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Pre-g*-continuous functions

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ABSTRACT

In this paper we introduce the concept of pre-g^* continuous functions using semiopen sets due to N. Levine (1963) and g^*-open sets due to Veera Kumar (2000). Also, we established the basic properties of pre-g^*-continuous functions and other related continuous functions.

Key words: *Semiopen sets, g^*-closed sets, gs-continuous functions, g^*-continuous functions, g^*-irresolute functions.*

1. INTRODUCTION

In 1963, N. Levine [11] introduced and studied the concepts of semiopen sets and semi-continuity in topological spaces. N. Levine [12] introduced the class of g-closed sets, a super class of closed sets in 1970. In 1987, P. Bhattacharya *et.al* [4] have defined and studied the concepts of sg-closed sets, sg-open sets and semi-$T_{\frac{1}{2}}$ spaces. In 1990, Arya *et.al* [2] have defined and studied the notions of sg-open sets and gs-closed sets in connection with the characterizations of s-normal spaces. In 1994, T. Noiri [17] have defined and studied the concepts of pre-sg-continuous functions and pre-sg-closed functions in connection with the study of semi-normal spaces. In, 1998, T. Noiri [18] have studied the notions of pre- gs -continuous functions and pre- gs -closed functions in connection with the study of s-normal spaces in topology. In 2000, M.K.R.S. Veera Kumar [22] has defined and studied the notions of g^* -closed sets, g^* -open sets, $T^*_{\frac{1}{2}}$-spaces, $*T_{\frac{1}{2}}$ –spaces, g^* -continuous functions and g^* -irresolute. In this paper we define and study the notions of pre-g^* -continuous functions and their basic characterizations using semiopen sets and g^* -open sets.

2. PRELIMINARIES

Throughtout this paper (X,τ) and (Y,σ) always represents non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. Let A be a subset of of X We denote the closure (resp. the interior) of A by $CI(A)\big(resp. Int(A)\big)$.

We need the following definitions in the sequel:

Definition 2.1: A subset A of a topological space (X,τ) is called
  (i) Semiopen[11] set, if $A \subset Cl(Int(A))$
  (ii) Semipreopen[1] set, if $A \subset ClIntCl(A)$

The complement of a semiopen (resp.semipreopen) sets is called semiclosed [5] (resp.semipreclosed [1]) sets of a space X. The family of all semiopen (resp.semipreopen) sets of a space X is denoted by $SO(X)\big(resp. SPO(X)\big)$.

Definition 2.2: A subset $A$of a topological space (X,τ) is called
  (i) Semi- interior [5] of A, if the union of all semiopen sets contained in A and is denoted by $sInt(A)$.
  (ii) semipre-interior[1] of A, if the union of all semipreopen sets contained in A, and is denoted by $spInt(A)$.
  (iii) semiclosure[5] of A if the intersection of all semi-closed sets containing A and is denoted by $sCI(A)$.
  (iv) semipreclosure[1] of A, if the intersection of all semipre-closed sets containing A and is denoted by $spCI(A.)$

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Definition 2.3: A function $f: X \to Y$ is called
(i) semi-continuity [11], if $f^{-1}(V)$ is semiopen in X for each open set V of Y.
(ii) semipre-contonuity[15], if $f^{-1}(V)$ is semipreopen in X for each open set V of Y.
(iii) irresolute [6],if $f^{-1}(V)$is semiopen set in X for each semiopen set V of Y.
(iv) semipre-irresolute[15], if $f^{-1}(V)$ is semipreopen set in X for each semipreopen set V of Y.

Definition 2.4: A subset A of a space X is said to be
(i) generalized closed(in brief g-closed)set[12], if $CI(A) \subseteq U$ whenever $A \subset U$and is U open in X.
(ii) semi generalized- closed(in brief sg-closed)set [4] , if $sCI(A) \subseteq U$whenever $A \subset U$ and U is semiopen in X.
(iii) generalized semi -closed(in brief gs-closed)set [2] , if $CI(A) \subseteq U$whenever $A \subset U$and U is open in X.
(iv) generalizedsemipre-closed(in brief gspclosed) set [9] , if $spCI(A) \subseteq U$ whenever $A \subset U$and U is open in X .
(v) g^*- closed set[22] , if $CI(A)U$ whenever $A \subset U$ and U is g-open in X.

The complement of g-closed (resp. sg-closed,gs closed, gsp-closed, g^*-closed) sets of X is called g-open (resp. sg –open, gs –open, gsp –open, g^*-open) set in X.

Definition 2.5: Afunction $f: X \to Y$ is said to be
(i) generalized continuous [3] (in brief g-continuous), if $f^{-1}(V)$ is g-closed in X for every closed set V of Y.
(ii) semi generalized continuous [21](in brief sg-continuous), if $f^{-1}(V)$ is sg-closed in X for every closed set V of Y.
(iii) generalized semi continuous [7] (in brief gs-continuous), if $f^{-1}(V)$is gs-closed in X for every closed set V of Y.
(iv) Generalized semipre continuous [9] (in brief gsp-continuous), if $f^{-1}(V)$is gsp-closed in X for every closed set V of Y.
(v) pre-sg- continuous [17], if $f^{-1}(V)$is sg-closed in X for every semiclosed set V of Y.
(vi) pre- gs – continuous [19], if $f^{-1}(V)$is gs-closed in X for every semiclosed set V of Y.
(vii) pre-gsp- continuous [16], if $f^{-1}(V)$ is gsp-closed in X for every semipreclosed set V of Y.
(viii) g^*-continuous [22], if $f^{-1}(V)$ is g^*-closed in X for every closed set V of Y.

Definition 2.7: A space (X, τ) is said to be
(i) s-normal space [13], if for every disjoint closed sets A and B of X, there exist disjoint $U, V \in SO(X)$ such that $A \subset U \text{ and } B \subset V$.
(ii) semi-normal space[10] , if for every disjoint semi-closed sets A and B of X , there exist disjoint $U, V \in SO(X)$ such that $A \subset U \text{ and } B \subset V$.
(iii) $T_{\frac{1}{2}}$ spaces [12], if every g-closed set in X is closed.
(iv) semi-$T_{\frac{1}{2}}$spaces [4], if every sg-closed set in X is semi-closed.
(v) Semipre- $T_{\frac{1}{2}}$ spaces [9], if every gsp-closed set in X is semipreclosed.
(vi) $T_{\frac{1}{2}}^*$–space [22], if every g^*-closed set of X is a closed set.
(vii) $*T_{\frac{1}{2}}$–space [22], if every g –closed set of X is a g^*-closed set.

3. Pre-g*-CONTINUOUS FUNCTIONS

We define the following:

Definition 3.1: A function $f: X \to Y$ is said to be pre-g^*--continuous if the inverse image of each g –open set of Y is g^*-open in X.
It is obvious that a function $f: X \to Y$ is said to be pre- g^*-continuous if the inverse image of each g –closed set of Y is g^*-closed in X.

Lemma 3.2: Every pre-g^*-continuous function is g^*-irresolute.

Proof: Let $f: X \to Y$ be pre-g^*-continuous function and V be any g^*-closedset in Y. But every g^*-closed is g –closed and hence V is any g –closed set in Y . Given that f is pre-g^*-continuous, $f^{-1}(V)$ is g^*-closed set in X. This shows that f is g^*-irresolute.

Lemma 3.3: Let $f: X \to Y$ be g^*-continuous function and X be $T_{\frac{1}{2}}^*$–space. Then f is continuous.

Proof: Let $f: X \to Y$ be g^*-continuous function and X be $T_{\frac{1}{2}}^*$–space. Let V be any closed set in Y. Then, $f^{-1}(V)$ is g^*-closed set in X, since by hypothesis. But, as X is $T_{\frac{1}{2}}^*$-space given, $f^{-1}(V)$ is closed in X. This shows that f is continuous.

We define the following:

Definition 3.4: A function $f: X \to Y$ is called strongly g –continuous if the inverse image of each g –closed set of Y is closed in X.

We prove the following:

Lemma 3.5: Every strongly g –continuous function is pre-g^*-continuous.

Proof is obvious, since every closed set is g^*-closed.

Lemma 3.6: Every gc-irresolute function is pre-g^*-continuous if X is $*_{T_{\frac{1}{2}}}$ –space.

Proof: Let $f: X \to Y$ be gc-irresolute function. Let V be any g-closed set in Y. As, f is gc-irresolute, $f^{-1}(V)$ is g-closed set in X. But X is given as $*_{T_{\frac{1}{2}}}$–space, then $f^{-1}(V)$ is g^*-closed in X. This shows that f is pre-g^*-continuous.

4. DECOMPOSITIONS OF pre-g^*-CONTINUOUS FUNCTIONS

Theorem 4.1: Let $f: X \to Y$ and $g: Y \to Z$ be functions. Then the composition $g.f: X \to Z$ is pre-g^*-continuous, if f and g satisfy one of the following conditions:

(i) f is pre-$\boldsymbol{g}^*$-continuous and g is gc-irresolute.

(ii) f is $\boldsymbol{g}^*$-continuous and g is strongly g-continuous.

Proof:

(i) Let V be any g-closed set in Z. Then $g^{-1}(V)$ is g-closed set in Y, since g is gc-irresolute function. Again, f is pre-g^*-continuous function and $g^{-1}(V)$ is g –closed in Y, then$f^{-1}\left(g^{-1}(V)\right) = (g.f)^{-1}(V)$ is g^*--closed set in X. Thus $g.f$ is pre-g^*-continuous functions.

(ii) Let V be any g –closed set in Z. Since g is strongly g –continuous, $g^{-1}(V)$ is closed set in Y. Again, f is g^*--continuous function and $g^{-1}(V)$ is closed set in Y, then $f^{-1}\left(g^{-1}(V)\right) = (g.f)^{-1}(V)$ is g^*--closed set in X. Hence $g.f$ is pre-g^*--continuous functions.

We recall the following:

Definition 4.2: A topological space X is said to be T_d space [8] if every gs-closed set in X is g –closed.

We define the following:

Definition 4.3: A function $f: X \to Y$ is said to be (gs, g^*)-continuous function if the inverse image of each gs-closed set of Y is g^*-closed set in X.

Clearly, every gs-irresolute function is (gs, g^*) –continuous if X is T_d space.

Theorem 4.4: Let $f: X \to Y$ be pre-g^*-continuous function with Y as T_d space and $g: Y \to Z$ gs-irresolute, then $g.f: X \to Z$ is (gs, g^*)-continuous function.

Proof is obvious.

We recall the following:

Definition 4.5: A function $f: X \to Y$ is called always-g^* -closed [20] if the image of eacg g^*-closed set of X is g^*-closed in Y.

Definition 4.6: A function $f: X \to Y$ is called g-closed [21] if for each closed set V of X, $f(V)$ is g-closed set in Y.

We define the following:

Definition 4.7: A function $f: X \to Y$ is called (g^*, g)-closed if the image of each g^*-closed set of X is g-closed in Y.

Definition 4.8: A function $f: X \to Y$ is called strongly g^*- closed if the image of each g^*-closed set of X is closed in Y.

Theorem 4.9: Let $f: X \to Y$ and $g: Y \to Z$ be functions and let composition $g.f: X \to Z$ be pre-g^*-continuous. Then the following hold:

(i) If $f$is always-g^*-closed surjection, then g is pre-g^*-continuous..
(ii) If f is strongly g^*-closed surjection, then g is strongly g-continuous.
(iii) If g is g-closed injection, then f is g^*-continuous.
(iv) If g is (g^*, g)-closed injection, then f is g^*-irresolute.

Proof:

(i) Let V be any g –closed subset of Z. As $g.f$ is pre-g^*-continuous, $(g.f)^{-1}(V)$ is g^*-closed set in X. As f is always g^*-closed surjection, $\left(f(f^{-1}(g^{-1}(V))\right) = g^{-1}(V)$ is g^*-closed set in Y. This shows that g is pre-g^*-continuous.
(ii) Let V be any g –closed subset of Z. As $g.f$ is pre-g^*-continuous, $(g.f)^{-1}(V)$ is g^*-closed set in X. As f is strongly g^*-closed surjection, $\left(f(f^{-1}(g^{-1}(V))\right) = g^{-1}(V)$ is g^*- closed set in Y. This shows that g is strongly g –continuous.
(iii) Let V be any closed subset of Y. Then $g(V)$ is g –closed set in Z, since g is g-closed function. Again $g.f$ is pre-g^*-continuous and g is injective, then $\left((g.f)^{-1}(g(V))\right) = \left(f^{-1}(g^{-1})(g(V))\right) = f^{-1}(V)$ is g^*-closed set in X. This shows that fis g^*-continuous.
(iv) Let V be any g^*- closed subset of Y. Then $g(V)$ is g-closed set in Z, since g is (g^*, g)-closed function. Again, $g.f$is pre-g^*-continuous and g is injective, then $\left((g.f)^{-1}(g(V))\right) = \left(f^{-1}(g^{-1})(g(V))\right) = f^{-1}(V)$ is g^*-closed set in X. This shows that f is g^*-irresolute.

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