

**CHAOS CONTROL IN THE PLANNER MAGNETIC BINARY PROBLEM
WITH VARIABLE MASS**

MOHD. ARIF*

**Department of Mathematics,
Zakir Husain Delhi College (Delhi University), New Delhi India - 11002.**

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ABSTRACT

This article addresses the complete synchronization and anti synchronization behavior of the magnetic binary problem when the charged particle has the variable mass. Here we have designed a non linear controller based on the Lyapunov stability theory. Numerical simulations are performed to plot time series analysis graphs of the master system and the slave system which further illustrate the effectiveness of the proposed control technique.

***Key words:** Space dynamics; magnetic binary problem; complete synchronization; Lyapunov stability theory; Jean's Law; variable mass.*

1. INTRODUCTION

In the last few decades, much effort has been devoted to the study of nonlinear dynamical systems and their various properties. Chaos control and synchronization are important research fields leveling to affect dynamics of chaotic systems in order to apply them for different kinds of applications that can be examined within many different scientific research. At present, there are different kinds of control methods and techniques that have been proposed for carrying out chaos control and synchronization of chaotic dynamical systems. Chaotic synchronization did not attract much attention until Pecora and Carroll introduced a method to synchronize two identical chaotic systems with different initial conditions in [17] and they demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device, the active control scheme proposed by E. W. Bai and K. E. Lonngren [2] has been received and successfully implemented in almost all the field of nonlinear sciences for synchronization for different systems with various techniques.

The synchronization problem via nonlinear control scheme is studied by Chen and Han [5], Chen [6], Ju H. Park [8], Amir Abbas Emadzadeh, and Mohammad Haeri [1], M. Mossa Al-sawalha, M.S.M. Noorani in [10] and Moh. Arif [14] and [15].

Jeans [9] has studied the two-body problem with variable mass. Omarov [16] has also discussed the restricted problem of perturbed motion of two bodies with variable mass. Shrivastava and Ishwar [18] have studied the circular restricted three body problem with variable mass with the assumption that the mass of the infinitesimal body varies with respect to time. Singh *et al.* [7] has discussed the non-linear stability of equilibrium points in the restricted three body with variable mass

The several cases of the magnetic binaries problem have been studied by A. Mavrnagais [11], [12] and [13], Bhatnagar *et al.* [3] and Bhatnagar and Mohd. Arif [4].

In this article we have discussed the complete synchronization and anti synchronization behavior of the magnetic-binaries problem when the charged particle has a variable mass, here we have designed a nonlinear controller based on the Lyapunov stability in both cases. The system under consideration is chaotic for some values of parameter involved in the system. Here two systems (master and slave) are synchronized and start with different initial conditions. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time.

***Corresponding Author: Mohd. Arif*, Department of Mathematics,
Zakir Husain Delhi College (Delhi University), New Delhi India - 11002.***

2. EQUATION OF MOTION

Two dipoles (the primaries), with magnetic fields move under the influence of gravitational forces and a charged particle P of charge q_1 and variable mass m moves in the vicinity of these dipoles. The question of the magnetic-binaries problem is to describe the motion of this particle. Here we assumed that the distance between the primaries as the unit of distance and the coordinate of one primary is $(\mu, 0, 0)$ then the other is $(\mu-1, 0, 0)$. We also assumed that the sum of their masses as the unit of mass. If mass of the one primaries μ then the mass of the other is $(1-\mu)$.

The equation of motion in the rotating coordinate system including the effect of the gravitational forces of the primaries on the charged particle P written as:

$$\ddot{x} + \frac{\dot{m}}{m} (\dot{x} - y) - 2\dot{y} f = -\frac{1}{m} U_x \tag{1}$$

$$\ddot{y} + \frac{\dot{m}}{m} (\dot{y} + x) + 2\dot{x} f = -\frac{1}{m} U_y \tag{2}$$

Where

$$f = 1 - \frac{1}{m} \left(\frac{1}{\rho_1^3} + \frac{\lambda}{\rho_2^3} \right), \quad U_x = \frac{\partial U}{\partial x} \quad \text{and} \quad U_y = \frac{\partial U}{\partial y}$$

$$U = -\frac{m}{2} (x^2 + y^2) - (x^2 + y^2) \left\{ \frac{1}{\rho_1^3} + \frac{\lambda}{\rho_2^3} \right\} - x \left\{ \frac{\mu}{\rho_1^3} - \frac{\lambda(1-\mu)}{\rho_2^3} \right\} - \frac{m(1-\mu)}{\rho_1} - \frac{m\mu}{\rho_2} \tag{3}$$

Here we assumed

1. Primaries participate in the circular motion around their centre of mass
2. Position vector of P at any time t be $\vec{p} = (xi + yj)$ referred to a rotating frame of reference $O(x, y)$ which is rotating with the same angular velocity $\vec{\omega} = (0, 0, 1)$ as those the primaries.
3. Initially the primaries lie on the x -axis.
4. The unit of time in such a way that the gravitational constant G has the value unity and $q_1 = c$ where c is the velocity of light.
 $\rho_1^2 = (x - \mu)^2 + y^2, \quad \rho_2^2 = (x + 1 - \mu)^2 + y^2, \quad \lambda = \frac{M_2}{M_1}$ (M_1, M_2 are the magnetic moments of the primaries which lies perpendicular to the plane of the motion).

The variation of mass of the charged particle P is given by (Jeans law)

$$\frac{dm}{dt} = -\alpha m^n \quad \text{i.e.} \quad \frac{\dot{m}}{m} = -\alpha m^{n-1} \tag{4}$$

Where α is a constant coefficient and $n \in [0.4, 4.4]$

Now introduce the space-time transformation as:

$$x = \xi \gamma^{-q}, \quad y = \eta \gamma^{-q}, \quad dt = \gamma^{-k} d\tau$$

$$\rho_1 = r_1 \gamma^{-q}, \quad \rho_2 = r_2 \gamma^{-q}, \quad \gamma = \frac{m}{m_0} < 1$$

Where m_0 is the mass of the charge particle P at time $t = 0$.

Differentiating x and y with respect to t twice and Putting the values of $x, y, \dot{x}, \dot{y}, \ddot{x}, \ddot{y}, U_x, U_y$ and $\frac{\dot{m}}{m}$ in equations (1) and (2) and after some simplification we get,

$$\xi'' + \beta \xi' (2q - k - 1) \gamma^{n-k-1} - \beta^2 q \xi (n - q) \gamma^{2(n-k-1)} - 2\eta' \gamma^{-k} \left[1 - \frac{\gamma^{3q}}{\gamma m_0} \left\{ \frac{1}{r_1^3} + \frac{\lambda}{r_2^3} \right\} \right]$$

$$- \beta \eta \gamma^{\frac{n-q-1}{2k-q}} \left[1 - 2q \left\{ 1 - \frac{\gamma^{3q}}{\gamma m_0} \left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3} \right) \right\} \right] = -\frac{\gamma^{2q-2k-1}}{m_0} \frac{\partial U}{\partial \xi} \tag{5}$$

$$\eta'' + \beta \eta' (2q - k - 1) \gamma^{n-k-1} - \beta^2 q \eta (n - q) \gamma^{2(n-k-1)} + 2\xi' \gamma^{-k} \left[1 - \frac{\gamma^{3q}}{\gamma m_0} \left\{ \frac{1}{r_1^3} + \frac{\lambda}{r_2^3} \right\} \right]$$

$$+ \beta \xi \gamma^{\frac{n-q-1}{2k-q}} \left[1 - 2q \left\{ 1 - \frac{\gamma^{3q}}{\gamma m_0} \left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3} \right) \right\} \right] = -\frac{\gamma^{2q-2k-1}}{m_0} \frac{\partial U}{\partial \eta} \tag{6}$$

To eliminate the non-variational factor from equations (5) and (6) we assume

$$2q - k - 1 = 0, \quad n - k - 1 = 0, \quad n = 1, \quad k = 0, \quad q = \frac{1}{2}, \quad \beta = \alpha.$$

Thus we have

$$\xi'' - 2\eta' \left[1 - \frac{\sqrt{\gamma}}{m_0} \left\{ \frac{1}{r_1^3} + \frac{\lambda}{r_2^3} \right\} \right] = \frac{\beta^2 \xi}{4} - \frac{\beta \eta \gamma^{\frac{3}{2}}}{m_0} \left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3} \right) - \frac{1}{m_0} \frac{\partial U}{\partial \xi} \tag{7}$$

$$\eta'' + 2\xi' \left[1 - \frac{\sqrt{\gamma}}{m_0} \left\{ \frac{1}{r_1^3} + \frac{\lambda}{r_2^3} \right\} \right] = \frac{\beta^2 \eta}{4} + \frac{\beta \xi \gamma^{\frac{3}{2}}}{m_0} \left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3} \right) - \frac{1}{m_0} \frac{\partial U}{\partial \eta} \tag{8}$$

Where

$$U = -\frac{m_0}{2} (\xi^2 + \eta^2) - (\xi^2 + \eta^2) \left\{ \frac{1}{r_1^3} + \frac{\lambda}{r_2^3} \right\} \gamma^{\frac{1}{2}} - \gamma \xi \left\{ \frac{\mu}{r_1^3} - \frac{\lambda(1-\mu)}{r_2^3} \right\} - \gamma^{\frac{3}{2}} \left(\frac{m_0(1-\mu)}{r_1} + \frac{m_0\mu}{r_2} \right) \tag{9}$$

3. COMPLETE SYNCHRONIZATION

Let

$$\xi = \xi_1, \xi' = \xi_2, \eta = \xi_3, \eta' = \xi_4$$

Then the equation (7) and (8) can be written as:

$$\xi'_1 = \xi_2 \tag{10}$$

$$\xi'_2 = 2\xi_4 + \xi_1 \left(\frac{\beta^2}{4} - 1\right) + A_1 \tag{11}$$

$$\xi'_3 = \xi_4 \tag{12}$$

$$\xi'_4 = -2\xi_2 + \xi_3 \left(\frac{\beta^2}{4} - 1\right) + A_2 \tag{13}$$

Where

$$A_1 = -\frac{1}{m_0} \left(\beta \xi_3 \gamma^{\frac{3}{2}} + 2 \xi_1 \sqrt{\gamma} \right) \left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3} \right) + \frac{3\sqrt{\gamma}}{m_0} (\xi_1^2 + \xi_3^2) \left\{ \frac{(\xi_1 - \mu\sqrt{\gamma})}{r_1^5} + \frac{\lambda(\xi_1 + \sqrt{\gamma} - \mu\sqrt{\gamma})}{r_2^5} \right\} \\ - \frac{\gamma}{m_0} \left\{ \frac{\mu}{r_1^3} - \frac{\lambda(1-\mu)}{r_2^3} \right\} + \frac{3\gamma\xi_1}{m_0} \left\{ \frac{\mu(\xi_1 - \mu\sqrt{\gamma})}{r_1^5} + \frac{\lambda(1-\mu)(\xi_1 + \sqrt{\gamma} - \mu\sqrt{\gamma})}{r_2^5} \right\} + \gamma^{\frac{3}{2}} \left\{ \frac{(1-\mu)(\xi_1 - \mu\sqrt{\gamma})}{r_1^3} \right\} \\ + \gamma^{\frac{3}{2}} \left\{ \frac{\mu(\xi_1 + \sqrt{\gamma} - \mu\sqrt{\gamma})}{r_2^3} \right\} - \frac{2\xi_4\sqrt{\gamma}}{m_0} \left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3} \right).$$

$$A_2 = \frac{1}{m_0} \left(\beta \xi_1 \gamma^{\frac{3}{2}} + 2\sqrt{\gamma} \xi_2 - 2 \xi_3 \sqrt{\gamma} \right) \left(\frac{1}{r_1^3} + \frac{\lambda}{r_2^3} \right) + \frac{3\xi_3\sqrt{\gamma}}{m_0} (\xi_1^2 + \xi_3^2) \left(\frac{1}{r_1^5} + \frac{\lambda}{r_2^5} \right) \\ + \frac{3\gamma\xi_1\xi_3}{m_0} \left\{ \frac{\mu}{r_1^3} - \frac{\lambda(1-\mu)}{r_2^3} \right\} + \gamma^{\frac{3}{2}}\xi_3 \left\{ \frac{(1-\mu)}{r_1^3} + \frac{\mu(\xi_1 + \sqrt{\gamma} - \mu\sqrt{\gamma})}{r_2^3} \right\}.$$

$$r_1^2 = (\xi_1 - \mu)^2 + \xi_3^2, \quad r_2^2 = (\xi_1 + 1 - \mu)^2 + \xi_3^2.$$

The system (10, 11, 12 and 13) is the master system. The state orbits and the surface of section of this system are shown in Figure (1) and Figure (2) respectively these figures shows that the system is chaotic.

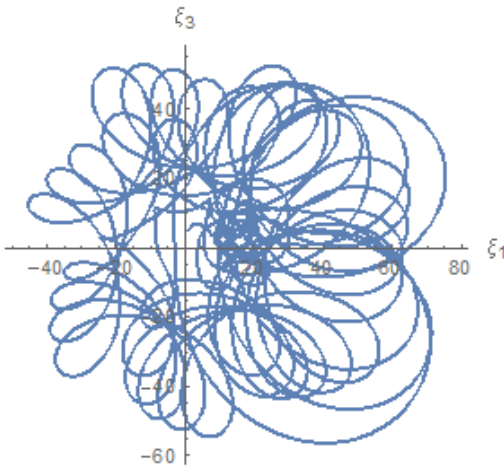


Figure-1

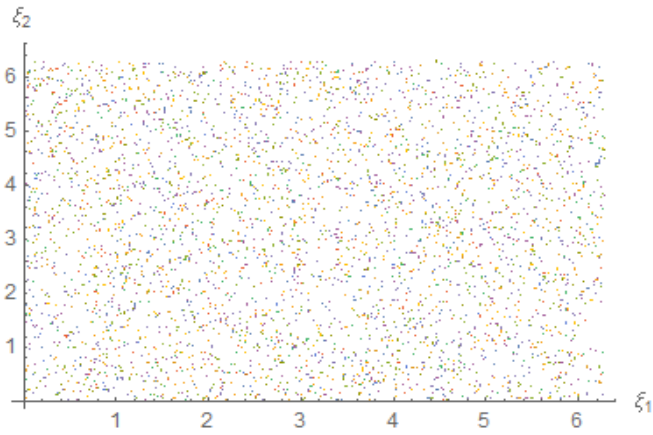


Figure-2

Corresponding to master system (10, 11, 12 and 13), the identical slave system is defined as:

$$\zeta'_1 = \zeta_2 + u_1 \tag{14}$$

$$\zeta'_2 = 2\zeta_4 + \zeta_1 \left(\frac{\beta^2}{4} - 1\right) + B_1 + u_2 \tag{15}$$

$$\zeta'_3 = \zeta_4 + u_3 \tag{16}$$

$$\zeta'_4 = -2\zeta_2 + \zeta_3 \left(\frac{\beta^2}{4} - 1\right) + B_2 + u_4 \tag{17}$$

Where

$$B_1 = -\frac{1}{m_0} \left(\beta \zeta_3 \gamma^{\frac{3}{2}} + 2 \zeta_1 \sqrt{\gamma} \right) \left(\frac{1}{r_{11}^3} + \frac{\lambda}{r_{12}^3} \right) + \frac{3\sqrt{\gamma}}{m_0} (\zeta_1^2 + \zeta_3^2) \left\{ \frac{(\zeta_1 - \mu\sqrt{\gamma})}{r_{11}^5} + \frac{\lambda(\zeta_1 + \sqrt{\gamma} - \mu\sqrt{\gamma})}{r_{12}^5} \right\} \\ - \frac{\gamma}{m_0} \left\{ \frac{\mu}{r_{11}^3} - \frac{\lambda(1-\mu)}{r_{12}^3} \right\} + \frac{3\gamma\zeta_1}{m_0} \left\{ \frac{\mu(\zeta_1 - \mu\sqrt{\gamma})}{r_{11}^5} + \frac{\lambda(1-\mu)(\zeta_1 + \sqrt{\gamma} - \mu\sqrt{\gamma})}{r_{12}^5} \right\} + \gamma^{\frac{3}{2}} \left\{ \frac{(1-\mu)(\zeta_1 - \mu\sqrt{\gamma})}{r_{11}^3} \right\} \\ + \gamma^{\frac{3}{2}} \left\{ \frac{\mu(\zeta_1 + \sqrt{\gamma} - \mu\sqrt{\gamma})}{r_{12}^3} \right\} - \frac{2\zeta_4\sqrt{\gamma}}{m_0} \left(\frac{1}{r_{11}^3} + \frac{\lambda}{r_{12}^3} \right).$$

$$B_2 = \frac{1}{m_0} \left(\beta \zeta_1 \gamma^{\frac{3}{2}} + 2\sqrt{\gamma} \zeta_2 - 2 \zeta_3 \sqrt{\gamma} \right) \left(\frac{1}{r_{11}^3} + \frac{\lambda}{r_{12}^3} \right) + \frac{3\xi_3\sqrt{\gamma}}{m_0} (\zeta_1^2 + \zeta_3^2) \left(\frac{1}{r_{11}^5} + \frac{\lambda}{r_{12}^5} \right) \\ + \frac{3\gamma\zeta_1\zeta_3}{m_0} \left\{ \frac{\mu}{r_{11}^3} - \frac{\lambda(1-\mu)}{r_{12}^3} \right\} + \gamma^{\frac{3}{2}}\zeta_3 \left\{ \frac{(1-\mu)}{r_{11}^3} + \frac{\mu(\zeta_1 + \sqrt{\gamma} - \mu\sqrt{\gamma})}{r_{12}^3} \right\}.$$

$$r_{11}^2 = (\zeta_1 - \mu)^2 + \zeta_3^2, \quad r_{12}^2 = (\zeta_1 + 1 - \mu)^2 + \zeta_3^2.$$

where $u_i(t)$; $i = 1, 2, 3, 4$ are control functions to be determined. Let $e_i = \zeta_i - \xi_i$; $i = 1, 2, 3, 4$ be the synchronization errors. From (10) to (17), we obtain the error dynamics as follows:

$$e_1' = e_2 + u_1 \tag{18}$$

$$e_2' = 2e_4 + \left(\frac{\beta^2}{4} - 1\right) e_1 + B_1 - A_1 + u_2 \tag{19}$$

$$e_3' = e_4 + u_3 \tag{20}$$

$$e_4' = -2e_2 + \left(\frac{\beta^2}{4} - 1\right) e_3 + B_2 - A_2 + u_4 \tag{21}$$

Lyapunov stability theory state that when controller satisfies the assumption with $V(e) = \frac{1}{2} e^t e$ a positive definite function and the first derivative of this function $V' < 0$ the chaos synchronization of two identical systems (master and slave) for different initial conditions is achieved.

The first derivative of $V(e)$ Will be

$$V' = e_1(e_2 + u_1) + e_2 \left\{ 2e_4 + \left(\frac{\beta^2}{4} - 1\right) e_1 + B_1 - A_1 + u_2 \right\} + e_3(e_4 + u_3) + e_4 \left\{ -2e_2 + \left(\frac{\beta^2}{4} - 1\right) e_3 + B_2 - A_2 + u_4 \right\}$$

Therefore, if we choose the controller u as follows,

$$u_1 = -e_1 - \frac{\beta^2}{4} e_2 \tag{22}$$

$$u_2 = -e_2 - B_1 + A_1 \tag{23}$$

$$u_3 = -e_3 - \frac{\beta^2}{4} e_4 \tag{24}$$

$$u_4 = -e_4 - B_2 + A_2 \tag{25}$$

Then

$$V' = -e_1^2 - e_2^2 - e_3^2 - e_4^2 < 0 \tag{26}$$

Hence the error state

$$\lim_{t \rightarrow \infty} \|e(t)\| = 0$$

which gives asymptotic stability of the system. This means that the controlled chaotic systems (master and slave) are synchronized for deferent initial conditions.

4. NUMERICAL SIMULATION

We select the parameters $\mu = .1, \gamma = .45, \beta = .1$ and $\lambda = 1$, with the different initial conditions for master and slave systems. Simulation results for uncoupled system are presented in figures. 3,5,7 and 9 and that of controlled system are shown in figures.4,6,8 and 10 respectively.

It can be seen from the figures that the chaos-synchronization errors converge to zero rapidly.

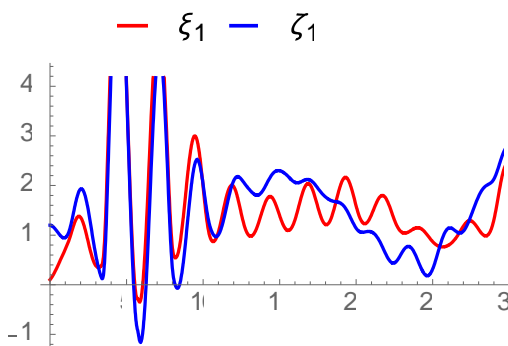


Figure-3

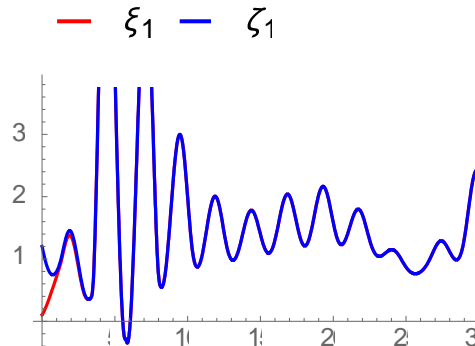


Figure-4

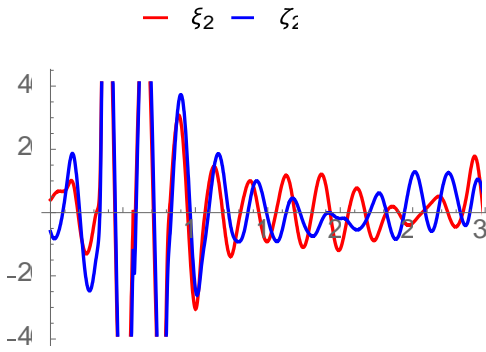


Figure-5

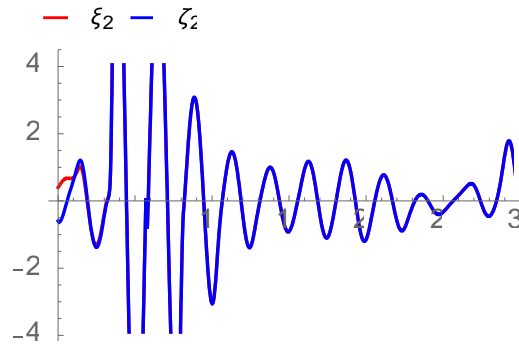


Figure-6

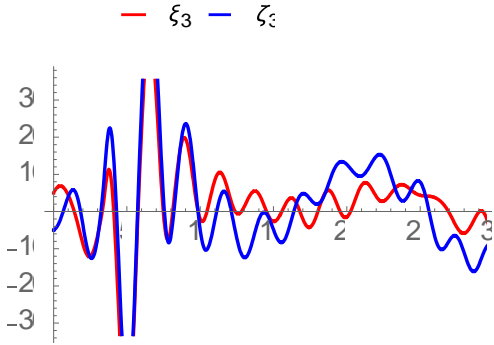


Figure-7

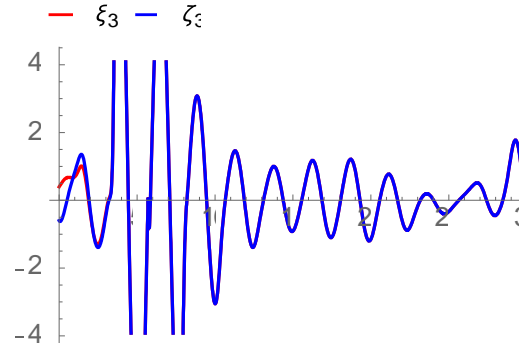


Figure-8

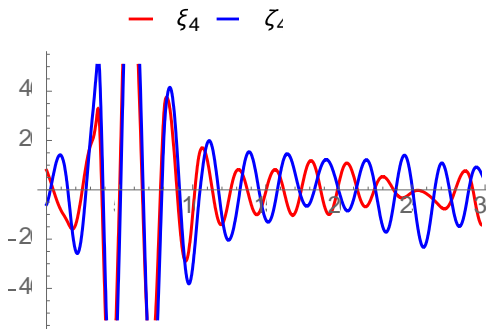


Figure-9

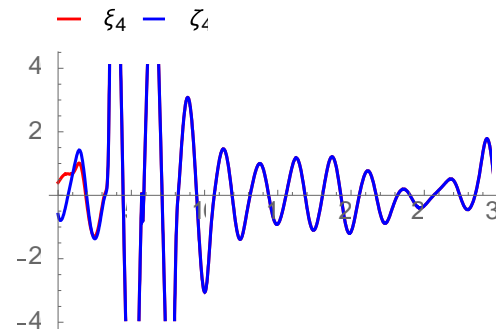


Figure-10

5. ANTI SYNCHRONIZATION

To observe anti-synchronization between the master and the slave system, let $E_i = \zeta_i + \xi_i$; $i = 1, 2, 3, 4$ be the synchronization errors. Now from (10) to (17), we obtain the error dynamics as.

$$E'_1 = E_2 + u_{11} \tag{27}$$

$$E'_2 = 2E_4 + \left(\frac{\beta^2}{4} - 1\right) E_1 + B_1 + A_1 + u_{12} \tag{28}$$

$$E'_3 = E_4 + u_{13} \tag{29}$$

$$E'_4 = -2E_2 + \left(\frac{\beta^2}{4} - 1\right) E_3 + B_2 + A_2 + u_{14} \tag{30}$$

Now the first derivative of $V(e)$ Will be

$$V' = E_1(E_2 + u_{11}) + E_2 \left\{ 2E_4 + \left(\frac{\beta^2}{4} - 1\right) E_1 + B_1 + A_1 + u_{12} \right\} + E_3(E_4 + u_{13}) + E_4 \left\{ -2E_2 + \left(\frac{\beta^2}{4} - 1\right) E_3 + B_2 + A_2 + u_{14} \right\}$$

Therefore, if we choose the controller u as follows,

$$u_{11} = -E_1 - \frac{\beta^2}{4} E_2 \tag{31}$$

$$u_{12} = -E_2 - B_1 - A_1 \tag{32}$$

$$u_{13} = -E_3 - \frac{\beta^2}{4} E_4 \tag{33}$$

$$u_{14} = -E_4 - B_2 - A_2 \tag{34}$$

Then

$$V' = -E_1^2 - E_2^2 - E_3^2 - E_4^2 < 0 \tag{35}$$

Hence the error state

$$\lim_{t \rightarrow \infty} \|E(t)\| = 0$$

which gives asymptotic stability of the system. This means that the controlled chaotic systems (master and slave) are Anti synchronized for deferent initial conditions.

6. NUMERICAL SIMULATION

We select the parameters $\mu = .1, \gamma = .45, \beta = .1$ and $\lambda = 1$, with the different initial conditions for master and slave systems and anti synchronization is achieved between the master and slave systems. Time Series Analysis graphs of the above are shown next to each via figures 11 to 14.

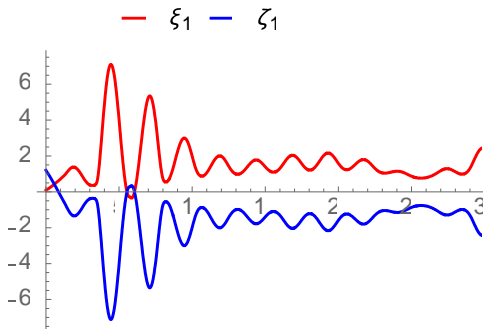


Figure-11

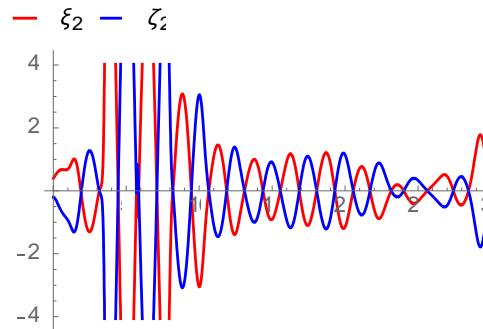


Figure-12

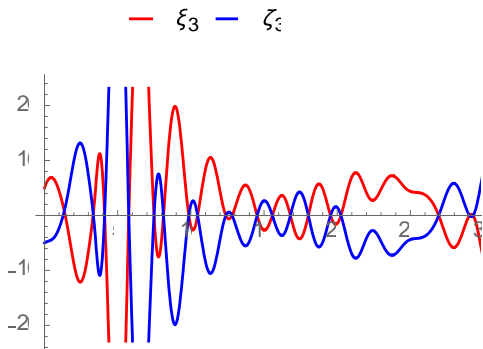


Figure-13

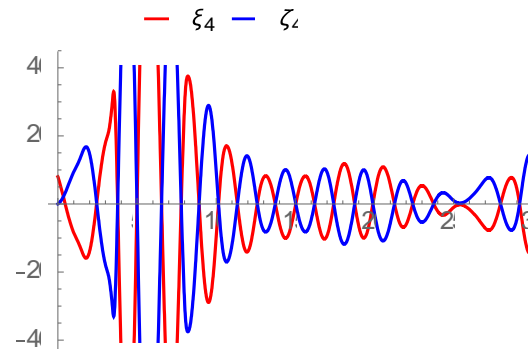


Figure-14

7. CONCLUSION

An investigation on complete synchronization and anti synchronization behavior of the magnetic binary problem when the charged particle has the variable mass via non linear controller based on the Lyapunov stability theory have been made. Here two systems (master and slave) are complete synchronized and start with different initial conditions. This problem may be treated as the design of control laws for chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with master system. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time. The results were validated by numerical simulations using Mathematica.

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