.It is evident from the Fig 15 that the mean mixed temperature increases with the increase in the values of the prandtl number 'p' and remains unaltered for large 'p' .It is observed that the mean mixed temperature decreases with the increasing values of 'h' Fig16).
6. The rate of heat transfer decreases as the porosity parameter α and prandtl number 'p' increases. Fig 17. Fig18 illustrates that rate of heat transfer on the bottom decreases for the increase in the prandtl number 'p'. The rate of heat transfer on the bottom plate increases with the increasing values of the heat source 'f' Fig19.
7. Heat transfer rate increases with the increase in the values of prandtl number 'p' and the presence of heat source increases the heat transfer rate fig20 With the supply of the constant heat source f=10 the rate of eat transfer increases with the increase in the prandtl number 'p' in the case of deep fluids fig2. Rate of heat transfer decreases with the increasing 'f' on the free surface fig22.

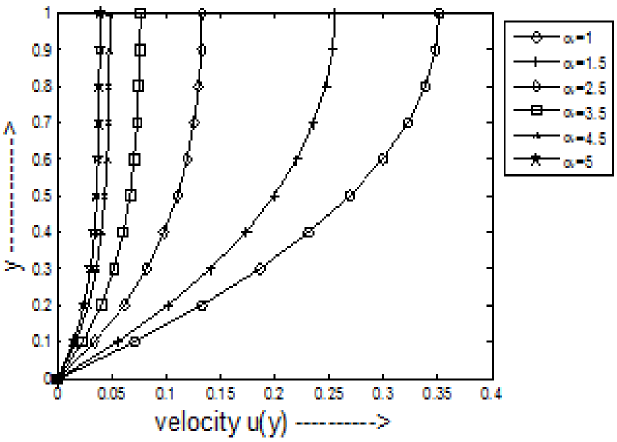


Figure-1: velocity profile for c_1=1 and h=1

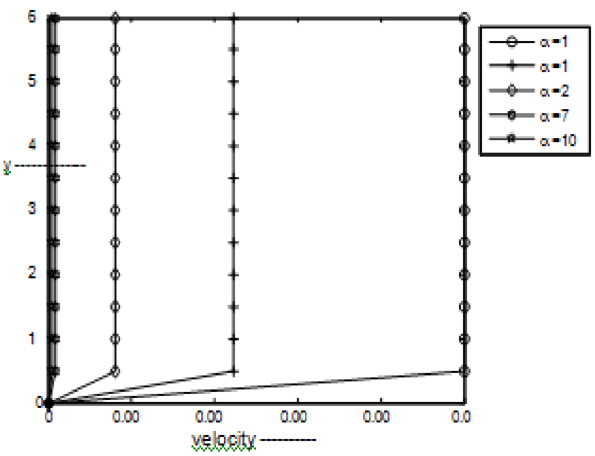


Figure-2: velocity profile for large

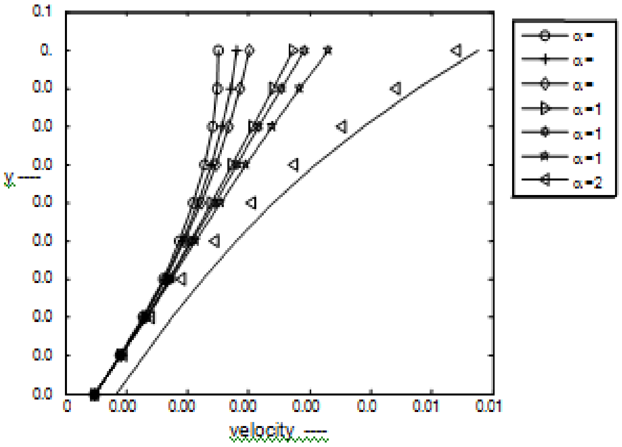


Figure-3: velocity for small

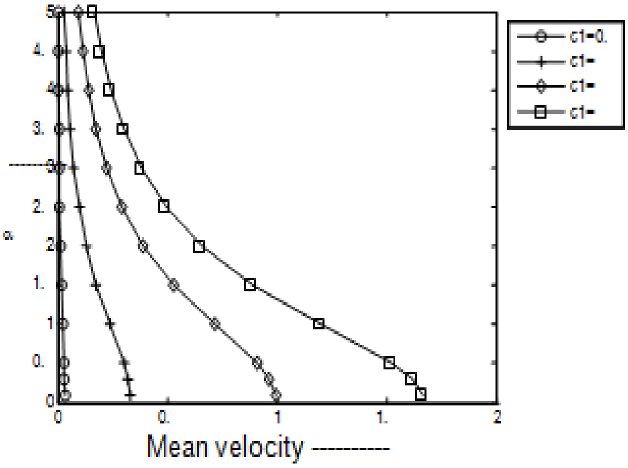


Figure-4: mean velocity for

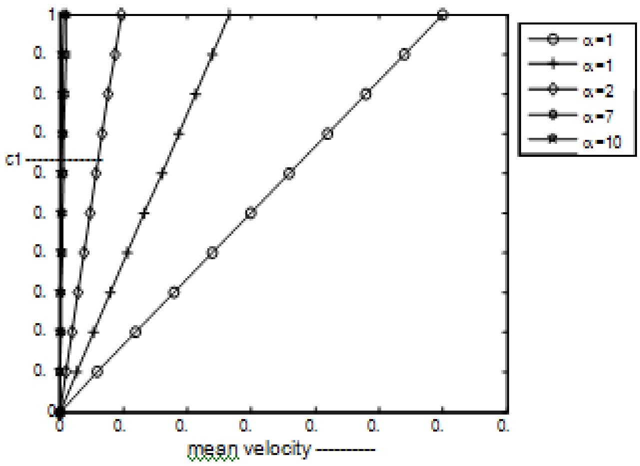


Figure-5: mean velocity profile for large

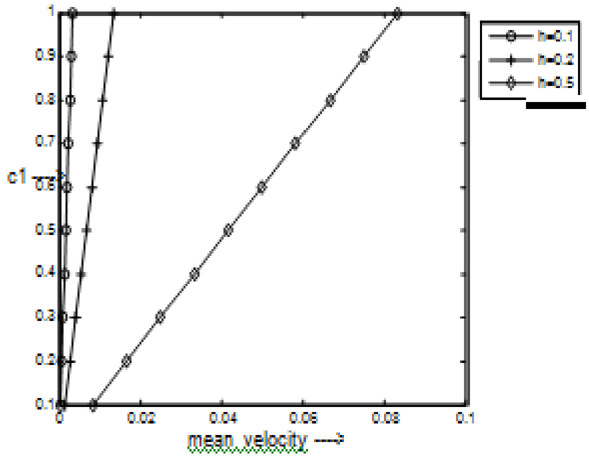


Figure-6: mean velocity for small h=.1,.2,.3

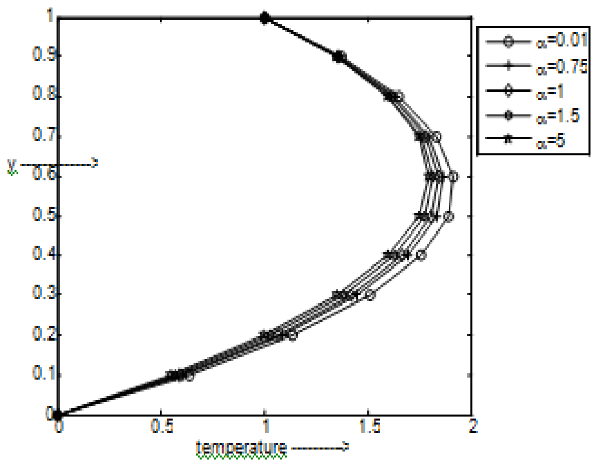


Figure-7: Temperature distribution for $h=1$, $E=5$, $c_1=1$, $c_2=1$, $p=1$, $f=10$,

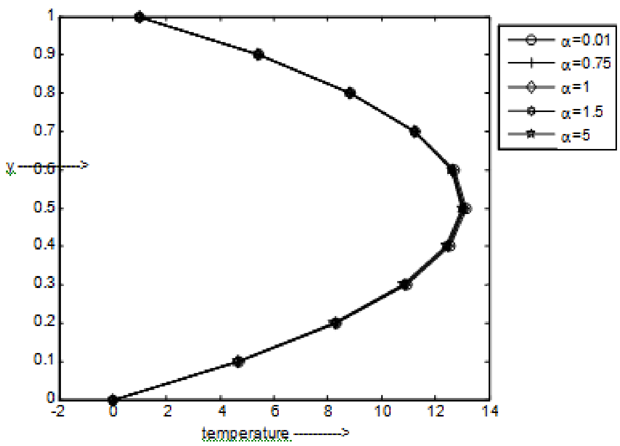


Figure-8: Temperature distribution for $h=1$, $E=5$, $c_1=1$, $c_2=1$, $p=1$, $f=100$

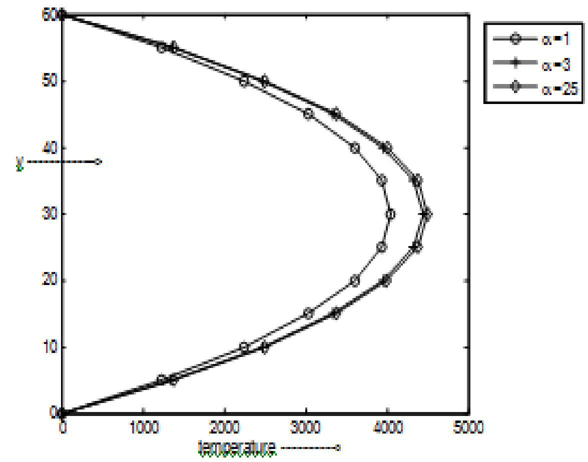


Figure-9: Temperature distribution for large h=30, E=5, c_1=1, c_2=1, p=1, f=10

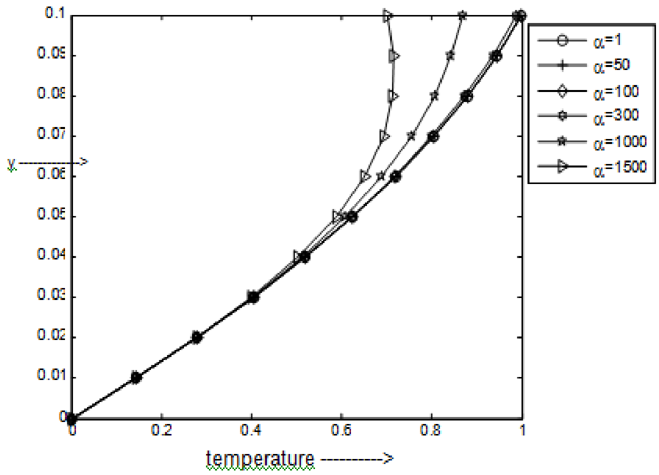


Figure-10: Temperature distribution for p=1, f=100, h=1, E=5, c_1=1, c_2=1

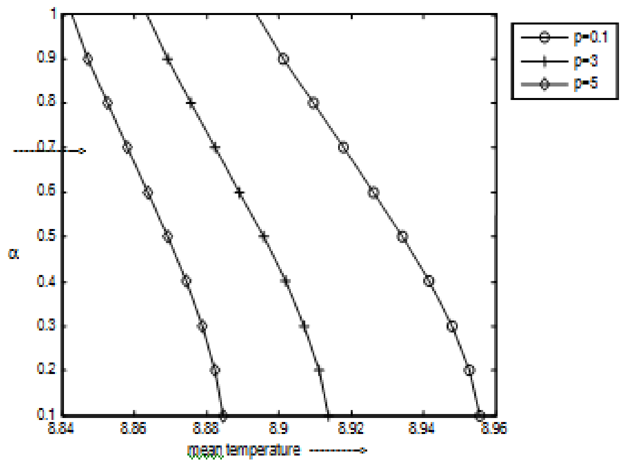


Figure-11: Mean Temperature for h=1, E=5, c_1=1, c_2=.5, f=100

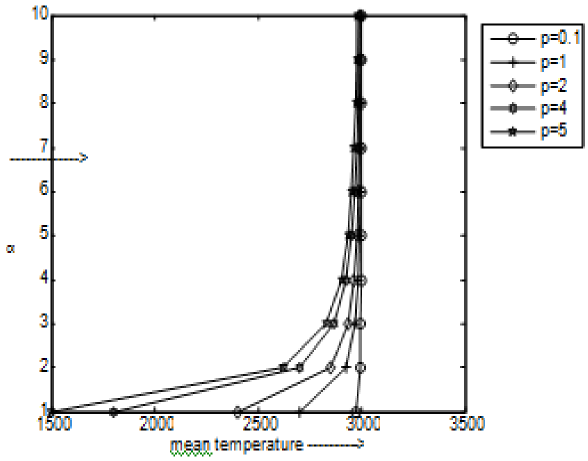


Figure-12: mean temperature distribution for large h=60, E=5, c_1=1, c_2=1, f=10

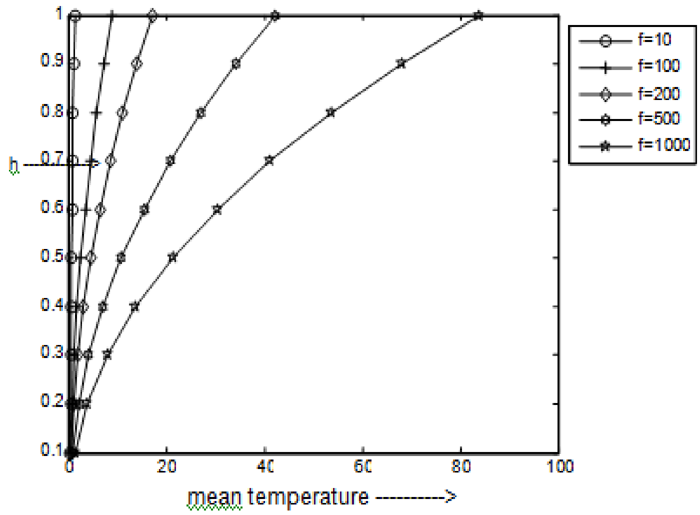


Figure-13: Mean temperature

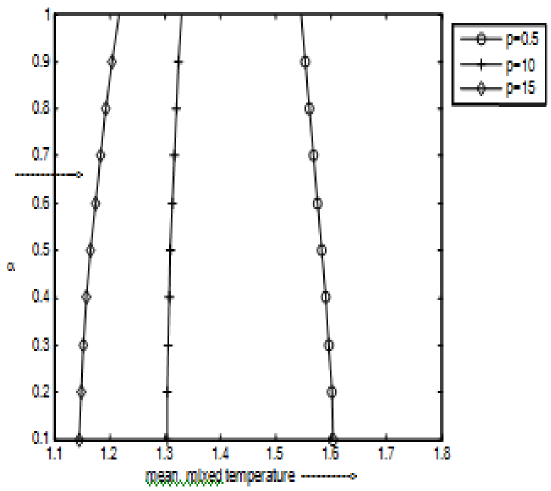


Figure-14: Mean Mixed Temperature for h=1, E=5, c_1=1, c_2=1, f=10

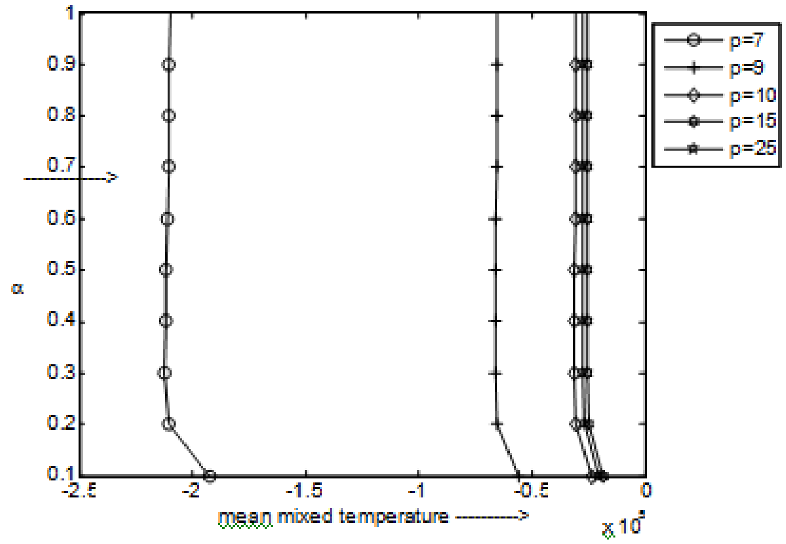


Figure-15: Mean mixed temperature for h=60, f=10; E=5, c_1=1, c_2=1

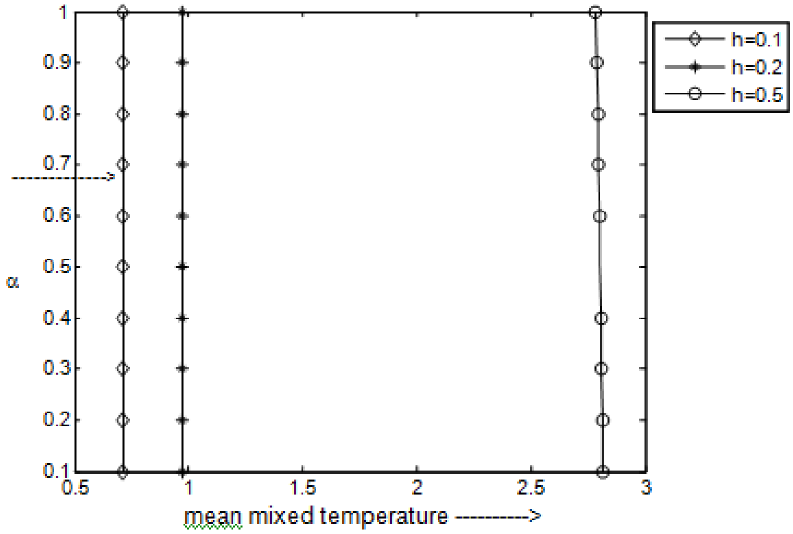


Figure-16: Mean mixed temperature for f=100

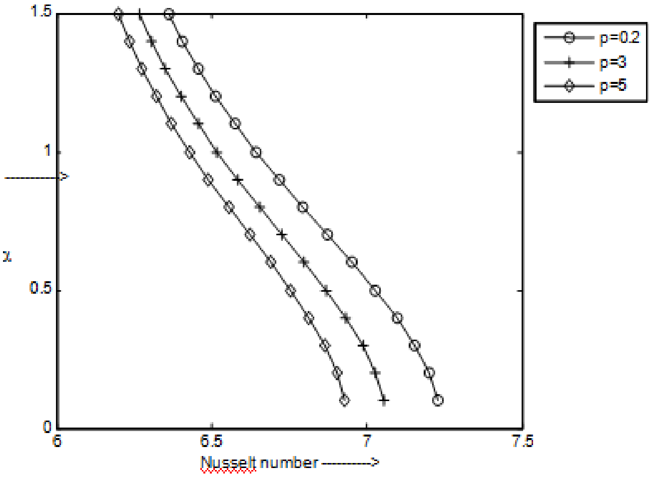


Figure-17: Nusselt number on the bottom for h=1, E=5, c1=1, c2=.5, f=10

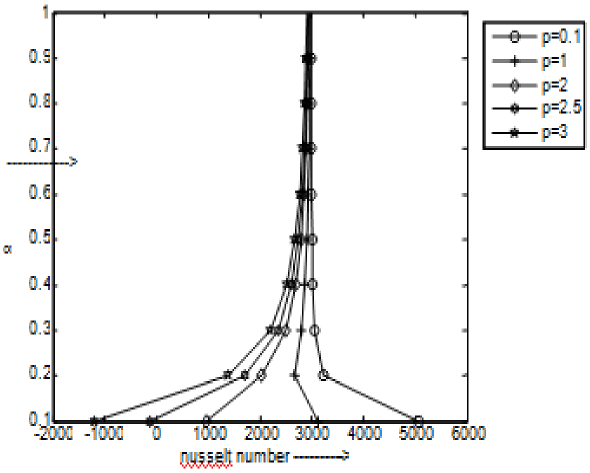


Figure-18: Nusselt number on the bottome for h=60, f=100; E=5, c_1=1, c_2=1

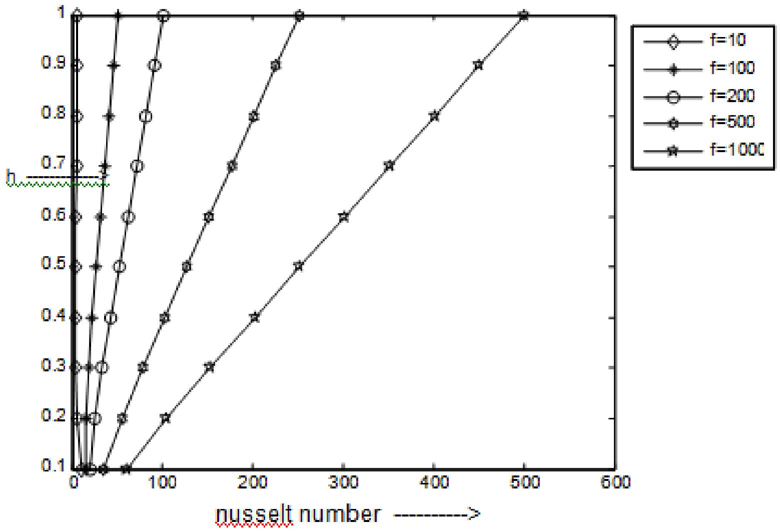


Figure-19: Nusselt number on the bottom

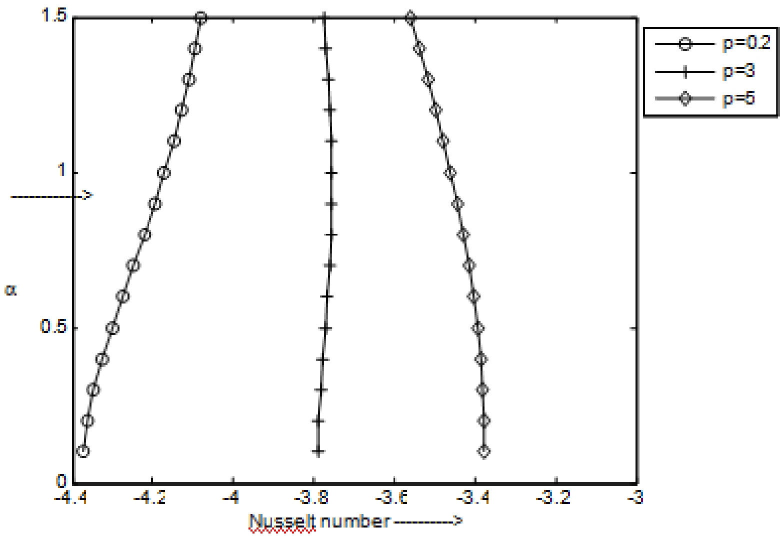


Figure-20: Nusselt number on the top for h=1, E=5, c_1=1, c_2=1, f=10

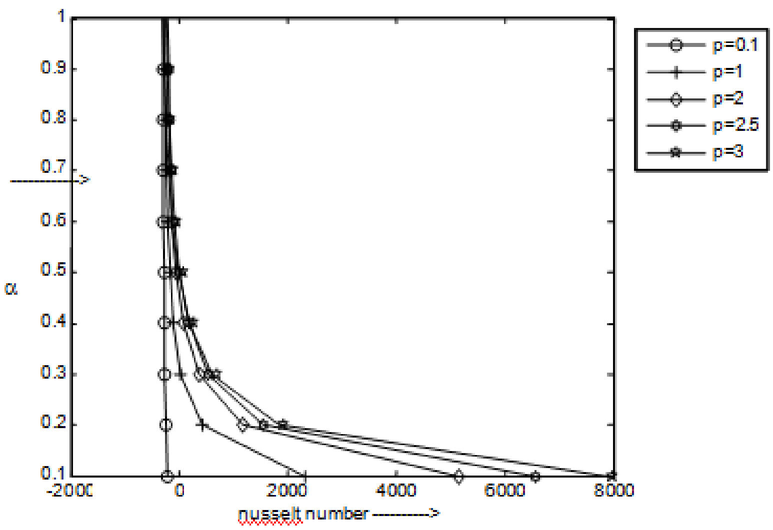


Figure-21: Nnusselt number on the top for h=60, f=10, E=5, c_1=1, c_2=1

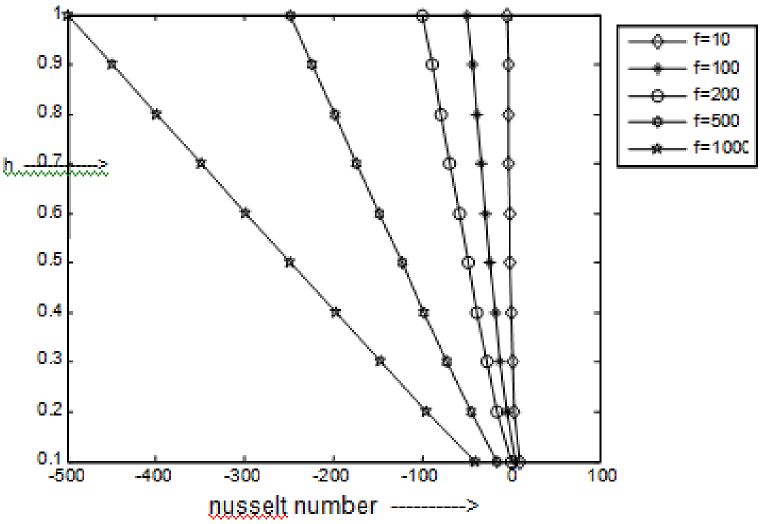


Figure-22: Nusselt number on the top

REFERENCES

1. K.Moinuddin and Prof.N.Ch.Pattabhi ramacharyulu., *Forced convective flow of a viscous liquid of finite depth in a porous medium over a fixed horizontal impermeable plate*, *Journal of Pure and Applied Physics,Vol.23,No.1, pp.99-111(2011).*
2. Mohammed Ameenuddin., *A problem of forced convective heat flow of a viscous liquid in a porous medium over a fixed horizontal impermeable plate for different depths of the channel, International Journal of Mathematical Sciences, Technology and Humanities 8(2016) vol.6, Issue 4,pp:67-80).*
3. K.Moinuddin.,*A fluid flow and heat transfer problem in porous media with a uniformly distributed constant heat source in the flow region, International e journal of Mathematics and Engineering 8(2016),Vol 7 Issue 4,pp 84-93.*
4. *K.Yama Moto and N.Iwamura, Flow with convective acceleration through a porous medium, J. Physical society Japan, Vol. 37,No.3(1976), pp.41-54.*

Source of support: Nil, Conflict of interest: None Declared.