International Journal of Mathematical Archive-8(10), 2017, 153-164
Available online through www.ijma.info ISSN 2229 – 5046

ON INTUITIONISTIC FUZZY n-NORM

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(Received On: 01-09-17; Revised & Accepted On: 18-09-17)

ABSTRACT

In this paper, we present a simple method to derive a intuitionistic fuzzy (n-1)-norm from intuitionistic fuzzy n-norm and then prove that any intuitionistic fuzzy n-normed linear space is an intuitionistic fuzzy (n-1)-normed linear space. Some results regarding convergence and completeness in the intuitionistic fuzzy n-normed linear spaces are obtained and use these results to prove a fixed point theorem in intuitionistic fuzzy n-Banach spaces.

1. INTRODUCTION

Gahler[17] introduced the theory of 2-norm and n-norm on a linear space. For a systematic development of n-normed linear spaces, one may refer to [1, 2, 8, 14]. The theory of fuzzy set was introduced by L. Zadeh in 1965[13]. T. Bag and S.K.Samanta [21] introduced the definition of fuzzy norm over a linear space. Further, Al. Narayanan and S.Vijayabalaji[4] defined the concept of fuzzy n-normed linear space. J.H.Park [9] introduced and studied a notion of intuitionistic fuzzy metric spaces. Further R.Saadati [15] defined the notion of intuitionistic fuzzy normed space. The definition of intuitionistic fuzzy n-normed linear space was given in the paper [20]. In this paper, we present a simple method to derive a intuitionistic fuzzy n-1-norm from intuitionistic fuzzy n-norm and then prove that any intuitionistic fuzzy n-normed linear space with $n \geq 2$ is an intuitionistic fuzzy (n-1)-normed linear space and hence by induction an fuzzy (n-r)-normed linear space for all r =1, 2,…..,n-1. Further some results regarding convergence and completeness in the intuitionistic fuzzy n-normed linear spaces are obtained and then used to prove a fixed point theorem in intuitionistic fuzzy n-Banach spaces.

2. PRELIMINARIES

Definition 2.1[17]: Let X be a real linear space of dimension greater than 1. Let $\|\bullet,\bullet\|$ be a real valued function on X ×X satisfying the following conditions:

1. $\|x, y\| = 0$ if any only if x, y are linearly dependent,
2. $\|x, y\|=\|y, x\|$
3. $\|ax, y\|=|a|\|x, y\|$, where $a \in R$(set of real numbers)
4. $\|x, y+z\| \leq \|x, y\|+\|x, z\|$.

$\|\bullet,\bullet\|$ is called a 2-norm on X and the pair $(X, \|\bullet,\bullet\|)$ is called a 2-normed linear space.

Definition 2.2[1]: Let $n \in N$ (natural numbers) and X be a real linear space of dimension greater than or equal to n. A real valued function $\|\bullet, \ldots, \bullet\|$ on $X \times \cdots \times X = X^n$ satisfying the following four properties:

(1) $\|x_1, x_2, \ldots, x_n\| = 0$ if any only if $x_1, x_2, \ldots, x_n$ are linearly dependent,
(2) $\|x_1, x_2, \ldots, x_n\|$ is invariant under any permutation,
(3) $\|x_1, x_2, \ldots, ax_n\| = |a|\, \|x_1, x_2, \ldots, x_n\|$, for any $a \in R$ (real),
(4) $\|x_1, x_2, \ldots, x_{n-1}, y + z\| \leq \|x_1, x_2, \ldots, x_{n-1}, y\| + \|x_1, x_2, \ldots, x_{n-1}, z\|$,
is called an n-norm on X and the pair $(X, \|\bullet, \ldots, \bullet\|)$ is called an n-normed linear space.

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Example 2.3: Let X be a space with inner product $\langle \bullet, \bullet \rangle$ Then

$$\|x_1, x_2, \ldots\ldots, x_n\|s = \begin{vmatrix} \langle x_1, x_1 \rangle & \langle x_1, x_2 \rangle & . & . & \langle x_1, x_n \rangle \\ \langle x_2, x_1 \rangle & \langle x_2, x_2 \rangle & . & . & \langle x_2, x_n \rangle \\ . & . & . & . & . \\ . & . & . & . & . \\ \langle x_n, x_1 \rangle & \langle x_n, x_2 \rangle & . & . & \langle x_n, x_n \rangle \end{vmatrix}^{\frac{1}{2}}$$

it defines an n-norm on X. This n-norm is called standard n-norm.

Definition 2.4[1]: A sequence { x_n }in an n-normed space (X, $\|\bullet, \ldots, \bullet\|$) is said to converge to $x \in X$ (in the n-norm) whenever

$$\lim_{t \to \infty} \|x_1, x_2, \ldots, x_{n-1}, x_n - x\| = 0.$$

Definition 2.5[1]: A sequence { x_n }in an n-normed space (X, $\|\bullet, \ldots, \bullet\|$) is called Cauchy sequence if

$$\lim_{n,k \to \infty} \|x_1, x_2, \ldots, x_{n-1}, x_n - x_k\| = 0.$$

Definition 2.6[1]: An n-normed linear space is said to be complete if every Cauchy sequence in it is convergent.

Definition 2.7[4]: Let X be a linear space over a real field F. A fuzzy subset N of $X^n \times R$ (R-set of real numbers) is called a fuzzy n-norm on X if and only if:
(N 1) For all $t \in R$ with $t \leq 0$, $N(x_1, x_2, \ldots, x_n, t) = 0$.
(N 2) For all $t \in R$ with $t > 0$, $N(x_1, x_2, \ldots, x_n, t) = 1$ if and only if $x_1, x_2, \ldots, x_n$ are linearly dependent.
(N 3) $N(x_1, x_2, \ldots, x_n, t)$ is invariant under any permutation of $x_1, x_2, \ldots, x_n$.
(N 4) For all $t \in R$ with $t > 0$,

$$N(x_1, x_2, \ldots, cx_n, t) = N\left(x_1, x_2, \ldots, x_n, \frac{t}{|c|}\right), \text{ if } c \neq 0, c \in F \text{ (field)}.$$

(N 5) For all $s, t \in R$,

$$N(x_1, x_2, \ldots, x_n + x_n', s + t) \geq \min\{N(x_1, x_2, \ldots, x_n, s), N(x_1, x_2, \ldots, x_n, t)\}.$$

(N 6) $N(x_1, x_2, \ldots, x_n, t)$ is a non-decreasing function of $t \in R$ and

$$\lim_{t \to \infty} N(x_1, x_2, \ldots, x_n, t) = 1.$$

Then (X, N) is called fuzzy n-normed linear space or in short f-n-NLS.

Example 2.8[4]: Let (X, $\|\bullet, \ldots, \bullet\|$) is called an n-normed linear space as in definition .Define

$$N(x_1, x_2, \ldots, x_n, t) = \begin{cases} \dfrac{t}{t + \|x_1, x_2, \ldots\ldots, x_n\|}, & when\ t > 0, t \in R, (x_1, x_2, \ldots\ldots, x_n) \in \underbrace{X \times X \times \ldots\ldots\ldots \times X}_{n} \\ 0, & when\ t \leq 0. \end{cases}$$

Then (X, N) is an f-n-NLS.

Definition 2.9[9]: A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $*$ satisfies the following conditions:

1. $*$ is commutative and associative
2. $*$ is continuous
3. $a * 1 = a$, for all $a \in [0,1]$
4. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0,1]$.

Definition 2.10[9]: A binary operation $\lozenge$: $[0, 1]\times[0, 1]\rightarrow[0, 1]$ is continuous t-co-norm if $\lozenge$ satisfies the following conditions:

1. $\lozenge$ is commutative and associative
2. $\lozenge$ is continuous
3. $a \lozenge 0=a$, for all $a\in[0,1]$
4. $a \lozenge b \leq c \lozenge d$ whenever $a\leq c$ and $b\leq d$ and $a,b,c,d \in [0,1]$.

Definition 2.11[10]: Let E any set. An intuitionistic fuzzy set A of E is an object of the form A={(x, $\mu_A(x),\gamma_A(x)$; x $\in$ **E},** where the functions μ_A :E$\rightarrow$[0,1] and γ_A :E$\rightarrow$[0,1] denote the degree of membership and non-membership of the element $x\in E$ respectively and for every $x\in E$, $0\leq \mu_A(x)+ \gamma_A(x) \leq 1$.

Definition 2.12[12]: If A and B are any two intuitionistic fuzzy sets of a non-empty set E, then $A\subseteq B$ if and only if for all $x\in E$, $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x)\geq \gamma_B(x)$; A=B if and only if for all $x\in E$, $\mu_A(x)= \mu_B(x)$ and $\gamma_A(x)= \gamma_B(x)$;
$\overline{A}$=**{(x,** $\gamma_A(x), \mu_A(x)$ **; x**$\in$E};

$A\bigcap B$= {(x, min($\mu_A(x)$, $\mu_B(x)$),max($\gamma_A(x)$, $\gamma_B(x)$)); $x\in E$};

$A\bigcup B$= {(x, max($\mu_A(x)$, $\mu_B(x)$),min($\gamma_A(x)$, $\gamma_B(x)$)); $x\in E$}.

INTUITIONISTIC FUZZY n-NORMED LINEAR SPACE

Definition 2.13[20]: Let X be a linear space over a realfield F, and fuzzy subsets N, M of $X^n \times (0,\infty)$, N denotes the degree of membership and M denotes the degree of non-membership of $(x_1, x_2, \ldots, x_n, t) \in X^n\times(0,\infty)$ satisfying the following conditions:

1. $N(x_1, x_2, \ldots, x_n, t) + M(x_1, x_2, \ldots, x_n, t) \leq 1$
2. For all $t \in R$ with $t\leq 0$, $N(x_1, x_2, \ldots, x_n, t) = 0$.
3. For all $t \in R$ with $t > 0$, $N(x_1, x_2, \ldots, x_n, t) = 1$ if and only if $x_1, x_2, \ldots, x_n$ are linearly dependent.
4. $N(x_1, x_2, \ldots, x_n, t)$ is invariant under any permutation of $x_1, x_2, \ldots, x_n$.
5. For all $t \in R$ with $t > 0$, $N(x_1, x_2, \ldots, cx_n, t) = N(x_1, x_2, \ldots, x_n, \frac{t}{|c|})$, if $c \neq 0$, $c \in F$ (field).
6. For all $s, t \in R$, $N(x_1, x_2, \ldots, x_n + x_n', s + t) \geq \min\{N(x_1, x_2, \ldots, x_n, s), N(x_1, x_2, \ldots, x_n, t)\}$.
7. $N(x_1, x_2, \ldots, x_n, t)$: $(0,\infty)\rightarrow[0,1]$ is continuous in t.
8. For all $t \in R$ with $t\leq 0$, $M(x_1, x_2, \ldots, x_n, t) = 1$.
9. For all $t \in R$ with $t > 0$, $M(x_1, x_2, \ldots, x_n, t) = 0$ if and only if $x_1, x_2, \ldots, x_n$ are linearly dependent.
10. $M(x_1, x_2, \ldots, x_n, t)$ is invariant under any permutation of $x_1, x_2, \ldots, x_n$.
11. For all $t \in R$ with $t > 0$, $M(x_1, x_2, \ldots, cx_n, t) = M(x_1, x_2, \ldots, x_n, \frac{t}{|c|})$, if $c \neq 0$, $c\in F$ (field).
12. For all $s, t \in R$, $M(x_1, x_2, \ldots, x_n + x_n', s + t) \leq \max\{M(x_1, x_2, \ldots, x_n, s), M(x_1, x_2, \ldots, x_n, t)\}$.
13. $M(x_1, x_2, \ldots, x_n, t)$: $(0,\infty)\rightarrow[0,1]$ is continuous in t.

   Then (X, N, M) is called a intuitionistic fuzzy n-normed linear space or in short i-f-n- NLS.

To strengthen the above definition, we present the following example.

Example 2.14 [20]: Let (X, || .,.,…,. ||) be an n-normed linear space and

$$N(x_1,\ldots,x_n,t) = \frac{t}{t+\| x_1, x_2,\ldots, x_n \|}$$

$$M(x_1,\ldots,x_n,t) = \frac{\| x_1,\ldots, x_n \|}{t+\| x_1, x_2,\ldots, x_n \|}$$

Then (X, N, M) is i-f-n-NLS.

Definition 2.15 [20]: A sequence $\{x_n\}$ in an i-f-n-NLS is said to x if given r>0, t>0, 0<r<1 there exists an integer $n_0 \in$ N such that $N(x_1, x_2, \ldots, x_{n-1}, x_n - x, t)>1-r$ and $M(x_1, x_2, \ldots, x_{n-1}, x_n - x, t)< r$, for all $n\geq n_0$.

Theorem 2.16 [20]: In an i-f-n-NLS, a sequence converges to x if and only if

$N(x_1, x_2, \ldots, x_{n-1}, x_n - x, t) \to 1$ and $M(x_1, x_2, \ldots, x_{n-1}, x_n - x, t) \to 0$, as $n \to \infty$.

Definition 2.17[20]: A sequence $\{x_n\}$in an i-f-n-NLS, is said to be Cauchy sequence if given $\varepsilon > 0$, with $0 < \varepsilon < 1$, $t > 0$ there exists an integer $n_0 \in N$ such that $N(x_1, x_2, \ldots, x_{n-1}, x_n - x_k, t) > 1 - \varepsilon$ and $M(x_1, x_2, \ldots, x_{n-1}, x_n - x_k, t) < \varepsilon$ for all $n, k \geq n_0$.

Theorem 2.18 [20]: In a i-f-n-NLS (X, N) a sequence $\{x_k\}$ is Cauchy if and only if

$$\lim_{k,\ell \to \infty} N(x_1, \ldots, x_{n-1}, x_k - x_\ell, x, t) = 1,$$

$$\lim_{k,\ell \to \infty} M(x_1, \ldots, x_{n-1}, x_k - x_\ell, x, t) = 0, \text{ for every } x_1, \ldots, x_{n-1} \in X.$$

Theorem 2.19[20]: In an i-f-n-NLS, every convergent sequence is a cauchy sequence.

3. MAIN RESULT

Suppose (X, N, M) is an i-f-n-NLS. Take a linearly independent set $\{a_1, \ldots, a_n\}$, define the following function $N_\infty(.,.,\ldots,.,.)$ and $M_\infty(.,.,\ldots,.,.)$ on $\underbrace{X \times X \times \ldots \times X}_{n-1} \times \mathbf{R}$ by

$N_\infty(x_1, x_2, \ldots, x_{n-1}, t) = \min\{N(x_1, x_2, \ldots, x_{n-1}, a_i, t);\ i = 1, .., n\}$

and $M_\infty(x_1, x_2, \ldots, x_{n-1}, t) = \max\{N(x_1, x_2, \ldots, x_{n-1}, a_i, t);\ i = 1, .., n\}$

Theorem 3.1: The function $N_\infty(.,.,\ldots,.,.)$ and $M_\infty(.,.,\ldots,.,.)$ defines an i-f-(n-1)-NLS on X.

Proof: We will verify that $N_\infty(.,.,\ldots,.,.)$ and $M_\infty(.,.,\ldots,.,.)$ satisfies the all properties of i-f-(n-1)-NLS.

(i) $N_\infty(x_1, x_2, \ldots, x_{n-1}, t) + M_\infty(x_1, x_2, \ldots, x_{n-1}, t) \leq 1$, since
$N(x_1, x_2, \ldots, x_{n-1}, a_i, t) + M(x_1, x_2, \ldots, x_{n-1}, a_i, t) \leq 1$, for each $i = 1, \ldots, n$.

(ii) for all $t \in \mathbf{R}$ with $t \leq 0$, we have
$N(x_1, x_2, \ldots, x_{n-1}, a_i, t) = 0$ for each $i = 1, \ldots, n$.
$\Rightarrow N_\infty(x_1, x_2, \ldots, x_{n-1}, t) = 0$

(iii) for all $t \in \mathbf{R}$ with $t > 0$, we have
$N_\infty(x_1, x_2, \ldots, x_{n-1}, t) = 1$
$\Leftrightarrow \min\{N(x_1, x_2, \ldots, x_{n-1}, a_i, t);\ i = 1, \ldots, n\} = 1$
$\Leftrightarrow N(x_1, x_2, \ldots, x_{n-1}, a_i, t) = 1$ for each $i = 1, \ldots, n$.
$\Leftrightarrow x_1, x_2, \ldots, x_{n-1}, a_i$ are linearly dependent for each $i = 1, \ldots, n$. But this can only happen when $x_1, \ldots, x_{n-1}$ are linearly dependent.

(iv) Since $N(x_1, \ldots, x_{n-1}, a_i, t)$ is invariant under any permutation of $x_1, \ldots, x_{n-1}$.
$\Rightarrow N_\infty(x_1, \ldots, x_{n-1}, t)$ is invariant under any permutation of $x_1, \ldots, x_{n-1}$.

(v) For all $t \in \mathbf{R}$ with $t > 0$ and $c \in F$, $c \neq 0$,
$N_\infty(x_1, \ldots, cx_{n-1}, t) = \min\{N(x_1, \ldots, cx_{n-1}, a_i, t);\ i = 1, \ldots, n\}$

$$N_\infty(x_1, \ldots, cx_{n-1}, t) = \min\{N(x_1, \ldots, x_{n-1}, a_i, \frac{t}{|c|});\ i = 1, \ldots, n\}$$

$$= N_\infty(x_1, \ldots, x_{n-1}, \frac{t}{|c|})$$

(vi) $N_\infty(x_1, \ldots, x_{n-2}, x_{n-1} + x'_{n-1}, t+s)$

$= \min\{N(x_1, \ldots, x_{n-2}, x_{n-1} + x'_{n-1}, a_i, t+s);\ i = 1, \ldots, n\}$

$\geq \min\{\min\{N(x_1, \ldots, x_{n-2}, x_{n-1}, a_i, t), N(x_1, \ldots x_{n-2}, x'_{n-1}, a_i, s;\ i = 1, \ldots, n\}$

$\geq \min\{\min\{N(x_1, \ldots, x_{n-2}, x_{n-1}, a_i, t);\ i = 1 \ldots n\}, \min\{N(x_1, \ldots x_{n-2}, x'_{n-1}, a_i, s\};\ i = 1\text{-}n\}\}$

$= \min\{N_\infty(x_1, \ldots, x_{n-1}, t), N_\infty(x_1, \ldots, x'_{n-1}, s)\}$

(vii) Since $N(x_1, \ldots, x_{n-1}, a_i, .)$ is continuous, so $N_\infty(x_1, \ldots, x_{n-1}, t)$ is continuous.

(viii) $M_\infty(x_1, x_2, \ldots, x_{n-1}, t) > 0$, for $M(x_1, x_2, \ldots, x_{n-1}, a_i, t) > 0$ for each $i = 1, 2, \ldots, n$.

(ix) for all $t \in \mathbf{R}$ with $t > 0$, we have
$\mathbf{M}_\infty(x_1, x_2, \ldots, x_{n-1}, t) = 0$
$\Leftrightarrow \max.\{M(x_1, x_2, \ldots, x_{n-1}, a_i, t);\ i = 1, \ldots, n\} = 0$
$\Leftrightarrow M(x_1, x_2, \ldots, x_{n-1}, a_i, t) = 0$ for each $i = 1, \ldots, n$.

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$\Leftrightarrow$ $x_1,x_2,\ldots,x_{n-1},a_i$ are linearly dependent for each i = 1, …,n. But this can only happen when $x_1, \ldots, x_{n-1}$ are linearly dependent

(x) $M_\infty(x_1,\ldots,x_{n-1}, t)$ is invariant under any permutation of $x_1,\ldots,x_{n-1}$, since $M(x_1,\ldots,x_{n-1},a_i,t)$ is invariant under any permutation of $x_1,\ldots,x_{n-1}$.

(xi) For all $t \in \mathbf{R}$ with $t > 0$ and $c \in F$, $c \neq 0$,

$M_\infty(x_1,\ldots,cx_{n-1},t) = \max. \{M(x_1,\ldots,cx_{n-1},a_i, t); i = 1,\ldots,n\}$

$$M_\infty(x_1,\ldots,cx_{n-1},t) = \max. \{M(x_1,\ldots,x_{n-1},a_i, \frac{t}{|c|}); i = 1,\ldots,n\}$$

$$= M_\infty(x_1,\ldots,x_{n-1}, \frac{t}{|c|})$$

(xii) $M_\infty(x_1,\ldots,x_{n-2}, x_{n-1} + x'_{n-1}, t+s) = \max.\{M (x_1,\ldots,x_{n-2}, x_{n-1}+ x'_{n-1}, a_i, t+s); i = 1,\ldots,n \}$

$\leq \max.\{\max.\{M(x_1,\ldots,x_{n-2},x_{n-1},a_i,t),M(x_1,\ldots x_{n-2},x'_{n-1},a_i,s\}; i = 1,\ldots,n \}$

$\leq \max.\{\max.\{M (x_1,\ldots,x_{n-2},x_{n-1},a_i,t); i = 1\ldots n\}, \text{Max}.\{M(x_1,\ldots x_{n-2},x'_{n-1},a_i,s\}; i = 1\text{-}n \}\}$

$= \max.\{M_\infty(x_1,\ldots,x_{n-1}, t), M_\infty(x_1,\ldots,x'_{n-1}, s)\}$

(xiii) Since $M(x_1,\ldots,x_{n-1}, a_i,.)$ is continuous function of t, so $M_\infty(x_1,\ldots,x_{n-1}, t)$ is continuous by definition.

Thus (X, N_∞, M_∞) becomes a i-f- (n-1)- NLS.

Corollary 3.2: Every i-f-n-normed space is i-f-(n-r)-normed space for all r=1,2,…,n-1. In particular, every i-f-n-normed space is a i-fuzzy normed linear space.

Example 3.3: Suppose (X, N, M) is a i-f-n-NLS define in example (2.13). Take a linearly independent set $\{a_1, a_2,\ldots,a_n\}$ in X. With respect to $\{a_1,\ldots,a_n\}$ define the following function

$$N_\infty(x_1,\ldots,x_{n-1},t) = \min\left\{\frac{t}{t+\|x_1,\ldots,x_{n-1},a_i\|}; i=1,..,n\right\}$$

and

$$M_\infty(x_1,\ldots,x_{n-1},t) = \max\left\{\frac{\|x_1,\ldots,x_{n-1},a_i\|}{t+\|x_1,\ldots,x_{n-1},a_i\|}; i=1,..,n\right\}$$

Then (X, N_∞, M_∞) becomes an i-f-(n-1) NLS.

Proof:

(i) Clearly $N_\infty (x_1,\ldots,x_{n-1}, t) + M_\infty (x_1,\ldots,x_{n-1}, t) \leq 1$;

(ii) Obviously $N_\infty (x_1,\ldots,x_{n-1}, t) > 0$;

(iii) $N (x_1,\ldots,x_{n-1}, t) = 1$

$$\Leftrightarrow \min\left\{\frac{t}{t+\|x_1,\ldots,x_{n-1},a_i\|}; i=1,..,n\right\} = 1$$

$$\Leftrightarrow \frac{t}{t+\max_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1},a_i\|} = 1$$

$$\Leftrightarrow t = t + \max_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1},a_i\|$$

$$\Leftrightarrow \max_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1},a_i\| = 0$$

But it is only possible, when $x_1,\ldots,x_{n-1}$ are linearly dependent.

(iv) $$N (x_1,\ldots, x_{n-2}, x_{n-1},t) = \min\left\{\frac{t}{t+\|x_1,\ldots,x_{n-2},x_{n-1},a_i\|}; i=1,..,n\right\}$$

$$= \min\left\{\frac{t}{t+\|x_1,\ldots,x_{n-1},x_{n-2},a_i\|}; i=1,..,n\right\}$$

$= N_\infty (x_1,\ldots,x_{n-1},x_{n-2},t)$

= ......

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(v) $N_\infty(x_1,x_2,\ldots,x_{n-1},\frac{t}{|c|}) = \min\left\{\frac{\frac{t}{|c|}}{\frac{t}{|c|}+\|x_1,\ldots,x_{n-1},a_i\|};i=1,..,n\right\}$

$$=\min\left\{\frac{\frac{t}{|c|}}{\frac{t+|c|\|x_1,\ldots,x_{n-1},a_i\|}{|c|}};i=1,..,n\right\}$$

$$=\min\left\{\frac{t}{t+|c|\|x_1,\ldots,x_{n-1},a_i\|};i=1,..,n\right\}$$

$$=\min\left\{\frac{t}{t+\|x_1,\ldots,cx_{n-1},a_i\|};i=1,..,n\right\}$$

$= N_\infty(x_1,x_2,\ldots,cx_{n-1},t)$

(vi) W.L.O.G. we assume that

$N_\infty(x_1,x_2,\ldots x'_{n-1},t) \le N_\infty(x_1,x_2,\ldots x_{n-1},s)$

$$\Rightarrow \min\left\{\frac{t}{t+\|x_1,\ldots,x'_{n-1},a_i\|};i=1,..,n\right\}\le\min\left\{\frac{s}{s+\|x_1,\ldots,x_{n-1},a_i\|};i=1,..,n\right\}$$

$$\Rightarrow \frac{t}{t+\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x'_{n-1},a_i\|}\le\frac{s}{s+\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1},a_i\|}$$

$$\Rightarrow t(s+\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1},a_i\|)\le s(t+\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x'_{n-1},a_i\|)$$

$$\Rightarrow \max\limits_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1},a_i\|\le\frac{s}{t}\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x'_{n-1},a_i\|.$$

$$\Rightarrow \max\limits_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1},a_i\|+\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x'_{n-1},a_i\|$$

$$\le\frac{s}{t}\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x'_{n-1},a_i\|+\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x'_{n-1},a_i\|$$

$$=\left(\frac{s}{t}+1\right)\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x'_{n-1},a_i\|$$

$$=\frac{s+t}{t}\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x'_{n-1},a_i\|.$$

But

$$\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1}+x'_{n-1},a_i\|\le\max\limits_{i=1,\ldots,n}\{\|x_1,\ldots,x_{n-1},a_i\|+\|x_1,\ldots,x'_{n-1},a_i\|\}$$

$$\le\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1},a_i\|+\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x'_{n-1},a_i\|$$

$$\le\frac{s+t}{t}\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x'_{n-1},a_i\|$$

$$\frac{\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1}+x'_{n-1},a_i\|}{s+t} \le \frac{\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1},x'_{n-1},a_i\|}{t}$$

$$1+\frac{\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1}+x'_{n-1},a_i\|}{s+t} \le 1+\frac{\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1},x'_{n-1},a_i\|}{t}$$

$$\frac{s+t+\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1}+x'_{n-1},a_i\|}{s+t} \le \frac{t+\max\limits_{i=1,\ldots,n}\|x_1,\ldots,x_{n-1},x'_{n-1},a_i\|}{t}$$

$$\min_{i=1,\ldots,n}\frac{s+t}{s+t+\|x_1,\ldots,x_{n-1}+x'_{n-1},a_i\|} \ge \min_{i=1,\ldots,n}\frac{t}{t+\|x_1,\ldots,x'_{n-1},a_i\|}$$

$\Rightarrow$ $N_\infty(x_1,\ldots,x_{n-1}+x'_{n-1},s+t) \ge \min\{N_\infty(x_1,\ldots,x_{n-1},s), N_\infty(x_1,\ldots,x'_{n-1},t)\}$

(vii) Clearly $N_\infty(x_1,\ldots,x_{n-1},t)$ is continuous in t.

(viii) By definition, we have $M_\infty(x_1,x_2,\ldots,x_{n-1},t) \ge 0$

(ix) $M_\infty(x_1,x_2,\ldots,x_{n-1},t) = 0$

$$M_\infty(x_1,\ldots,x_{n-1},t) = \max\left\{\frac{\|x_1,\ldots,x_{n-1},a_i\|}{t+\|x_1,\ldots,x_{n-1},a_i\|}; i=1,..,n\right\} = 0$$

$\Leftrightarrow \dfrac{\|x_1,\ldots,x_{n-1},a_i\|}{t+\|x_1,\ldots,x_{n-1},a_i\|} = 0$ for each i=1,……,n.

$\Leftrightarrow \|x_1,x_2,\ldots,x_{n-1},a_i\| = 0$ for each i=1,……,n.

$\Leftrightarrow$ $x_1, x_2, \ldots, x_{n-1}$ are linearly dependent.

(x)
$$M_\infty(x_1,x_2,\ldots,x_{n-1},t) = \max\left\{\frac{\|x_1,.x_2,\ldots,x_{n-2},x_{n-1},a_i\|}{t+\|x_1,x_2,\ldots,x_{n-2},x_{n-1},a_i\|}; i=1,..,n\right\}$$

$$= \max\left\{\frac{\|x_1,.x_2,\ldots,x_{n-1},x_{n-2},a_i\|}{t+\|x_1,x_2,\ldots,x_{n-1},x_{n-2},a_i\|}; i=1,..,n\right\}$$

$= M_\infty(x_1, x_2, \ldots, x_{n-1},x_{n-2},t)$

$= \ldots$

(xi)
$$M_\infty(x_1,x_2,\ldots,cx_{n-1},t) = \max\left\{\frac{\|x_1,\ldots,cx_{n-1},a_i\|}{t+\|x_1,\ldots,cx_{n-1},a_i\|}; i=1,..,n\right\}$$

$$= \max\left\{\frac{|c|\|x_1,\ldots,x_{n-1},a_i\|}{t+|c|\|x_1,\ldots,x_{n-1},a_i\|}; i=1,..,n\right\}$$

$$= \max\left\{\frac{\|x_1,\ldots,x_{n-1},a_i\|}{\frac{t}{|c|}+\|x_1,\ldots,x_{n-1},a_i\|}; i=1,..,n\right\}$$

$= M_\infty(x_1,\ldots,x_{n-1},\dfrac{t}{|c|})$.

(xii) Without loss of generality assume,

$M_\infty(x_1,\ldots,x_{n-1},s) \le M_\infty(x_1,\ldots,x'_{n-1},t)$

$$\max\left\{\frac{\|x_1,...,x_{n-1},a_i\|}{s+\|x_1,...,x_{n-1},a_i\|};i=1,..,n\right\} \leq \max\left\{\frac{\|x_1,...,x'_{n-1},a_i\|}{t+\|x_1,...,x'_{n-1},a_i\|};i=1,..,n\right\}$$

$$\Rightarrow \left\{\frac{\|x_1,...,x_{n-1},a_i\|}{s+\|x_1,...,x_{n-1},a_i\|}\right\} \leq \left\{\frac{\|x_1,...,x'_{n-1},a_i\|}{t+\|x_1,...,x'_{n-1},a_i\|}\right\} \quad \text{for each i=1,...,n}$$

$$\Rightarrow \frac{\|x_1,......,x_{n-1}+x'_{n-1},a_i\|}{s+t+\|x_1,.....,x_{n-1}+x'_{n-1},a_i\|} \leq \frac{\|x_1,......,x'_{n-1},a_i\|}{t+\|x_1,.....,x'_{n-1},a_i\|} \quad \text{for each i=1,...,n}$$

$$\Rightarrow \max\left\{\frac{\|x_1,...,x_{n-1}+x'_{n-1},a_i\|}{s+t+\|x_1,...,x_{n-1}+x'_{n-1},a_i\|};i=1,..,n\right\} \leq \max\left\{\frac{\|x_1,...,x'_{n-1},a_i\|}{t+\|x_1,...,x'_{n-1},a_i\|};i=1,..,n\right\}$$

$\Rightarrow M_\infty(x_1,...,x_{n-1}+x'_{n-1},s+t) \leq M_\infty(x_1,x_2,...,x'_{n-1},t)$

Similarly,

$M_\infty(x_1,...,x_{n-1}+x'_{n-1}, s+t) \leq M_\infty(x_1, x_2,...,x_{n-1}, s)$

$\Rightarrow M_\infty(x_1,...,x_{n-1}+x'_{n-1}, s+t) \leq \max\{M_\infty(x_1, x_2,...,x_{n-1}, s), M_\infty(x_1, x_2,...,x'_{n-1}, t)\}$

(xiii) Clearly
$M_\infty (x_1,...,x_{n-1},t)$ is continuous in t.
Thus (X, N_∞, M_∞) is an i-f-(n-1) NLS.

Example 3.4: Let $(X, \| .,.,...,. \|_s)$ be standard n-norm space and

$$N_s (x_1, x_2,...,x_n, t) = \frac{t}{t+\| x_1, x_2,..., x_n \|_s}$$

and

$$M_s (x_1, x_2,...,x_n, t) = \frac{\| x_1, x_2,..., x_n \|_s}{t+\| x_1, x_2,..., x_n \|_s}$$

Then (X, N_s , M_s) is an i-f-n-NLS space and the space (X, N_s, M_s) is called standard i-f-n-NLS space.

Proposition 3.5: On a i-f-n-NLS X, the derived i-f-(n-1)-NLS $N_\infty(.,.,\ldots,.,.)$ and $M_\infty(.,.,\ldots,.,.)$ defined with respect to $\{e_1,\ldots,e_n\}$ and $N_S(.,.,\ldots,.,.)$, $M_S(.,.,\ldots,.,.)$ standard i-f-(n-1)-norm. The, we have

$$N_\infty(x_1,\ldots,x_{n-1},t) \geq N_S(x_1,\ldots,x_{n-1},t) \geq N_\infty(x_1,\ldots,x_{n-1}, \frac{t}{\sqrt{n}})$$

and

$$M_\infty(x_1,\ldots,x_{n-1},t) \leq M_S(x_1,\ldots,x_{n-1},t) \leq M_\infty(x_1,\ldots,x_{n-1}, \frac{t}{\sqrt{n}})$$

Proof: Assume that $x_1,\ldots,x_{n-1}$ are linearly independent. For each i = 1,….,n write $e_i = e_i^{\,0} + e_i^{\,\perp}$ where $e_i^o \in$ span $\{x_1,\ldots,x_{n-1}\}$ and $e_i^{\,\perp} \perp$ span$\{x_1,\ldots,x_{n-1}\}$. Then we have

$$N_S (x_1,\ldots,x_{n-1}, e_i, t) = \frac{t}{t+\| x_1,..., x_{n-1}, e_i \|_S}$$

As $\left\|x_1,..., x_{n-1}, e_i^{\,0}\right\|_s = 0$,

And
$$\left\|x_1,..., x_{n-1}, e_i\right\|_s = \left\|x_1,..., x_{n-1}, e_i^{\,0} + e_i^{\,\perp}\right\|_s \leq \left\|x_1,..., x_{n-1}, e_i^{\,0}\right\|_s + \left\|x_1,..., x_{n-1}, e_i^{\,\perp}\right\|_s$$
$$= \left\|x_1,..., x_{n-1}, e_i^{\,\perp}\right\|_s$$

Therefore,

$$N_S (x_1,\ldots,x_{n-1}, e_i, t) \geq \frac{t}{t+\left\|x_1,..., x_{n-1}, e_i^{\,\perp}\right\|_s}$$
$$\geq \frac{t}{t+\| x_1,..., x_{n-1} \|_S}$$
$$= N_S (x_1,\ldots,x_{n-1},t)$$

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$\Leftrightarrow \min N_S(x_1,\ldots,x_{n-1},e_i,t) \geq N_S(x_1,..,x_{n-1},t)$

$\therefore N_\infty(x_1,\ldots,x_{n-1},t) \geq N_S(x_1,..,x_{n-1},t)$ (1)

Next, take a unit vector $e = \alpha_1 e_1 + \ldots + \alpha_n e_n$ such that $e \perp \text{span}\{x_1,\ldots,x_{n-1}\}$. (We still assume that $x_1,\ldots,x_{n-1}$ are linearly independent). We have

$$N_S(x_1,\ldots,x_{n-1},t) = \frac{t}{t+\| x_1,\ldots,x_{n-1}\|_S}$$

$$= \frac{t}{t+\| x_1,\ldots,x_{n-1},e\|_S}$$

$$\geq \frac{t}{t+|\alpha_1|\|x_1,\ldots\ldots,x_{n-1},e_1\|_s+\ldots\ldots\ldots+|\alpha_2|\|x_1,\ldots.,x_{n-1},e_n\|_s}$$

as $|\alpha_1|+|\alpha_2|+\ldots.+|\alpha_n| \leq \sqrt{n}$, therefore,

$$N_S(x_1,\ldots,x_{n-1},t) \geq \frac{t}{t+\sqrt{n}\max \|x_1,\ldots,x_{n-1},e_i\|_S}$$

$$= \min \frac{\frac{t}{\sqrt{n}}}{\frac{t}{\sqrt{n}}+\|x_1,\ldots,x_{n-1},e_i\|_S}$$

$$= N_\infty\left(x_1,\ldots,x_{n-1},\frac{t}{\sqrt{n}}\right)$$

Hence we obtain

$$N_S(x_1,\ldots,x_{n-1},t) \geq N_\infty\left(x_1,\ldots,x_{n-1},\frac{t}{\sqrt{n}}\right). \quad (2)$$

Hence by (1) and (2), we get

$$N_\infty(x_1,\ldots,x_{n-1},t) \geq N_S(x_1,\ldots,x_{n-1},t) \geq N_\infty(x_1,\ldots,x_{n-1},\frac{t}{\sqrt{n}})$$

Now consider, by (1)

$$\min\left\{\frac{t}{t+\|x_1,\ldots..,x_{n-1,}e_i\|_s};i=1,\ldots..,n\right\} \geq \frac{t}{t+\|x_1,\ldots.,x_{n-1}\|_s}$$

$$\Rightarrow \mathbf{1}\text{-}\min\left\{\frac{t}{t+\|x_1,\ldots..,x_{n-1,}e_i\|_s};i=1,\ldots..,n\right\} \leq \mathbf{1}\text{-}\frac{t}{t+\|x_1,\ldots.,x_{n-1}\|_s}$$

$$\Rightarrow \max\left\{1-\frac{t}{t+\|x_1,\ldots..,x_{n-1,}e_i\|_s};i=1,\ldots..,n\right\} \leq \frac{t+\|x_1,\ldots..,x_{n-1}\|_s-t}{t+\|x_1,\ldots.,x_{n-1}\|_s}$$

$$\Rightarrow \max\left\{\frac{\|x_1,\ldots..,x_{n-1}\|_s}{t+\|x_1,\ldots..,x_{n-1,}e_i\|_s};i=1,\ldots..,n\right\} \leq \frac{\|x_1,\ldots..,x_{n-1}\|_s}{t+\|x_1,\ldots.,x_{n-1}\|_s}$$

$$\Rightarrow M_\infty(x_1,\ldots,x_{n-1},t) \leq M_S(x_1,\ldots,x_{n-1},t) \quad (3)$$

And by (2),

$$\frac{t}{t+\|x_1,\ldots.,x_{n-1}\|_s} \geq \frac{\frac{t}{\sqrt{n}}}{\frac{t}{\sqrt{n}}+\|x_1,\ldots.,x_{n-1},e_i\|_s}$$

$$\Rightarrow\ 1-\frac{t}{t+\|x_1,....,x_{n-1}\|_s} \le \max\left\{1-\frac{\frac{t}{\sqrt{n}}}{\frac{t}{\sqrt{n}}+\|x_1,......,x_{n-1},e_i\|_s};i=1,....,n\right\}$$

$$\Rightarrow\ \frac{\|x_1,......,x_{n-1}\|_s}{t+\|x_1,....,x_{n-1}\|_s} \le \max\left\{\frac{\|x_1,.......,x_{n-1}\|_s}{\frac{t}{\sqrt{n}}+\|x_1,......,x_{n-1},e_i\|_s};i=1,....,n\right\}$$

$$\Rightarrow\ M_S(x_1,\dots,x_{n-1},t) \le M_\infty(x_1,\dots,x_{n-1},\frac{t}{\sqrt{n}}). \tag{4}$$

Thus we obtain

$$M_\infty(x_1,\dots,x_{n-1},t) \le M_S(x_1,\dots,x_{n-1},t) \le M_\infty(x_1,\dots,x_{n-1},\frac{t}{\sqrt{n}}).$$

The finite-dimensional case 3.6:

For finite-dimensional i-f-n-NLS (X, N,M), we can derive an i-f-(n-1)-norm from the i-f-n-norm by taking $N_\infty(x_1,\dots,x_{n-1},t) = \min\{N(x_1,\dots,x_{n-1},a_i,t);\ i = 1,\dots,m\}$ and $M_\infty(x_1,\dots,x_{n-1},t) = \max.\{M(x_1,\dots,x_{n-1},a_i,t);\ i = 1,\dots,m\}$ and where the set $\{a_1,\dots\dots,a_n\}$ is linearly independent in X with $n \le m \le d$ (where d is the dimension of X) Then, as in theorem [1.6], the function $N_\infty(.,.,\dots,.,.)$ and $M_\infty(.,.,\dots,.,.)$ defines i-f- (n-1)- norm on X.

Theorem 3.7: If $\{x_k\}$ converges to $x \in X$ in i-f-n-norm. Then $\{x_k\}$ also converges to x in the derived i-f-(n-1)-norm N_∞ and M_∞.

Proof: Let $x_k \to x$ in i-f-n-norm then

$$\lim_{k\to\infty} N(x_1,\dots,x_{n-2},x_k-x,a_i,t) = 1$$

and $$\lim_{k\to\infty} M(x_1,\dots,x_{n-2},x_k-x,a_i,t) = 0 \text{ for every } x_1,\dots,x_{n-2} \text{ and } i = 1,\dots,n.$$

Thus we have

$$\lim_{k\to\infty} N(x_1,\dots,x_{n-2},x_k-x,t) = 1$$

$$\lim_{k\to\infty} M(x_1,\dots,x_{n-2},x_k-x,t) = 0$$

Proposition 3.8: A sequence in a standard i-f-n normed space X is convergent in i-f-n-norm if and only if it is convergent in the derived i-f-(n-1)-norm N_∞ and M_∞.

Proof: Suppose $x_k \to x$ in the derived i-f-(n-1)-norm. Then

$$N_S(x_1,\dots,x_{n-2},x_{n-1},x_k-x,t)$$
$$\ge N_S(x_1,\dots,x_{n-2},x_k-x,\frac{t}{\|x_{n-1}\|_S})$$
$$\ge N_\infty(x_1,\dots,x_{n-2},x_k-x,\frac{t}{\sqrt{n}\,\|x_{n-1}\|_S})$$

Here $\|.\|_s$ on right-hand side denote the usual norm on X.

But $$\lim_{k\to\infty} N_\infty(x_1,\dots,x_{n-2},x_k-x,\frac{t}{\sqrt{n}\,\|x_{n-1}\|_S}) = 1$$

So,

$$\lim_{k\to\infty} N_s(x_1,\ldots,x_{n-2},x_k-x,t)=1$$

And

$$M_s(x_1,x_2,\ldots,x_{n-2},x_{n-1},x_k-x,t) \le M_\infty\left(x_1,\ldots,x_{n-2},x_k-x, \frac{t}{\sqrt{n}\,\|x_{n-1}\|_S}\right)$$

But $$\lim_{k\to\infty} M_\infty\left(x_1,\ldots,x_{n-2},x_k-x, \frac{t}{\sqrt{n}\,\|x_{n-1}\|_S}\right)=0$$

So, $$\lim_{k\to\infty} M_S(x_1,\ldots,x_{n-1},x_k-x,t) = 0$$

i.e. $x_k \to x$ in i-f-n-norm.

Remark 3.9: A sequence in a standard i-f-n-normed space is convergent in the i-f-n-norm if and if only it is convergent in the standard i-f-(n-1)-norm and, by induction, in the standard i-f-(n-r)-norm for all r=1, 2,……,n-1. In particular, a sequence in a standard n-normed space is convergent in the i-f-n-norm if and only if it is convergent in i-f-n-norm if and only if it is convergent in the standard intuitionistic fuzzy norm.

Now, for finite-dimensional cases, we can obtain a better i-f-(n-1)-norm by using a set of d vectors, rather than just n, linearly independent vectors in X (that is, by using a basis for X). Let $\{b_1,\ldots,b_d\}$ be a basis for X and we define the following function $\mathbf{N}_{\infty'}(.,.,\ldots,.,.)$ and $\mathbf{M}_{\infty'}(.,.,\ldots,.,.)$ on $X^{n-1} \times \mathbf{R}$ by

$$\mathbf{N}_{\infty'}(x_1,\ldots,x_{n-1},t) = \min\{N(x_1,\ldots,x_{n-1},b_i,t);\ i = 1,\ldots,d\}$$
$$\mathbf{M}_{\infty'}(x_1,\ldots,x_{n-1},t) = \max.\{M(x_1,\ldots,x_{n-1},b_i,t);\ i = 1,\ldots,d\}$$

Then, the function $N_{\infty'}(.,.,\ldots,.,.)$ and $M_{\infty'}(.,.,\ldots,.,.)$ defines an i-f-(n-1)- norm on X with respect to $\{b_1,\ldots,b_d\}$. With this derived i-f- (n-1)- norm, we have the following result.

Theorem 3.10: A sequence in the finite-dimensional i-f-n-normed space X is convergent in the i-f-n-norm if and only if it is convergent in the derived i-f- (n-1)- norm $\mathbf{N}_{\infty'}(.,.,\ldots,.,.)$, $\mathbf{M}_{\infty'}(.,.,\ldots,.,.)$.

Proof: If a sequence in X is convergent in the i-f-n-norm, then it will certainly be convergent in the i-f-(n-1)-norm $\mathbf{N}_{\infty'}(.,.,\ldots,.,.)$,$M_{\infty'}(.,.,\ldots,.,.)$. Conversely suppose $\{x_k\}$ converges to an $x \in X$ in $\mathbf{N}_{\infty'}(.,.,\ldots,.,.)$,$M_{\infty'}(.,.,\ldots,.,.)$. Take $x_1, \ldots, x_{n-1} \in X$. Writing $x_{n-1} = \alpha_1 b_1 +\ldots+ \alpha_d b_d$ We get

$$N(x_1,\ldots,x_{n-1}, x_k-x, t) \ge N_{\infty'}\left(x_1,\ldots,x_{n-2}, x_k-x, \frac{t}{|\alpha_1|+\ldots\ldots+|\alpha_d|}\right)$$

But $$\lim_{k\to\infty} N_{\infty'}\left(x_1,\ldots,x_{n-2},x_k-x, \frac{t}{|\alpha_1|+\ldots\ldots+|\alpha_d|}\right) = 1 \text{ and so}$$

We obtain

$$\lim_{k\to\infty} N(x_1,\ldots,x_{n-1},x_k-x,t) = 1$$

And $$M(x_1,\ldots,x_{n-1}, x_k-x, t) \le M_{\infty'}\left(x_1,\ldots,x_{n-2}, x_k-x, \frac{t}{|\alpha_1|+\ldots\ldots+|\alpha_d|}\right)$$

But $$\lim_{k\to\infty} M_{\infty'}\left(x_1,\ldots,x_{n-2},x_k-x, \frac{t}{|\alpha_1|+\ldots\ldots+|\alpha_d|}\right) = 0 \text{ and so}$$

We obtain

$$\lim_{k\to\infty} M(x_1,\ldots,x_{n-1},x_k-x,t) = 0$$

that is, $\{x_k\}$ converges to x in the i-f-n-norm.

CAUCHY SEQUENCES, COMPLETENESS AND FIXED POINT THEOREM

The results for Cauchy sequences for standard and finite dimensional cases can be obtained similarly as the results (theorem 3.7-3.10) obtained above for convergent sequences by replacing "x_k converges to x" with "x_k is Cauchy" and "x_k-x with x_k-x_ℓ".

Hence we obtain:

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Theorem 3.11:

(a) A standard i-f-n-NLS is complete if and only if it is complete with respect to one of the three i-f-(n-1) norms (N_∞, M_∞) $(N_{\infty'}, M_{\infty'})$ or (N_S, M_s).

(b) A finite dimensional i-f-n-NLS is complete if and only if it is complete with respect to the derived i-f-(n-1)-norm $N_{\infty'}$ (.,.,…,.,.), $M_{\infty'}$**(.,.,….,.)**

Using the above theorem (3.10) we obtained the following fixed point theorem

Fixed Point Theorem 3.12: Let (X, N) be a standard or finite dimensional complete i-f-n-NLS and T a contractive mapping of X into itself, that is there exist a constant $k \in (0, 1)$ s.t.

$N(x_1,\ldots,x_{n-1}, Ty-Tz, kt) \geq N(x_i,\ldots,x_{n-1},y-z,t)$

$M(x_1,\ldots,x_{n-1}, Ty-Tz, kt) \geq M(x_i,\ldots,x_{n-1},y-z,t)$, for all $x_1,\ldots,x_{n-1}$, y, z in X. Then T has a unique fixed point in X.

Proof: First consider the case n=2. By above proposition, we know that X is complete with respect to the derived i-f-norm N_∞, M_∞ or $N_{\infty'}$, $M_{\infty'}$. Since the mapping T is also contractive with respect to N_∞, M_∞ or $N_{\infty'}$, $M_{\infty'}$ we conclude by the fixed point theorem for intuitionistic Fuzzy Banach space that T has a unique fixed point is X. For n > 2, the result follows by induction.

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Source of support: Nil, Conflict of interest: None Declared.

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