International Journal of Mathematical Archive-2(9), Sept. - 2011, Page: 1523-1527
Available online through www.ijma.info ISSN 2229 – 5046

On Strongly Generalized b-Closed Sets

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(Received on: 17-08-11; Accepted on: 31-08-11)

ABSTRACT

*In this paper, we study a new class of generalized sets called strongly generalized b-closed sets, briefly g*b-closed sets. We study some of their properties. These sets are placed between the class of gs-closed sets and gp-closed sets.*

***2000 Mathematics Subject Classification**: 54A10, 54C8, 54C10, 54D10.*

***Keywords and phrases**: g-closed, g*-closed,b-closed, g*b-closed, Tg*b-space.*

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1. INTRODUCTION AND PRELIMINARIES:

Generalized closed sets form a strong tool in the characterization of topological spaces satisfying weak separation axioms. The concept of generalization was first intiated by Levine [4] in 1963. Since then this method of generalizing sets was adopted by many topologists. Andrijevic [1] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. The class of b-open sets is contained in the class of semi-preopen sets and contains all semi-open sets and preopen sets. The class of b-open sets generates the same topology as the class of preopen sets. Extensive research on generalizing closedness was done in recent years as the notions of a generalized closed, generalized semi-closed, α-generalized closed, generalized semi-preopen closed sets were investigated in [2,3,5].

The aim of this paper is to continue the study of generalized closed sets. In particular, the notion of strongly generalized b-closed sets and its various characterizations are given in this paper. All through this paper, all spaces X and Y (or (X,τ) and (Y,σ)) stand for topological spaces with no separation axioms assumed, unless otherwise stated. Let $A \subseteq X$, the closure of A and the interior of A will be denoted by Cl(A) and Int(A), respectively.

Definition: 1.1 [7]A subset A of a space X is said to be:
(1) α-open if $A \subseteq Int(Cl(Int(A)))$;
(2) Semi-open if $A \subseteq Cl(Int(A))$;
(3) Preopen or nearly open if $A \subseteq Int(Cl(A))$;
(4) β-open or semi-preopen if $A \subseteq Cl(Int(Cl(A)))$;
(5) b-open or sp-open -open if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$.

The complement of a b-open set is said to be b-closed [2]. The intersection of all b-closed sets of X containing A is called the b-closure of A and is denoted by bCl(A). The union of all b-open sets of X contained in A is called b-interior of A and is denoted by bInt(A). The family of all b-open (resp. α-open, semi-open, preopen, β-open,, b-closed, preclosed) subsets of a space X is denoted by bO(X) (resp. αO(X), SO(X), PO(X), βO(X), bC(X), PC(X)) and the collection of all b-open subsets of X containing a fixed point x is denoted by bO(X, x). The sets SO(X, x), αO(X, x), PO(X, x), βO(X, x) are defined analogously.

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Definition: 1.2[6] A subset A of a space (X, τ) is called
(1) a generalized closed set (briey g-closed) if Cl(A) ⊆U whenever A ⊆U and U is open;
(2) a generalized preclosed set (briey gp-closed) if pCl(A) ⊆U whenever A ⊆U and U is open;
(3) a generalized semi-preclosed set (briey gsp-closed) if spCl(A)⊆ U whenever A ⊆U and U is open;
(5) a generalized b- closed set (briey g*b -closed) [15] if bCl(A) ⊆U whenever A ⊆U and U is open.

Complements of g-closed (resp.gp-closed, etc.) sets are called g-open (resp. gp-open, etc.)

2. STRONGLY GENERALIZED b-CLOSED SETS:

Definition: 2.1 A subset of a topological space (X, τ) is said to be g*b-closed set in (X, τ) if bcl(A) ⊆G whenever A ⊆ G where G is g-open. The collection of all g*b-closed sets of (X,τ) is denoted by G*bC(X, τ).

Theorem 2.2: If a subset A of a topological space (X, τ) is closed , then it is g*b-closed.

Proof: Let G be a g-open set containing A. Then G ⊇ A = cl(A) as A is closed. Also cl(A) ⊇ bcl(A). Thus G ⊇ bcl(A). Hence A is a g*b-closed set in (X, τ).

The converse of the above theorem need not be true as seen from the following example.

Example: 2.3 Let X= {a, b, c} and $\tau = \{X, \phi, \{a\}\}$, then the subset {c} is g*b-closed but not closed in (X, τ).

Corollary: 2.4 If a subset A of a topological space (X, τ) is regular closed , then it is g*b-closed but not conversely.

Proof: Since every regular closed set is closed but not conversely. By theorem 2.2 every closed set is g*b-closed but not conversely. Hence every regular closed set is g*b-closed but not conversely.

Theorem: 2.5 If a subset A of a topological space (X, τ) is g*b-closed , then it is gb-closed.

Proof: Let G be an open set containing A. Then G ⊇ A and G ⊇ bcl(A) as A is g*b-closed set. Hence A is gb-closed set in (X, τ).

The converse of theorem 2.5 need not be true as seen from the following example.

Example: 2.6 Let X={a, b, c} and $\tau = \{X, \phi, \{a\}\{b\}\{a,b\}\{a,b,c\}\}$,then the subset {b, c} is gb-closed but not g*b-closed set in (X, τ).

Theorem: 2.7 If a subset A of a topological space (X, τ) is b-closed, then it is g*b-closed.

Proof: Let G be an open set containing A. Then G ⊇ A = bcl(A),as A is b-closed. Thus G ⊇ bcl(A). Hence A is g*b-closed in (X, τ).

The converse of the above theorem need not be true and it can be seen from the following example.

Example: 2.8 Let X= {a, b, c} and $\tau = \{X, \phi, \{a\}, \{a,b\}\}$, then the subset {b} is g*b- closed but not b-closed in (X, τ).

Theorem: 2.9 If a subset A of a topological space (X, τ) is g*b-closed, then it is bg-closed.

Proof: Let G be an open set containing A. Then G is b-open set containing A, as every open set is b-open. Thus G ⊇ bcl(A), as A is g*b-closed set. Hence A is bg-closed in (X, τ).

The converse of the above theorem is not true and can be seen from the following example.

Example 2.10: Let X= {a, b, c} and $\tau = \{X, \phi, \{a\}, \{a,b\}\}$, then the subset {a, c} is bg-closed but not g*b-closed.

Theorem: 2.11 Let A be a subset of a topological space (X, τ). If A is g*b-closed, then it is gs-closed.

Proof: Let G be an open set containing A. Then G $\supseteq$ bcl(A), as A is g*b-closed. Thus G $\supseteq$ bcl(A) $\supseteq$ Scl(A).

Therefore A is gs-closed in (X, τ).

The following example shows that the converse of the above theorem need not be true.

Example: 2.12 Let X= {a, b, c} and $\tau = \{X, \phi, \{a\}, \{a,b\}\}$, then the subset {a} is gs-closed but is not g*b-closed in (X, τ).

Remark: 2.13 The following example shows that every g-closed set is g*b-closed but not conversely.

Example: 2.14 Let X= {a, b, c} and $\tau = \{X, \phi, \{b\}, \{a,b\}\}$, then the subset {a} is g*b closed but not g-closed.

Remark: 2.15 The following examples shows that the concept of semi-closed and g*b-closed sets are independent.

Example: 2.16 (a) Let X= {a, b, c} and $\tau = \{X, \phi, \{a\}, \{a,b\}\}$, then the subset {a, c} is g*b-closed but not semi-closed.

Example: 2.16 (b) Let X= {a, b, c, d} and $\tau = \{X, \phi, \{a\}\{b\}, \{a,b\}\{a,b,c\}\}$, then the subset {b} is semi closed but not g*b-closed.

Remark: 2.17 The following examples shows that the concept of pre-closed and g*b-closed sets are independent.

Example: 2.18 Let X= {a, b, c, d} and $\tau = \{X, \phi, \{a\}\{b\}, \{a,b\}\{a,b,c\}\}$, then the subset {a, c, d} is g*b closed but not pre-closed.

Remark: 2.20 The following examples shows that the concept of g-closed and g*b-closed sets are independent.

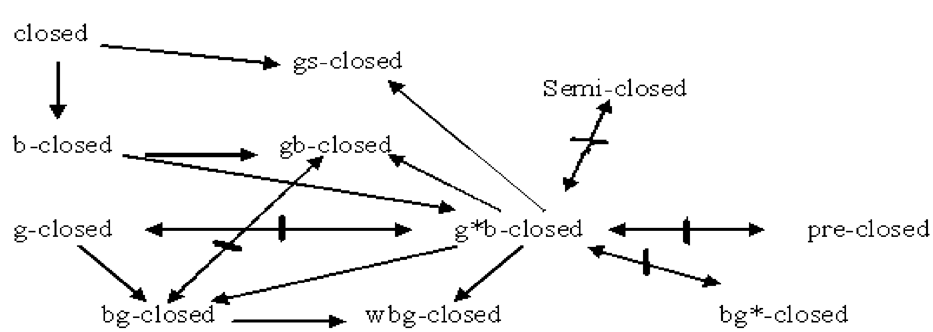
Example: 2.21 Let X= {a, b, c} and $\tau = \{X, \phi, \{a\}\}$, then the subset {a, b} is g-closed but not g*b-closed.

Example: 2.22 Let X= {a, b, c} and $\tau = \{X, \phi, \{a\}, \{a,b\}\}$, then the subset {b} is g*b-closed but not g-closed.

Remark: 2.23 The following examples shows that the concept of bg*-closed and g*b-closed sets are independent.

Example: 2.24 Let X= {a, b, c} and $\tau = \{X, \phi, \{a\}, \{a,b\}\}$, then the subset {a, c} is g*b-closed but not bg*-closed in (X, τ).
Example: 2.25 Let X= {a, b, c} and $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$, then the subset {a, b} is bg*-closed but not g*b-closed in (X, τ).

Remark: 2.26 The following diagram shows the relationship between g*b-closed sets with various sets.



where $A \to B$ (resp. A $\not\leftrightarrow$ B) represents a implies B and B need not implies A (resp. A & B are independent of each other).

3. PROPERTIES OF g*b-CLOSED SETS IN TOPOLOGICAL SPACES:

Theorem: 3.1 (i) Let A $\subset$ (X, τ) be g*b - closed. Then Cl(A)\A contains no non empty bg-closed set.
(ii) If A is g*b - closed A $\subset$ B $\subset$ cl(A) , then cl(B)\B contains no non empty b closed sets.

Proof: (i) Let us suppose that A is g*b closed and F is any b closed subset of Cl(A)\A. Then F $\subset$ X\A => A $\subset$ X\F is b open. Since A is g*b closed, bcl(A) $\subset$ X\F. That is F $\subset$ X \ bcl(A).

We already have F $\subset$ bCl(A) .So F $\subset$ bcl(A) $\cap$ X \ bcl(A) = ϕ. Thus F = ϕ. Hence cl(A) \ A contains no non empty bg- closed set.

(ii) Let A be g*b- closed and A $\subset$ B $\subset$ cl(A), then we have cl(B) $\cap$ X\B $\subset$ cl(A) $\cap$ X\A. That is cl(B)\B $\subset$ cl(A) \A. By (i) cl(A) \A has no nonempty b - closed set. Hence cl(B) \ B contains no nonempty b - closed set.

Theorem: 3.2 A g*b-closed set A is b-closed if and only if bcl(A) – A is bg-closed.

Proof: Necessity: Since a is b-closed, we have bcl(A) = A. Then bcl(A) –A = ϕ is b-closed and hence bg-closed. Sufficiency: By theorem 3.1, bcl(A)-A contains no non empty bg-closed set. That is bcl(A)-A = ϕ.

Therefore bcl(A) = A. Hence A is b-closed.

Theorem: 3.3 If A is a g*b-closed set and B is any set such that A $\subseteq$ B $\subseteq$ bcl(A), then B is a g*b-closed set.

Proof: Let B $\subseteq$ U where U is g-open set. Since A is g*b-closed set and A $\subseteq$ U, then bcl(A) $\subseteq$ U and also bcl(A) = bcl(B).Therefore bcl(B) $\subseteq$ U and hence B is a g*b-closed set.

Theorem: 3.4 (i) The intersection of a g*b-closed set and a b closed sets is always a g*b-closed set.
(ii) If A is a g*b-closed set and A $\subset$ B $\subset$ cl(A) , then B is g*b-closed set.

Proof: (i) Let A be g*b-closed set and let F be a b-closed set. Suppose G is a g-open set with A $\cap$ F $\subset$ G, then A $\subset$ G U Fc where G $\cup$ F is b-open.

Therefore bcl(A) $\subset$ G $\cup$ F. Now bcl(A $\cap$ F) $\subset$ bcl(A) $\cap$ bcl(F) = bcl(A) $\cap$ F $\subset$G.

Hence A $\cap$ F is a g*b-closed set.

(iii) Let A be g*b-closed and B $\subset$ G where G is a g-open set. Then A $\subset$ G. Since A is g*b-closed, bcl(A) $\subset$ G.

Hence by assumption bcl(B) $\subset$ bcl(A) $\subset$ G. Thus bcl(B) $\subset$ G implies that B is g*b-closed.

Theorem: 3.5 Let $\{A_i : i\in I\}$ be a locally finite family of g*b-closed sets. Then $A = \bigcup A_i$ is g*b -closed for every i$\in$I.

Proof: Since $\{A_i : i\in I\}$ is locally finite, $cl(\bigcup A_i) = \bigcup cl(A_i)$. Assume that for some b-open set we have $A = \bigcup A_i \subset U$. Then $cl(\bigcup A_i) = \bigcup cl(A_i) \subset$ U, since each A_i is g*b-closed. Thus A is g*b-closed.

Remark: 3.6 The spaces g*bTg* and space g*bTb are independent as seen from the following examples:

Example: 3.7 Let X= { a, b, c} with topology $\tau = \{X, \phi, \{a,b\}\}$. Then (X, τ) is a g*bTg* -space but not a g*bTb –space , since {a,c}is g*-closed but not b-closed in (X, τ).

Example: 3.8 Let X= {a, b, c} with topology $\tau = \{X, \phi, \{a\}, \{a,b\}\}$. Then (X, τ) is a g*bTb -space but not a g*bTg* –space , since {b}is g*b-closed but not g*-closed in (X, τ).

Theorem: 3.9 If (X, τ) is both b-space and g*bTb –space, then it is a g*bTg* –space.

Proof: Let A be a g*b-closed set in (X, τ).Since (X, τ) is a g*bTb –space, A is b-closed in (X, τ).Since (X, τ) is a b-space, every b-closed set is closed and hence A is closed in (X, τ).We know that every closed set is g*-closed in

(X, τ), A is g*-closed. Hence (X, τ) is a g*bTg* –space.

Theorem 3.10: If (X,τ) is both $T^*_{1/2}$ –space and g*bTg* –space, then it is a g*bTb –space.

Proof: Let A be a g*b-closed set in (X,τ).Since (X,τ) is a g*bTg* –space, a is g*-closed. Since (X,τ) is a $T^*_{1/2}$-space, A is closed in (X,τ). Since every closed set is b-closed, A is b-closed in (X,τ). Hence it is a g*bTb –space.

Remark: 3.11 In a semiregular space T1/2 –space , the concepts of g*b - closed , g-closed and closed sets coincide.

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