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SOLVING FUZZY TRANSPORTATION PROBLEM USING SUBINTERVAL AVERAGE METHOD OF RANKING

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ABSTRACT

In this paper a new ranking method called Subinterval Average Method is proposed to solve a fuzzy transportation problem. The basic feasible solution has been determined and Modified Distribution method have been utilized to check the optimality of the solution without converting the fuzzy transportation problem into crisp one using the new ranking method. A relevant numerical example is given to clarify the method.

Keywords: *Defuzzification, Fuzzy Number, Ranking of Fuzzy Numbers, Fuzzy Transportation Problem.*

1. INTRODUCTION

The process of production management involves increase of profit by reducing the cost with the values imprecise, the fuzzy set theory could be applied appropriately. Linear programming problems play a vital role as a pioneering tool. Fuzzy sets were introduced by Zadeh and Dieter Klaua in 1965 to represent, manipulate data and information possessing nonstatistical uncertainties [13]. Since the parameters engaged in this process uncertainly, we can use fuzzy linear programming problems. Bellman and Zadeh proposed the concepts of decision making in fuzzy environments [1]. The idea of fuzzy linear programming was first initiated by H.O'heigeartaigh [5].

The process of production management involves maximizing of profit by minimizing the cost with the values imprecise; the fuzzy set theory could be suitably fit. A fuzzy transportation problem is a transportation problem formulated with fuzzy quantities of transportation costs, supply and demand. Many of the existing techniques provide only crisp solutions for the fuzzy transportation problem. Chanas and Kutcha [2] proposed a method to find the optimal solution to the transportation problem with fuzzy coefficients in 1996. Saad and Abbas [7] discussed an algorithm to solve a transportation problem in fuzzy environment in 2003. Gani and Razak [3] discussed a two stage cost minimizing fuzzy transportation in which the demand and supply quantities are trapezoidal fuzzy numbers in 2006. Dinagar and Palanivel [4] studied FTP with trapezoidal fuzzy numbers and they developed a method to find optimal solution in terms of fuzzy numbers in 2009. Pandian and Natarajan [6] proposed a new algorithm namely fuzzy zero point method to find optimal solution of a FTP with trapezoidal fuzzy numbers in 2010.

Since fuzzy numbers are denoted by possibility distribution, it is tough to order clearly the ascending or descending order. A right method for ordering the fuzzy numbers is by the use of a ranking function. The ranking function maps each fuzzy number into the real line. A ranking function is a function $R:(R) \rightarrow R$ which maps each fuzzy number into the real line, where a natural order exists. There are so many ranking methods available, nowadays. Among them, the notable procedures are Lexicographic screening procedure discovered by Wang, M. L., Wang, H. F., Lung, L. C. (2005) [11], Area between centroid and its original point method [12] by Wang, Y. J., Lee, H. S. (2008); SD of PILOT procedure [8]; and Area method [9] and A Revised approach of PILOT ranking procedure [10] by Stephen Dinagar, D., and Kamalanathan, S.

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There are many methods to find the basic feasible solution, such as North–West Corner Rule, Row Minima Method Column Minima Method, Matrix Minima Method (Lowest Cost Entry Method), Vogel's Approximation Method (Unit Cost Penalty Method) (VAM), Russell's method. Here we use (VAM) to find the basic feasible solution and Modified Distribution (MODI) method to find the optimal solution. We solve the problem by using the new ranking method Subinterval Average method without changing the fuzzy number to crisp form and the solution is given as the fuzzy number. The rest of the article is organized as follows: the next section provides the definitions of fuzzy numbers; Section 3 explains the ranking procedure; the algorithm is explained in Section 4; Section 5 is provided with the numerical examples for solving fuzzy transportation problems.

2. PRELIMINARIES

Definition 1: Fuzzy set: A fuzzy set A in a nonempty set X is categorized by its membership function $\mu_A(x) \rightarrow [0,1]$ and $\mu_A(x)$ is meant as the degree of membership of element *x* in fuzzy set A for each *x* belongs to *X*.

Definition 2: A fuzzy number A is a fuzzy set of the real line with a normal, convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by F.

Definition 3: A trapezoidal fuzzy number denoted by $A = (a_1, a_2, a_3, a_4)$ which has the membership function as

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases}$$

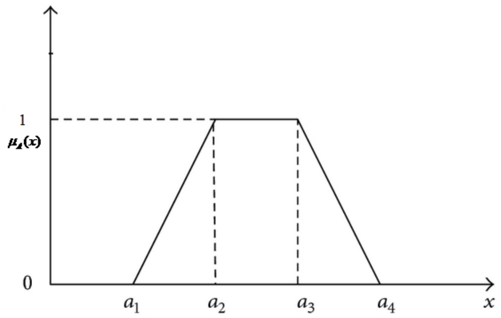

Figure-2: Trapezoidal Fuzzy number $A^4 = (a_1, a_2, a_3, a_4)$

Definition 5: A pentagonal fuzzy number is represented as $A = (a_1, a_2, a_3, a_4, a_5)$ whose membership function is

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \left[\dfrac{x-a_1}{a_2-a_1}\right], & a_1 \le x \le a_2 \\ \left[\dfrac{x-a_2}{a_3-a_2}\right], & a_2 \le x \le a_3 \\ 1, & x = a_3 \\ \left[\dfrac{a_4-x}{a_4-a_3}\right], & a_3 \le x \le a_4 \\ \left[\dfrac{a_5-x}{a_5-a_4}\right], & a_4 \le x \le a_5 \\ 0, & x > a_5 \end{cases}$$

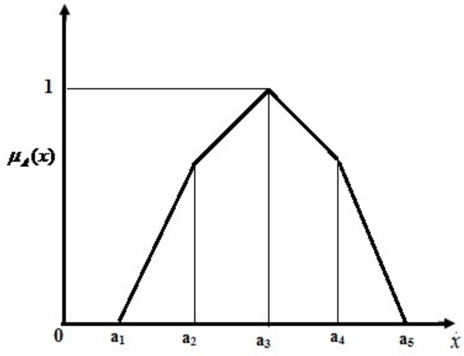

Figure-3: Pentagonal Fuzzy number $A^5 = (a_1, a_2, a_3, a_4, a_5)$.

3. RANKING PROCEDURE

The x-axis has the discrete real points a_i, $i = 1, 2, ..., n$. It has x_n number of intervals between the discrete points. Each sub interval has a discrete upper and lower limit (i.e., least upper bound and greatest lower bound of each interval), say, a_j, a_k, $j \le k$, where $1 \le j \le k \le n$ and they could be averaged as $(a_j + a_k)/2$. The average of average of all such intervals is our ranking function. If $i = n$, then the number of intervals is x_n, where i is the corresponding triangular number and it is given by $x_n = n(n+1)/2$.

The ranking function is $\Re\left(A^i\right) = \frac{1}{i}\sum_{i=1}^{n} a_i$ (1)

If $\Re\left(A_1^i\right)$ and $\Re\left(A_2^i\right)$ are two fuzzy numbers, then

(i) $\Re\left(A_1^i\right) < \Re\left(A_2^i\right) \Rightarrow A_1^i \prec A_2^i$
(ii) $\Re\left(A_1^i\right) > \Re\left(A_2^i\right) \Rightarrow A_1^i \succ A_2^i$
(iii) $\Re\left(A_1^i\right) = \Re\left(A_2^i\right) \Rightarrow A_1^i \approx A_2^i$
(iv) $\Re\left(A_1^i\right) + \Re\left(A_2^i\right) \Rightarrow \Re\left(A_1^i + A_2^i\right)$
(v) $\Re\left(A_1^i\right) - \Re\left(A_2^i\right) \Rightarrow \Re\left(A_1^i - A_2^i\right)$
(vi) $\Re\left(A_1^i\right) \bullet \Re\left(A_2^i\right) \Rightarrow i^2\Re\left(A_1^i \bullet A_2^i\right)$

Working Procedure

(i) Determine the value of i according to what type of fuzzy number has to be dufuzzified, e.g., if $i = 3$, A^i is triangular fuzzy number; if $i = 4$, A^i is trapezoidal fuzzy number, etc.
(ii) Using the value of i in the relation (1).
(iii) If $\text{Average}\left(A_1^i\right) < \text{Average}\left(A_2^i\right)$, then $A_1^i \prec A_2^i$
(iv) If $\text{Average}\left(A_1^i\right) > \text{Average}\left(A_2^i\right)$, then $A_1^i \succ A_2^i$, Otherwise $A_1^i \approx A_2^i$

4. METHOD OF SOLVING FUZZY TRANSPORTATION PROBLEM

Transportation dealt with the transporting a commodity from some sources to some destinations. The objective is to determine the amount to be shifted from each source to each destination such that the total transportation cost is minimum. The French mathematician Gaspard Monge initiated the concept in 1781. At first it was studied mathematically by A.N. Tolstoi in 1920.

The transportation problem can be modelled as a standard linear programming problem, which can then be solved by the simplex method. Typically, a commodity is to be transported from m sources to n destinations and their capacities are $\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_m$ and $\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n$ respectively. $\tilde{c}_{ij}$ be the unit transportation cost from source i to destination j associated with transporting unit of product from source i to destination j. This penalty may be cost or delivery time or safety of delivery etc. A variable $\tilde{x}_{ij}$ represents the unknown quantity to be shipped from source i to destination j. Let $\tilde{a}_i$ be the amount of the product available at origin i and $\tilde{b}_{ij}$ be the amount of the product required at destination j. If shipping cost, are assumed to be proportional to the amount shipped from each origin to each destination so as to minimize total shipping cost turns out to be a linear programming problem.

In this paper we solve the fuzzy transportation problem with supplies and demands using trapezoidal fuzzy quantities. We solve the problem without converting it into crisp quantities. Consider a fuzzy transportation with m sources and n destinations with triangular fuzzy numbers. Let $\tilde{a}_i\left(\tilde{a}_i \geq \tilde{0}\right)$ be the fuzzy availability at source *i* and $\tilde{b}_j\left(\tilde{b}_j \geq \tilde{0}\right)$ be the fuzzy requirement at destination *j*. Let $\tilde{x}_{ij}$ denote the number of fuzzy units to be transported from source *i* to destination *j*. Then the problem is to determine a feasible way of transporting the available amount at each source to satisfy the demand at each destination so that the total transportation cost is minimized.

The mathematical formulation of the fuzzy transportation problem whose parameters are triangular fuzzy numbers under the case that the total supply is equivalent to the total demand is given by:

$$\min \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\tilde{x}_{ij}$$

Subject to $\sum_{j=1}^{n}\tilde{x}_{ij} = \tilde{a}_i,\ i = 1, 2, \ldots, m$

$$\sum_{i=1}^{m}\tilde{x}_{ij} = \tilde{b}_j,\ j = 1, 2, \ldots, n$$

$$\sum_{i=1}^{m}\tilde{a}_i \approx \sum_{j=1}^{n}\tilde{b}_j,\ i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n$$

and $\tilde{x}_{ij} \geq \tilde{0}$

This problem can also be represented as follows:

Destinations

		1	……………..	n	Supply
Sources	1	$\tilde{c}_{11}$	……………..	$\tilde{c}_{1n}$	$\tilde{a}_1$
	⋮ :			⋮ :	⋮ :
	M	$\tilde{c}_{m1}$	……………..	$\tilde{c}_{mn}$	$\tilde{a}_m$
	Demand	$\tilde{b}_1$	……………..	$\tilde{b}_n$	

Algorithm to find the basic feasible solution and checking optimality

Step-1: Check if $\sum_{i=1}^{m}\tilde{a}_i \approx \sum_{j=1}^{n}\tilde{b}_j,\ i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. If not add a row or column with zero cost and convert this into a balanced one by adjusting source or destination whichever has to be added.

Step-2: Find the difference (penalty) between the smallest and next smallest costs in each row (column) and write them in brackets against the corresponding row (column).

Step-3: Identify the row (or column) with largest penalty. If a tie occurs, break the tie arbitrarily. Choose the cell with smallest cost in that selected row or column and allocate as much as possible to this cell and cross out the satisfied row or column and go to step (4).

Step-4: Again compute the row and column penalties for the reduced transportation table and then go to step (3). Repeat the procedure until all the rim requirements are satisfied.

Modified Distribution (MODI) Method

Step-1: Find the initial basic feasible solution of the given problem by Northwest Corner rule (or) Least Cost method (or) VAM.

Step-2: Check the number of occupied cells. If there are less than $m + n - 1$, there exists degeneracy and we introduce a very small positive assignment of ϵ (≈0) in suitable independent positions, so that the number of occupied cells is exactly equal to $m + n - 1$.

Step-3: Find the set of values u_i, v_j (i = 1, 2, 3, … , m; j = 1, 2, 3, … , n) from the relation $c_{ij} = u_i + v_j$ for each occupied cell (i, j), by starting initially with $u_i = 0$ or $v_j = 0$ preferably for which the corresponding row or column has maximum number of individual allocations.

Step-4: Find $u_i + v_j$ for each occupied cell (i, j) and enter at the upper right corner of the corresponding cell (i, j).

Step-5: Find the cell evaluations $\tilde{d}_{ij} = \tilde{c}_{ij} - (u_i + v_j)$ for each occupied cell (i, j).

Step-6: Examine the cell evaluations d_{ij} for all unoccupied cells (i, j) and conclude that

(i) If all $d_{ij} > 0$, then the solution is optimal and unique.

(ii) If all $d_{ij} > 0$, with at least one $d_{ij} = 0$, then the solution under the test is optimal and an alternative optimal solution exists.

(iii) If at least one $d_{ij} < 0$, then the solution is not optimal.

5. NUMERICAL EXAMPLE

We take a fuzzy transportation problem as example with pentagonal fuzzy number. Without converting the fuzzy numbers in to crisp one with the aid of the ranking function we found the basic feasible solution and then the optimality was checked using the MODI method. In ranking function equation (1) provided, if t = 2, then the FLPP is of interval fuzzy number; if t = 3, then the FLPP is of triangular fuzzy number, etc.

Example 1: Solve the transportation problem:

(9,16,20,27,33)	(4,11,16,22,27)	(10,16,25,32,42)	(3,6,11,21,24)	(2, 5, 9, 13, 26)
(5,9,16,23,32)	(7,12,15,24,32)	(2,7,11,20,30)	(7,12,23,32,41)	(4, 7, 11, 17, 26)
(8,21,35,42,54)	(9,16,28,39,43)	(4,10,18,24,34)	(23,34,42,49,57)	(5, 9, 16, 28, 37)
(1,3,5,9,12)	(2,6,10,14,18)	(2,8,10,16,24)	(2,5,14,21,33)	

Solution to the problem is as follows:

The table with the demand and supply allotted in the cost cell is enclosed separately.

No. of allocations = 6
m + n – 1 = 6

Since number of allocations = m + n -1, the initial solution is non-degenerate. We are to check the optimality further using MODI method.

To check the optimality

Finding u_i and v_j from occupied cells using the relation $c_{ij} = u_i + v_j$:

Since second row has more number of allocations, put $u_2 = 0$.

$c_{21} = u_2 + v_1$ = (0,0,0,0,0) + v_1 = (5,9,16,23,32); v_1 = (5,9,16,23,32)
$c_{22} = u_2 + v_2$ = (0,0,0,0,0) + v_2 = (7,12,15,24,32); v_2 = (7,12,15,24,32)
$c_{24} = u_2 + v_4$ = (0,0,0,0,0) + v_4 = (7,12,23,32,41); v_4 = (7,12,23,32,41)
$c_{14} = u_1 + v_4$ = (3,6,11,21,24); u_1 + (7,12,23,32,41) = (3,6,11,21,24); u_1 = (-38,-26,-12,9,17)
$c_{32} = u_3 + v_2$ = (9,16,28,39,43); u_3 +(7,12,15,24,32) = (9,16,28,39,43); u_3 = (-23,-18,13,27,36)
$C_{33} = u_3 + v_3$ = (4,10,18,24,34); (-23,-18,13,27,36) + v_3 = (4,10,18,24,34); v_3 = (-32,-17,5, 42,57)

After finding the values of u_i and v_j, the transportation table is:

(9,16,20,27,33)	(4,11,16,22,27)	(10,16,25,32,42)	(3,6,11,21,24) (2,5,9,13,26)	u_1 = (-38, -26, -12, 9, 17)
(5,9,16,23,32) (1,3,5,9,12)	(7,12,15,24,32) (-39,-8,1,22,49)	(2,7,11,20,30)	(7,12,23,32,41) (-24,-8,5,16,31)	u_2 = (0, 0, 0, 0, 0)
(8,21,35,42,54)	(9,16,28,39,43) (-19,-7,6,20,35)	(4,10,18,24,34) (2,8,10,16,24)	(23,34,42,49,57)	u_3 = (-23,-18, 13, 27, 36)

v_1 = (5,9,16,23,32) v_2 =(7,12,15,24,32) v_3 = (-32,-17,5, 42,57) v_4 =(7,12,23,32,41)

Finding d_{ij} using the relation $d_{ij} = c_{ij} - (u_i + v_j)$ in unoccupied cells:

$\tilde{d}_{11} = \tilde{c}_{11} - (\tilde{u}_1 + \tilde{v}_1)$ = (9,16,20,27,33) – [(-38,-26,-12,9,17) + (5,9,16,23,32)] = (-40,-16,16,44,66)

$\tilde{d}_{12} = \tilde{c}_{12} - (\tilde{u}_1 + \tilde{v}_2)$ = (4,11,16,22,27) – [(-38,-26,-12,9,17) + (7,12,15,24,32)] = (-45,-22,13,36,58)

$\tilde{d}_{13} = \tilde{c}_{13} - (\tilde{u}_1 + \tilde{v}_3)$ = (10,16,25,32,42) – [(-38,-26,-12,9,17) + (-32,-17,5, 42,57)] = (-64,-35,32,75,112)

$\tilde{d}_{23} = \tilde{c}_{23} - (\tilde{u}_2 + \tilde{v}_3)$ = (2,7,11,20,30) – [(0,0,0,0,0) + (-32,-17,5, 42,57)] = (-55,-35,6,37,62)

$\tilde{d}_{31} = \tilde{c}_{31} - (\tilde{u}_3 + \tilde{v}_1)$ = (8,21,35,42,54) – [(-23,-18,13,27,36) + (5,9,16,23,32)] = (-60,-29,6,51,72)

$\tilde{d}_{34} = \tilde{c}_{34} - (\tilde{u}_3 + \tilde{v}_4)$ = (23,34,42,49,57) – [(-23,-18,13,27,36) + (7,12,23,32,41)] = (-54,-25,6,55,73)

Since all $d_{ij} \geq 0$, the optimal solution is reached.

The solution is $\tilde{x}_{14}$ = (2,5,9,13,26); $\tilde{x}_{21}$ = (1,3,5,9,12); $\tilde{x}_{22}$ = (-39,-8,1,22,49); $\tilde{x}_{24}$ = (-24,-8,5,16,31);
$\tilde{x}_{32}$ = (-19,-7, 6, 20, 35); $\tilde{x}_{33}$ = (2, 8, 10, 16, 24)

Transportation cost = (3, 6, 11, 21, 24) × (2, 5, 9, 13, 26) + (5, 9, 16, 23, 32) × (1, 3, 5, 9, 12) + (7, 12, 15, 24, 32) × (-39,-8, 1, 22, 49) + (7, 12, 23, 32, 41) × (-24, -8, 5, 16, 31) + (9, 16, 28, 39, 43) × (-19, -7, 6, 20, 35) + (4, 10, 18, 24, 34) × (2, 8, 10, 16, 24) = (237, 460, 796, 1178, 1489)
which is Rs. 832.

6. CONCLUSION

In this work, the fuzzy transportation problem without converting in to a crisp valued transportation problem using a new ranking method is solved, which is very easy for computation. The same could be solved for any fuzzy number. The ranking method could be applied to some other field involving fuzzy number, in future.

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