International Journal of Mathematical Archive-8(10), 2017, 219-223
Available online through www.ijma.info ISSN 2229 – 5046

FUZZY COVERING SPACE AND FUZZY LIFT ON FUZZY BANACH MANIFOLD

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(Received On: 19-07-17; Revised & Accepted On: 10-10-17)

ABSTRACT

In this paper, we define fuzzy smooth covering map on fuzzy Banach manifold as a natural development of fuzzy smooth maps and prove some results. Further we define fuzzy lift and prove unique existence of lift and lifting criteria on fuzzy Banach manifold.

Subject Classification: *58B05, 54A40.*

Keywords: *Fuzzy smooth map, Fuzzy smooth covering map, Fuzzy Lift.*

1. INTRODUCTION

The concept of fuzzy sets and fuzzy set operations were introduced by L. A. Zadeh in 1965[8]. Subsequently many basic concepts from general topology were applied to fuzzy sets. Similarly, in recent developments of fuzzy set theory many concepts from algebraic topology are defined and studied with different perception.

In our previous papers we have introduced the concept of Fuzzy Banach Manifold [12] and studied some topological properties [13][14] and geometrical properties[15]. Further in [16] we have defined fuzzy smooth homotopy and studied some related properties. A covering space is a locally trivial map with discrete fibres which are classified by algebraic data related to fundamental group and lift.

In this paper we define fuzzy smooth covering map and study some of its properties and further we define fuzzy lift and prove the unique existence and lifting criteria.

Some preliminary definitions referred are as follows:

Definition 1.1: Let X be a set. A fuzzy subset A of X is defined to be a function $\mu_A: X \rightarrow [0,1]$. Thus we have the following:
$A = \{(x, \mu_A): \forall\, x \in X\} = \mu_A$. [8]

Definition 1.2: A fuzzy subset in X is called a fuzzy point iff it takes the value 0 for all $y \in X$ except one say, $x \in X$. If its value at x is $\lambda(0 < \lambda \leq 1)$ we denote this fuzzy point by x_λ where the point x is called its support. [11]

Definition 1.3: A fuzzy topology on a set X is a family τ of fuzzy subsets in X which satisfies the following conditions:
(i) $k_0, k_1 \in \tau$
(ii) If $A, B \in \tau$ then $A \cap B \in \tau$
(iii) If $A_j \in \tau\ \forall\, j \in J$ (where J is index set) then $\bigcup_{j\in J} A_j \in \tau$
The pair (X, τ) is called a fuzzy topological space and the members of τ are called open fuzzy sets. [1]

Definition 1.4: Let X, Y be fuzzy topological spaces. A bijection f of X onto Y is said to be a fuzzy continuous map if for each open fuzzy subset A in Y the inverse image $f^{-1}(A)$ is open fuzzy set in X. [10]

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Definition 1.5: A fuzzy set A in (X,τ) is called a neighborhood of a fuzzy point (x,α) if there exists a fuzzy open set $B \in \tau$ such that $(x,\alpha) \in B \leq A$, a neighborhood A is said to be open if A is open. The family consisting of all the neighborhoods of (x,α) is called as the system of neighborhoods of (x,α). [11]

Definition 1.6: A fuzzy subset A of X is said to be quasi-coinsident with a fuzzy subset B of X if there exists $x \in X$ such that $A(x) + B(x) > 1$ OR $A(x) > B^C(x)$, and is denoted by AqB. [11]

Definition 1.7: A fuzzy subset A in (X,τ) is called a Q-neighborhood of (x,α) if there exists a open fuzzy set B such that $(x,\alpha)qB \leq A$. The family consisting of all the Q-neighborhoods of (x,α) is called the system of Q-neighborhoods of (x,α). [11]

Definition 1.8: Let (X,τ) be a fuzzy topological space. $\alpha: (I,\hat{\epsilon}) \to (X,\tau)$ is a fuzzy continuous function and a fuzzy set A is connected in $(I,\hat{\epsilon})$ with $A(0) > 0$ and $A(1) > 0$, then the fuzzy set $\alpha(A)$ in (X,τ) is called a fuzzy path in (X,τ).

The fuzzy point $(\alpha(0))_{A(0)} = \alpha(0_{A(0)})$ and $(\alpha(1))_{A(1)} = \alpha(1_{A(1)})$ are called the initial point and the terminal point of the fuzzy path $\alpha(A)$, respectively. [2]

Definition 1.9: Let M be a fuzzy topological space, U be a fuzzy subset of M such that $sup\{\mu_U(x)\} = 1 \; \forall \; x \in M$ and φ is a fuzzy homeomorphism defined on the support of $U = \{x \in M : \mu_U(x) > 0\}$, which maps U onto an open fuzzy set $\varphi(U)$ in some fuzzy Banach space E_i. Then the pair (U,φ) is called as fuzzy Banach chart. [12]

Definition 1.10: A fuzzy Banach atlas A of class C^k on M is a collection of pairs (U_i,φ_i) (i $\in I$) subjected to the following conditions:

(i) $\bigcup_{i \in I} U_i = M$ that is the domain of fuzzy Banach charts in A cover M.

(ii) Each fuzzy homeomorphism φ_i, defined on the support of $U_i = \{ x \in M : \mu_U(x) > 0 \}$ which maps U_i onto an open fuzzy subset $\varphi_i(U_i)$ in some fuzzy Banach space E_i, and for each $\forall \; i,j \in I, \varphi_i(U_i \cap U_j)$ and $\varphi_j(U_i \cap U_j)$ are open fuzzy subset in E_i.

(iii) The maps $\varphi_i \circ {\varphi_j}^{-1}$ which maps $\varphi_j(U_i \cap U_j)$ onto $\varphi_i(U_i \cap U_j)$ is fuzzy diffeomorphism of class C^k $(k \geq 1)$ for each pair of indices i, j.

The maps $\varphi_i \circ {\varphi_j}^{-1}$ and $\varphi_j \circ {\varphi_i}^{-1}$ for $i,j \in I$ are called fuzzy transition maps. [12]

Definition 1.11: Let M and N be fuzzy Banach manifolds with corresponding maximal fuzzy Banach atlases A_M and A_N. We say that a map $f : M \to N$ is of class C^r (r times continuously fuzzy differentiable) at $p \in M$ if there exists a fuzzy Banach chart (V,ψ) in A_N with $f(p) \in V$, and a fuzzy Banach chart (U,φ) from A_M with $p \in M$, such that $f(U) \subset V$ and such that $f(U) \subset V$ and $\psi \circ f \circ \varphi^{-1}$ is of class C^r. If f is of class C^r at every point $p \in M$, then we say f is of class C^ron M. Maps of class C^∞ are called fuzzy smooth maps. [14]

Definition 1.12: Let M and N be fuzzy Banach manifolds with corresponding maximal fuzzy Banach atlases A_M and A_N. We say that a map $f : M \to N$ is called fuzzy diffeomorphism if both f and f^{-1} are fuzzy smooth. [16]

2. FUZZY COVERING SPACE

In this section we define fuzzy covering space and prove some results referring [4] [5] [17].

Definition 2.1: Let $\widetilde{M}$ and M be fuzzy Banach manifolds. A surjective map $\wp: \widetilde{M} \to M$ is called as a fuzzy smooth covering map if every point $p \in M$ there is a fuzzy open Q-neighborhood U such that each component $\widetilde{U_\iota}$ of $\wp^{-1}(U)$ is fuzzy diffeomorphism to U by a restriction map ${}^{\wp}/_{\widetilde{U_\iota}} : \widetilde{U_\iota} \to U$, then U is evenly covered by covering map.

The triplet $(\widetilde{M}, \wp, M)$ is called a fuzzy covering space; M is a base space and $\widetilde{M}$ is the total space. The fuzzy open Q-neighborhood $\widetilde{U_\iota}$ is called the fuzzy sheet over the evenly covered by fuzzy open Q-neighborhood U.

The set of all fuzzy smooth covering map between fuzzy smooth Banach manifold is the set of objects of a category. A fuzzy diffeomorphism between fuzzy smooth covering spaces, $(\widetilde{M_1}, \wp_1, M_1)$ and $(\widetilde{M_2}, \wp_2, M_2)$ is a pair of fuzzy smooth maps $(f, \tilde{f})$ such that $f \circ \wp_1 = \wp_2 \circ \tilde{f}$.

Similarly, the coverings of a fixed space M are the objects of a cateogory where the fuzzy diffeomorphisms are $\varphi: \widetilde{M_1} \to \widetilde{M_2}$ such that $\wp_1 = \wp_2 \circ \varphi$.

Remark 2.2:

1) For each $x_\lambda \in M$ the preimage $\wp^{-1}(x_\lambda) = \{\widetilde{x_\lambda} \in \widetilde{M} : \wp(\widetilde{x_\lambda}) = x_\lambda\}$ is the fibre over x_λ.
2) Any fuzzy smooth covering map is a local fuzzy diffeomorphism.
3) An injective fuzzy smooth covering map is the fuzzy diffeomorphism.

Example 2.3: Let $\wp: M \to M$ be a surjective identity map on fuzzy Banach manifold M, then clearly $\wp$ is fuzzy covering map. That is every fuzzy Banach manifold is covered by itself.

Definition 2.4: If $\wp: \widetilde{M} \rightarrow M$ is fuzzy continuous map, a fuzzy section of $\wp$ is fuzzy continuous map $\sigma: M \rightarrow \widetilde{M}$, such that $\wp \circ \sigma: M \rightarrow$ is Id_M.

A local fuzzy section is a fuzzy continuous map $\sigma: U \rightarrow \widetilde{M}$ defined on some fuzzy open Q-neighborhood $U \subset M$ satisfying the condition $\wp \circ \sigma : Id_U$.

Theorem 2.5: Suppose $\wp: \widetilde{M} \rightarrow M$ is a fuzzy smooth covering map, for any $q \in \widetilde{M}$ there is a fuzzy open Q-neighborhood U of $p = \wp(q)$ and a unique fuzzy smooth local section $\sigma: U \rightarrow \widetilde{M}$ such that $\sigma(p) = q$.

Proof: Let $U \subset M$ be an evenly covered fuzzy open Q-neighborhood of p. If $\widetilde{U}$ is the component of $\wp^{-1}(U)$ containing q then by definition ${}^{\wp^{-1}}\!/_{\widetilde{U}} : \widetilde{U} \rightarrow U$ is fuzzy diffeomorphism hence there exists a fuzzy local section σ: ${}^{\wp^{-1}}\!/_{\widetilde{U}} : \widetilde{U} \rightarrow U$ of $\wp$ such that $\sigma(p) = q$.

Now we shall show that σ is unique. Let σ' is another fuzzy local section of $\wp$ on U such that $\sigma'(p) = q$ and $\circ\, \sigma' = Id_U$. Since σ is local section for $\wp$ on U by definition we have $\wp \circ \sigma = Id_U$. Thus by above discussion it is clear that σ is unique.

Theorem 2.6: Suppose $\wp: \widetilde{M} \rightarrow M$ is a fuzzy smooth covering map and N is any fuzzy Banach manifold, then a map $F: M \rightarrow N$ is fuzzy smooth if and only if $F \circ \wp: \widetilde{M} \rightarrow M$ is fuzzy smooth.

Proof: Let $\wp: \widetilde{M} \rightarrow M$ be a fuzzy smooth covering map and N be fuzzy Banach manifold. If $F: M \rightarrow N$ is a fuzzy smooth map then by Proposition 2.2 the composition $F \circ \wp$ is fuzzy smooth.

Conversely, if $F \circ \wp$ is fuzzy smooth map and $p \in M$ then by theorem 3.1 there is a fuzzy open Q-neighborhood U of p and a fuzzy smooth local section $\sigma: U \rightarrow \widetilde{M}$ such that $\wp \circ \sigma = Id_U$ then we have ${}^{F}\!/_{U} = {}^{F}\!/_{U} \circ Id_U = {}^{F}\!/_{U} \circ (\wp \circ \sigma) = (F \circ \wp) \circ \sigma$, which is a composition of fuzzy smooth maps. Thus F is fuzzy smooth map on U. Hence F is fuzzy smooth in a fuzzy open Q-neighborhood of each point.

Theorem 2.7: Let M be a fuzzy Banach manifold and $\wp: \widetilde{M} \rightarrow M$ is any fuzzy covering map then $\widetilde{M}$ has a smooth manifold structure such that $\wp$ is a smooth covering map.

Proof: Let M be fuzzy Banach manifold and $\wp: \widetilde{M} \rightarrow M$ be fuzzy smooth covering map then any point $p \in M$ has an evenly covered fuzzy open Q-neighborhood U such that ${}^{\wp^{-1}}\!/_{\widetilde{U}_\iota} : \widetilde{U}_\iota \rightarrow U$ is fuzzy diffeomorphism. Since M is fuzzy Banach Manifold let (U, φ) be fuzzy Banach chart of M and $\widetilde{U}$ be component of $\wp^{-1}(U)$. Thus we can consider $\widetilde{U}$ as the domain of the co-ordinate map $\tilde{\varphi}: \varphi \circ \wp: \widetilde{U} \rightarrow E_i$ where $sup\left(\mu_{\widetilde{U}}(x)\right) = 1\ \forall\, x \in M$. Let $\widetilde{U}$ be a component of $\wp^{-1}(U)$ and $\tilde{\varphi}: \varphi \circ \wp: \widetilde{U} \rightarrow E_i$ be co-ordinate map then clearly, $\left(\widetilde{U}, \tilde{\varphi}\right)$ is fuzzy Banach chart on $\widetilde{M}$. If two such charts $\left(\widetilde{U}, \tilde{\varphi}\right)$ and $\left(\widetilde{V}, \tilde{\psi}\right)$ overlap then the fuzzy transition maps can be defined as:

$$\psi \circ \varphi^{-1} = \left(\psi \circ {}^{\wp}\!/_{(\widetilde{U} \cap \widetilde{V})}\right) \circ \left(\phi \circ {}^{\wp}\!/_{(\widetilde{U} \cap \widetilde{V})}\right)^{-1}$$

$$= \psi \circ {}^{\wp}\!/_{(\widetilde{U} \cap \widetilde{V})} \circ {}^{\wp^{-1}}\!/_{(\widetilde{U} \cap \widetilde{V})} \circ \phi^{-1}$$

$$= \psi \circ \varphi^{-1}$$

which is fuzzy diffeomorphism. Thus the collection of all such fuzzy Banach charts defines a smooth structure on $\widetilde{M}$.

Remark 2.8: Since the co-ordinate map on $\widetilde{M}$ is given by $\tilde{\varphi}: \varphi \circ \wp: \widetilde{U} \rightarrow E_i$, it can be easily shown that the smooth structure defined on $\widetilde{M}$ is unique by existence of unique fuzzy local section.

Theorem 2.9: Let $\wp_1: \widetilde{M} \rightarrow M$ and $\wp_2: \widetilde{M'} \rightarrow M$ be covering maps with $\widetilde{M}, \widetilde{M'}$ be connected and M is locally fuzzy path connected. If $\tilde{\wp}: \widetilde{M} \rightarrow \widetilde{M'}$ is a fuzzy smooth map such that $\wp_1 = \wp_2 \tilde{\wp}$ and $\tilde{\wp}$ is also a fuzzy smooth covering map.

Proof: Let M and $\widetilde{M}$ are locally path connected fuzzy Banach manifolds and U be a path connected open Q-neighborhood of M which is evenly covered by both the covering maps $\wp_1$ and $\wp_2$. Then the fuzzy sheets above U for $\wp_1$ and $\wp_2$ are the fuzzy path component of ${\wp_1}^{-1}(U) = \cup_{i \in I} W_i$ and ${\wp_2}^{-1}(U) = \cup_{j \in J} V_j$ where ${}^{\wp_1}\!/_{W_i}$ and ${}^{\wp_2}\!/_{V_j}$ are fuzzy diffeomorphisms. Since each W_i is path connected and $\tilde{p}$ is fuzzy smooth then $\tilde{p}(W_i) \subseteq V_k$ where $k \in I$ relating i and j. Since ${p_1}/_{W_i}$ and ${p_2}/_{V_k}$ are fuzzy diffeomorphisms it follows that ${\tilde{p}}/_{W_i}$ is a fuzzy diffeomorphism onto V_k. Thus V_k is evenly covered by $\tilde{p}$.

If $\tilde{p}(\widetilde{M})$ is open fuzzy set in $\tilde{p}(\widetilde{M'})$. Let x_λ be any point of $\widetilde{M'}$ and V be a open Q- neighborhood of $x_\lambda$which is evenly covered by $\tilde{p}$. Then $V \cap \tilde{p}(\widetilde{M})$ is nonempty and so, $V \subset \tilde{p}(\widetilde{M})$. Since $\widetilde{M'}$ is connected it follows that $\tilde{p}(\widetilde{M}) = \widetilde{M'}$ that is $\tilde{p}$ is surjective. Hence $\tilde{p}$ is fuzzy smooth covering map.

3. FUZZY LIFTING PROPERTIES ON FUZZY BANACH MANIFOLD

In this section we define fuzzy lift and prove unique existence and lifting criteria referring [4] [7] [17].

Definition 3.1: Let $p: \widetilde{M_1} \to M_1$ be a fuzzy smooth covering map and let $f: M_2 \to M_1$ be a fuzzy smooth map. A map $\tilde{f}: M_2 \to \widetilde{M_1}$ is said to be a fuzzy smooth lift of the map f if $p \circ \tilde{f} = f$.

Theorem 3.2: Let $f: M_2 \to M_1$ be a fuzzy smooth map with connected domain M_2. If $f_1, f_2: M_2 \to \widetilde{M_1}$ are two fuzzy lifts of f which agree at at least one point then $f_1 = f_2$.

Proof: Let $f: M_2 \to M_1$ be a fuzzy smooth map with connected domain M_2. If $f_1, f_2: M_2 \to \widetilde{M_1}$ are two fuzzy lifts of f which agree at at least one point then we have $W = \{x_\lambda \in M_2 / f_1(x_\lambda) = f_2(x_\lambda)\}$. Let $x_\lambda \in M_2$ and U be an evenly covered Q-neighborhood of $f(x_\lambda)$.

If $x_\lambda \in W$ then let V is a fuzzy sheet which is covering over U which contains x_λ such that $f_1 = f_2$ then ${f_1}^{-1}(V) \cap {f_2}^{-1}(V)$ is an open fuzzy set contained in W. Since $pf_1 = pf_2$, W is open fuzzy set.

If $x_\lambda \notin W$ then we choose disjoint sheets V_1 and V_2 over U such that $f_1(x_\lambda) \in V_1$ and $f_2(x_\lambda) \in V_2$ then ${f_1}^{-1}(V) \cap {f_2}^{-1}(V)$ is an open fuzzy set contained in $M_1 \setminus W$. Thus we have $f_1 = f_2$.

Lemma 3.3: If $p: \widetilde{M_1} \to M_1$ is a fuzzy smooth covering space, $\alpha: (I, \widetilde{\varepsilon_I}) \to (M_1, \tau_1)$ is any fuzzy path and $x_\lambda \in p^{-1}\left(\alpha_A\left(0_{A(0)}\right)\right)$ then there exists a unique map $\tilde{\alpha}: (I, \widetilde{\varepsilon_I}) \to \left(\widetilde{M_1}, \widetilde{\tau_1}\right)$ such that $p \circ \tilde{\alpha}_A = \alpha_A$ and $\tilde{\alpha}_A\left(0_{A(0)}\right) = x_\lambda$.

Proof: Let $p: \widetilde{M_1} \to M_1$ is a fuzzy smooth covering space, $\alpha: (I, \widetilde{\varepsilon_I}) \to (M_1, \tau_1)$ is any fuzzy path and $x_\lambda \in p^{-1}\left(\alpha_A\left(0_{A(0)}\right)\right)$ then by definition of fuzzy smooth covering map, for every point $y_\delta \in \alpha(A)$ there is an open Q-neighborhood U_{y_δ} whose inverse image under p is disjoint Q-neighborhood each of which is mapped fuzzy diffeomorphically by p on to U_{y_δ}, the set of all such open fuzzy Q-neighborhood covers $\alpha(A)$ and since $\alpha(A)$ is a-compact where $a \in [0;\ 1]$, which has finite subcovers that we denote by $U_0, U_1, \cdots U_k$ that is $\alpha(A) \subseteq \bigcup_{i=0}^{k} U_i$.

Let $x_\lambda = \alpha_A(0_A(0)) \in U_0$ and $\widetilde{U_0}$ be the component of $p^{-1}(U_0)$ that contains x_λ then we can lift a part of the path α_A contained in U_0 to $\widetilde{U_0}$.

Let $z_\rho \in \alpha_A$ be a fuzzy point contained in both U_0 and U_1 and ${z_\rho}' \in {p}/_{U_1}(z_\rho)$ and $\widetilde{U_1}$ be the component of the $p^{-1}(U_1)$ containing ${z_\rho}'$.

We now extend the process of lifting of the fuzzy path to its part contained in U_1 by using ${p^{-1}}/_{\widetilde{U_1}}$ which is fuzzy diffeomorphism between $\widetilde{U_1}$ to U_1. Continuing the process of lifting for finite number of steps the entire fuzzy path α_A is lifted to a fuzzy path $\tilde{\alpha}_A$ in $\widetilde{M_1}$ and from above theorem we can show that the lift obtained is unique.

Theorem 3.4: Let $p: \widetilde{M_1} \to M_1$ be fuzzy smooth covering map, $f: M_2 \to M_1$ be fuzzy smooth map where M_2 is locally fuzzy path connected and $w_\rho \in \widetilde{M_1}$ and $y_\delta \in M_2$ such that $(w_\rho) = f(y_\delta) = x_\lambda$. If f has a fuzzy lift such that $\tilde{f}: M_2 \to \widetilde{M_1}$ such that $\tilde{f}(y_\delta) = w_\rho$ then $f_*\left(\pi_1(M_2, y_\delta)\right) \subseteq p_*\left(\pi_1\left(\widetilde{M_1},\ w_\rho\right)\right)$.

Proof: For every $z_\eta \in M_2$ choose a fuzzy path α_A from y_δ to $z_\eta \in M_2$ then clearly $(f\alpha)_E$ is a fuzzy path from x_λ to $f(z_\eta) \in M_1$. Let $\tilde{f}(z_\eta)$ be the terminal point of $\widetilde{(f\alpha)}_E$ which is the lift of a fuzzy path $(f\alpha)_E$ starting at w_ρ. If ${\alpha_A}'$ is another fuzzy path from y_δ to z_η in M_2 then $\alpha_A \cdot \overline{{\alpha_A}'}$ is a fuzzy loop at y_δ hence $f(\ \alpha_A \cdot \overline{{\alpha_A}'})$ is a fuzzy loop at $x_\lambda \in M_1$ which by hypothesis lifts to a fuzzy loop at w_ρ. Since $(f\alpha)_E$ is fuzzy smooth homotopic to $f(\ \alpha_A \cdot \overline{{\alpha_A}'} \cdot {\alpha_A}') = f(\ \alpha_A \cdot \overline{{\alpha_A}'}) \cdot f({\alpha_A}')$ as a fuzzy path from x_λ to $f(z_\eta)$ and their fuzzy lifts have the same end points and so $\widetilde{(f\alpha)}_E(1) = \left(\widetilde{f\alpha'}\right)_F(1)$. Thus $\tilde{f}$ is well defined.

Let V be an open fuzzy Q-neighborhood of $\tilde{f}(z_\eta)$ we may assume that V is a fuzzy sheet above an evenly covered open fuzzy subset i.e., $U = \wp(V)$ of M_1. Let S be the fuzzy path connected open fuzzy Q-neighborhood of z_η in $f^{-1}(U)$. Let $u_\zeta \in S$ and β_B be a fuzzy path from y_δ to u_ζ so $\tilde{f}(u_\zeta) = \tilde{f}(\alpha_A \cdot \beta_B)(1)$ which is the end point of $(\widetilde{f\beta})_C$ the fuzzy lift of the fuzzy path $(f\beta)_C$ begining at $\tilde{f}(z_\eta)$ in V , but $\left(\wp/_V\right)^{-1}((f\alpha)_E)$ is also a fuzzy lift of same kind thus $\widetilde{f\beta_B} = \left(\wp/_V\right)^{-1}((f\alpha)_E)$ therefore $\tilde{f}(u_\zeta) = (\widetilde{f\beta})_F$ in V . Hence $\tilde{f}(S) \subseteq V$ and so $\tilde{f}$ is fuzzy continuous at z_η. Since z_η is arbitrary we have $f_*(\pi_1(M_2, y_\delta)) \subseteq \wp_*\left(\pi_1(\widetilde{M_1},\ w_\rho)\right)$.

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Source of support: Nil, Conflict of interest: None Declared.