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FIXED POINT THEOREMS IN TRI-D-METRIC SPACES

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ABSTRACT

In this some fixed point theorems for the contraction mappings in a D-metric space with three D-metrics called Tri –D-metric space on the lines initiated by Maia [6] and Dhage [1] have been established.

1. INTRODUCTION

Dhage [1], [2], [3] has given the foundation of a new structure of D-metric space and proved some basic results concerning topology, completeness and compactness etc, of the D- metric space.

Definition1.1: A function D on X x X x X into R is said to be a D-metric on nonempty set X if it satisfies the following properties

(M1): $D(x, y, z) \geq 0$; for all x, y, z Є X (Non negativity)
(M2): $D(x, y, z) = 0$ if and only if $x = y = z$
(M3): $D(x, y, z) = D(x, z, y) =….$ (Symmetry)
(M4): $D(x, y, z) \leq D(x, y, a) + D(x, a, z) + D(a, y, z)$, for all x, y, z, a Є X (Rectangle inequality),

A nonempty set X together with a D- metric is called generalized metric space or Dhage metric space or D-metric space and is denoted by (X, D).

We give some examples of D-metrics paces

Example 1.1: Define a function D_1 on X By

$D_1(x, y, z) = \max\{d(x, y), d(y, z), d(x, z)\}$ for x, y, z Є X, and d is an ordinary metric on X. Then D_1 is D-metric and (X,D) is D-metric space.

Example 1.2: Define a function D_2: X x X xX →R by

$D_2(x, y, z) = d(x, y) + d(y, z) + d(x, z)$, for x, y, z Є X and d is ordinary metric on X. Then (X, D_2) is D- metric space

2. D-CONTRACTION PRINCIPLE

The fundamental and most interesting result in D- metric spaces is D-contraction principle due to Dhage [2] & it is proved by same author that the Banach contraction principle is the particular case of this theorem. Dhage [2] proved the following fixed point theorem for D-contraction mapping in D-metric spaces called D-contraction principle.

Theorem 2.1: Let f be a self mapping of a complete and bounded D-metric space X satisfying

$$D(fx, fy, fz) \leq \alpha \, D(x, y, z) \qquad (2.1.1)$$

for all x, y, z ЄX and α<1.Then f has unique fixed point.

The following lemma and theorem of [1] is useful to prove main result.

Lemma 2.1: Let $\{x_r\}$ be a sequence of bounded D-metric space X such that

$$D(x_n, x_{n+1}, x_{n+2}) \leq q \, D(x_{n-1}, x_n, x_{n+1}) \qquad (2.1.2)$$

for all n ϵ N, where $0 \leq q < 1$. Then $\{x_r\}$ is D- cauchy.

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Theorem 2.2: Let f be a self-map of a complete and bounded D-metric space X satisfying

$$D(fx, fy, fz) \leq \alpha \max\{D(x, fx, fy), D(y, fy, fz)\} \quad (2.1.3)$$

For all x, y, z Є X and $0 \leq q < 1$. Then f has a unique fixed point.

3. MAIN RESULT

Theorem 3.1: Let X be a Tri-D-metric space with three D-metrics D, D_1, D_2. Let f: X → X be a mapping and suppose that the following conditions hold in X.

(i) X is bounded w.r.to D
(ii) $D_2(x, y, z) \leq D_1(x, y, z) \leq D(x, y, z)$ for all x, y, z ЄX
(iii) X is complete w. r. to D_1
(iv) X is continuous w. r. to D_2
(v) f satisfies the condition (2.1.3) w. r. to D.
Then X has a unique fixed point.

Proof: Suppose $x = x_0$ Є X is an arbitrary point and consider a sequence $\{x_n\}$ in X defined by

$$x_0 = x,\ x_{n+1} = f x_n,\ n \in N \cup \{0\} \quad (3.1.1)$$

where N denotes the set of natural numbers.

If $x_r = x_{r+1}$ for some r Є N then $x_r = u$ is a fixed point of f. Therefore we assume that $x_n \neq x_{n+1}$ for each n Є N, we show that $\{x_n\}$ be a D-Cauchy sequence in X.

Let $x = x_0$, $y = x_1$, $z = x_2$ then by condition (2.1.3) we get

$$D(fx_0, fx_1, fx_2) \leq \alpha \max\{D(x_0, fx_0, fx_1), D(x_1, fx_1, fx_2)\}$$

i .e, $D(x_1, x_2, x_3) \leq \alpha \max\{D(x_0, x_1, x_2), D(x_1, x_2, x_3)\}$

since, $D(x_1,x_2,x_3) \leq \alpha \max D(x_1, x_2, x_3)$ is not possible, we have

$$D(x_1, x_2, x_3) \leq \alpha D(x_0, x_1, x_2) \quad (3.1.2)$$

Similarly letting $x = x_1$, $y = x_2$, $z = x_3$ in condition (2.1.3) we obtain

$$D(fx_1, fx_2, fx_3) \leq \alpha \max\{D(x_1, fx_1, fx_2), D(x_2, fx_2, fx_3)$$

i.e, $D(x_2, x_3, x_4) \leq \alpha \max\{D(x_1, x_2, x_3), D(x_2, x_3, x_4)\}$

since, $D(x_2, x_3, x_4) \leq \alpha \max D(x_2, x_3, x_4)$ is not possible, we have

$$D(x_2, x_3, x_4) \leq \alpha D(x_1, x_2, x_3) \quad (3.1.3)$$

Proceeding in this way by induction we obtain

$$D(x_n, x_{n+1}, x_{n+2}) \leq \alpha D(x_{n-1}, x_n, x_{n+1}) \quad (3.1.4)$$

for all n, n=1, 2,… .Then by Lemma (2.1.1) $\{x_n\}$ is D- Cauchy sequence.

i.e, $\lim\limits_{m, n, p \to \infty} D(x_n, x_m, x_p) = 0$

The hypothesis (ii) implies that

$$\lim_{m, n, p \in \infty} D(x_n, x_m, x_p) \leq \lim_{m, n, p \to \infty} D(x_m, x_n, x_p) \to 0$$

This shows that $\{x_n\}$ is a D-cauchy sequence w,r,t. D_1, there is a point u Є X such that

$$\lim D_1(x_m, x_n, u) = 0$$

i.e, $\lim\limits_{m, n, \to \infty\ n \in \infty} x_n = 0$

w.r.to D_1. Again $D_2 \leq D_1$ on X^3, we get $x_n \to u$ w.r.to D_2.

$$u = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} fx_n = f \lim_{n\to\infty} x_n = fu$$

showing that u is a fixed point of f.

To prove uniqueness, let v (≠ u) be another fixed point of f then by condition (2.1.3) we obtain

$$\begin{aligned} D(u, u, v) &= D(fu, fu, fv) \\ &\leq \alpha \max\{D(u\ fu, fu), D(u, fu, fv)\} \\ &= \alpha \max\{D(u, u, u), D(u, u, v)\} \\ &= \alpha \max\{0, D(u, u, v)\} \end{aligned}$$

$$D(u, u, v) \leq \alpha D(u, u, v)$$

Which is contradiction since $\alpha < 1$. Hence $u = v$. Therefore f has a unique fixed point.

Corollary 3.1: Let X be a Tri-D- metric space with three D-metrics D, D_1, D_2. Let f: X → X be a mapping and suppose that following conditions are satisfied.

(i) The conditions (i) - (iv) of Theorem 3.1

(ii) There exists a positive intiger p such that f^p satisfies condition

$$D(f^px, f^py, f^pz) \leq \alpha \max\{D(x, f^px, f^py), D(y, f^py, f^pz)\} \quad (3.1.5)$$

For all x, y, z ∈X and $0 \leq p < 1$.

Then f has a unique fixed point.

Proof: Let $T = f^p$, then T is continuous no X w .r. to D_2, and since f and consequently f^p is continuous on X w. r. to D_2.

Now by an application of Theorem 3.1 implies that T has a unique fixed point, say u in X. i. e, it is a point such that $Tu = f^pu = u$

But $fu = f(f^pu) = f^p(fu) = T(fu)$, which shows that fu is again a fixed point of T. By uniqueness of u, we get fu = u. Again the uniqueness of u follows from the condition (2.1.3).

This completes the proof.

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