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DETERIORATING INVENTORY MODEL WITH POWER DEMAND AND PARTIAL BACKLOGGING

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ABSTRACT

In this study, a deterministic inventory model has been developed for which items are subject to constant deterioration. Here shortages are allowed and are partially backlogged. The demand follows power pattern. The backlogging rate is taken to be inversely proportional to the waiting time for the next replenishment. We have established the optimal order quantity by minimizing the total inventory cost.

***Keywords**: Inventory, deteriorating items, shortages, Power Demand, Partial Backlogging.*

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1. INTRODUCTION

Inventory management is one of the key decision areas in designing an effective working capital mechanism in organization. In recent years, mathematical ideas have been used in different areas of inventory management. The important problem of the inventory manager is to decide when and how much to order or to manufacture so that the total cost associated with the inventory system should be minimum. This is somewhat more important, when the inventory undergoes decay or deterioration. Deterioration is defined as change, damage, decay, spoilage obsolescence and loss of utility or loss of original value in a commodity that results in the decreasing usefulness from the original one. Often we encounter products such as fruits, milk, drug, vegetables and photographic films etc that have a defined period of life time. Such items are referred as deteriorating items. Due to deterioration, inventory system faces the problem of shortages and loss of good will or loss of profit. Shortage is a fraction of those customers whose demand is not satisfied in the current period and reacts to this by not returning the next period. Researchers assume that during shortage period all demand is completely backlogged, lost or partially backlogged.

Inventory in deteriorating items was first considered by Whitin [7], he considered the deterioration of fashion goods at the end of prescribed shortage period. Ghare and Schrader [4] extended the classical EOQ formula with exponential decay of inventory due to deterioration and gave a mathematical modeling of inventory in deteriorating items. Dave and Patel [2] developed the first deteriorating inventory model with linear trend in demand. He considers demand as a linear function of time. Goel and Aggarwal [5] formulated an order-level inventory system with power-demand pattern for deteriorating items. A power demand pattern inventory model for deteriorating items was discussed by Dutta and Pal [3]. Chang and Dye [1] developed an EOQ model with power demand and partial backlogging. He considered that if longer the waiting time smaller the backlogging rate would be. So the proportion of the customer who would like to accept backlogging at time t is decreasing with the waiting time for the next replenishment. To take care of this situation he defined the backlogging rate as

$$B(t) = \frac{1}{1+\alpha\left(t_i - t\right)} \tag{1.1}$$

Where t_i is the time at which the i^{th} replenishment is being made and α is backlogging parameter.

Goyal and Giri [6] gave recent trends of modeling in deteriorating items inventory. They classified inventory models on the basis of demand variations and various other conditions or constraints. Ouyang, Wu and Cheng [10] developed an inventory model for deteriorating items with exponential declining demand and partial backlogging. Joaquin Sicilia Rodriguez, Jaime Febles-Acosta, Manuel Gonzalez-Del Rosa Nusa Dua [9] investigated order-level-lot size inventory systems with power demand pattern. Vinod Kumar Mishra, Lal Sahab Singh [12] developed deteriorating inventory model with time dependent demand and partial backlogging.

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Classical inventory models deal with constant demand. But most practical inventory management systems come across demand which is increasing with time during the development phase of the business. Hence a power demand pattern suits the practical situation. In the present paper the deterministic inventory model with power demand pattern is developed in which inventory is depleted not only by demand but also by deterioration. Deterioration rate is assumed to be constant and shortages are allowed and partially backlogged. The effect due to change in various parameters have been considered in the model numerically.

2. ASSUMPTIONS AND NOTATIONS

The mathematical model is based on the following notations and assumptions.

Notations

- A The ordering cost per inventory cycle.
- C The purchase cost per unit.
- h The inventory holding cost per unit per time unit.
- π_b The backordered cost per unit short per time unit.
- π_l The cost of lost sales per unit.
- t_1 The time at which the inventory level reaches zero, $t_1 \geq 0$.
- t_2 The length of period during which shortages are allowed, $t_2 \geq 0$.
- T $(= t_1+t_2)$ The length of cycle time.
- I_m The maximum inventory level during [0, T].
- I_b The maximum backordered units during stock out period.
- Q $(= I_m+I_b)$ The order quantity during a cycle of length T.
- $I_1(t)$ The level of positive inventory at time t, $0 \leq t \leq t_1$.
- $I_2(t)$ The level of negative inventory at time t, $t_1 \leq t \leq T$.
- TCPT The total cost per time unit.

Assumptions

- The inventory system deals with single item.
- The demand rate follows a power demand pattern expressed as $\frac{dt^{(1-n)/n}}{nT^{1/n}}$ at any time t, where d is a positive

constant, n may be any positive number denoting pattern index, T is the planning horizon.

- The deterioration rate 'θ' is constant. Where $0 < \theta << 1$.
- The replenishment rate is infinite.
- The lead-time is zero or negligible.
- The planning horizon is infinite.
- During the stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. The proportion of the customers who would like to accept the backlogging at time "t" is with the waiting time (T-t) for the next replenishment i.e., for the negative inventory the backlogging rate is $B(t) = \frac{1}{1+\delta(T-t)}$; $\delta > 0$ denotes the backlogging parameter and $t_1 \leq t \leq T$.

MATHEMATICAL MODEL

Case (i): Inventory level in the period [0, t_1] without shortages

Under above assumptions, the on – hand inventory level at any instant of time is exhibited in figure 1.

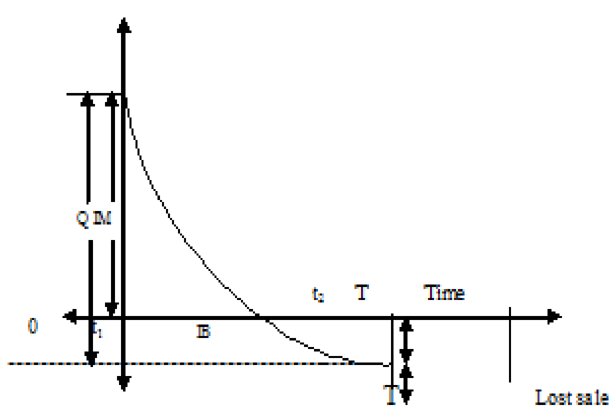


Figure: 1 Representation of inventory system

During the period $[0, t_1]$ the inventory depletes due to the deterioration and demand. Hence the inventory level at any instant of time during $[0, t_1]$ is described by the differential equation,

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -\frac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}, \; 0 \le t \le t_1 \tag{3.1}$$

with the boundary condition $I_1(t_1) = 0$ at $t = t_1$.

The solution of equation (1) is given by

$$I_1(t) = \frac{d}{T^{\frac{1}{n}}}\left[(1-\theta t)\left(t_1^{\frac{1}{n}} - t^{\frac{1}{n}}\right) + \frac{\theta}{1+n}\left(t_1^{\frac{1+n}{n}} - t^{\frac{1+n}{n}}\right)\right], \; 0 \le t \le t_1 \tag{3.2}$$

Case (ii): Inventory level in the period $[t_1, T]$ with shortage

During the interval $[t_1, T]$ the inventory level is zero and a fraction of demand during this period is backlogged. The state of inventory during $[t_1, T]$ can be represented by the differential equation,

$$\frac{dI_2(t)}{dt} = -\frac{\left(\frac{dt^{(1-n)/n}}{nT^{(1/n)}}\right)}{1+\delta(T-t)}, \; t_1 \le t \le T \tag{3.4}$$

with the boundary condition $I_2(t_1) = 0$ at $t = t_1$.

The solution of equation (3) is given by

$$I_2(t) = -\frac{d}{T^{\frac{1}{n}}}\left[(1-\delta T)\left(t^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right) + \frac{\delta}{1+n}\left(t^{\frac{1+n}{n}} - t_1^{\frac{1+n}{n}}\right)\right], \; t_1 \le t \le T \tag{3.5}$$

The maximum backordered units are

$$I_b = -I_2(T) = \frac{d}{T^{\frac{1}{n}}}\left[(1-\delta T)\left(T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right) + \frac{\delta}{1+n}\left(T^{\frac{1+n}{n}} - t_1^{\frac{1+n}{n}}\right)\right] \tag{3.6}$$

Hence, the order quantity for each cycle is $Q = I_m + I_b$.

$$Q = \frac{d}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}} + \delta T t_1^{\frac{1}{n}} + \frac{1}{1+n}\left((\theta-\delta)t_1^{\frac{1+n}{n}} - \delta n T^{\frac{1+n}{n}}\right)\right] \tag{3.7}$$

The total cost per replenishment cycle consists of the following cost components:

Ordering cost: $I_{OC} = A$ (3.8)

Holding cost:

$$I_{HC} = h\int_0^{t_1} I_1(t)dt$$

$$= \frac{hd}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1+n}{n}}}{1+n} + \frac{\theta t_1^{\frac{1+2n}{n}}}{2(1+2n)}\right] \tag{3.9}$$

Backordered cost:

$$I_{BC} = \pi_b \int_{t_1}^{T} -I_2(t)dt$$

$$= \frac{\pi_b d}{T^{\frac{1}{n}}}\left[(\delta T^2 - T)t_1^{\frac{1}{n}} + \frac{nT^{\frac{1+n}{n}}}{1+n} + \frac{(1-2\delta T)t_1^{\frac{1+n}{n}}}{1+n} - \frac{2\delta n^2 T^{\frac{1+2n}{n}}}{(1+n)(1+2n)} + \frac{\delta t_1^{\frac{1+2n}{n}}}{1+2n} \right] \tag{3.10}$$

Cost due to lost sales:

$$I_{LS} = \pi_l \int_{t_1}^{T} \left[1 - \frac{1}{1+\delta(T-t)} \right] \left(\frac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}} \right) dt$$

$$= \frac{\pi_l d\delta}{T^{\frac{1}{n}}} \left[\frac{nT^{\frac{1+n}{n}}}{1+n} - Tt_1^{\frac{1}{n}} + \frac{t_1^{\frac{1+n}{n}}}{1+n} \right] \tag{3.11}$$

Purchase cost:

$$I_{PC} = C \times Q$$

$$= \frac{Cd}{T^{\frac{1}{n}}} \left[T^{\frac{1}{n}} + \delta T t_1^{\frac{1}{n}} + \frac{1}{1+n}\left((\theta-\delta)t_1^{\frac{1+n}{n}} - \delta n T^{\frac{1+n}{n}} \right) \right] \tag{3.12}$$

Hence, the total cost per time unit is given by

$$TCPT = \frac{1}{T}[I_{OC} + I_{HC} + I_{BC} + I_{LS} + I_{PC}]$$

$$\begin{aligned} &= \frac{1}{T}\Bigg\{ A + \frac{hd}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1+n}{n}}}{1+n} + \frac{\theta t_1^{\frac{1+2n}{n}}}{2(1+2n)} \right] + \frac{\pi_b d}{T^{\frac{1}{n}}}\Bigg[(\delta T^2 - T)t_1^{\frac{1}{n}} + \frac{nT^{\frac{1+n}{n}}}{1+n} \\ &+ \frac{(1-2\delta T)t_1^{\frac{1+n}{n}}}{1+n} - \frac{2\delta n^2 T^{\frac{1+2n}{n}}}{(1+n)(1+2n)} + \frac{\delta t_1^{\frac{1+2n}{n}}}{1+2n} \Bigg] + \frac{\pi_l d\delta}{T^{\frac{1}{n}}}\left[\frac{nT^{\frac{1+n}{n}}}{1+n} - Tt_1^{\frac{1}{n}} + \frac{t_1^{\frac{1+n}{n}}}{1+n} \right] \\ &+ \frac{Cd}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}} + \delta T t_1^{\frac{1}{n}} + \frac{1}{1+n}\left((\theta-\delta)t_1^{\frac{1+n}{n}} - \delta n T^{\frac{1+n}{n}} \right) \right] \Bigg\} \end{aligned} \tag{3.13}$$

To minimize total average cost per unit time (*TCPT*), the optimal value of $t_1$can be obtained by solving the equation

$$\frac{dTCPT}{dt_1} = 0. \tag{3.14}$$

The condition $\frac{d^2TCPT}{dt_1^2} > 0$, is also satisfied for the above value of t_1. (3.15)

The value of t_1 obtained from equation (3.14) is used to obtain the optimal value of Q and TCPT respectively. Since the equation (3.14) is nonlinear, it is solved using MATLAB.

To illustrate and validate the proposed model, numerical data that fits to realistic situations is considered in the following section and the sensitivity analysis is carried out with respect to backlogging parameter and deterioration rate to test the model.

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

The following parametric values are considered for an inventory system with appropriate units. d = 50 units, n = 2 units, T = 1 year, A = \$ 250 per order, C = \$ 8.0 per unit, h = \$ 0.50 per unit per year, π_b = \$ 12.0 per unit per year, π_l = \$ 15.0 per unit.

Optimum Solution

For the above numerical values, when deterioration rate is 5%, the optimum time t_1 at which positive inventory is zero is 0.965018 time units and stock out period t_2 is of length is 0.034982 time units. This advices the retailer to buy 51 units which will cost a minimum \$ 666.73 (by rounding off Q*).

Effect of backlogging parameter (δ):

The backlogging parameter has initially been taken as 2. Now varying the backlogging parameter between 1 and 3 the following table is obtained.

Table 4.1 Variation in backlogging parameter 'δ'

Parameter value (δ)	% Change	t_1 (Year)	Q (Units)	TCPT (\$)
1.0	-50	0.952864	50.746891	664.59
1.5	-25	0.959803	50.752884	664.67
2.0	0	0.965018	50.759033	664.73
2.4	+25	0.969064	50.764744	664.77
3.0	+50	0.972287	50.769863	664.81

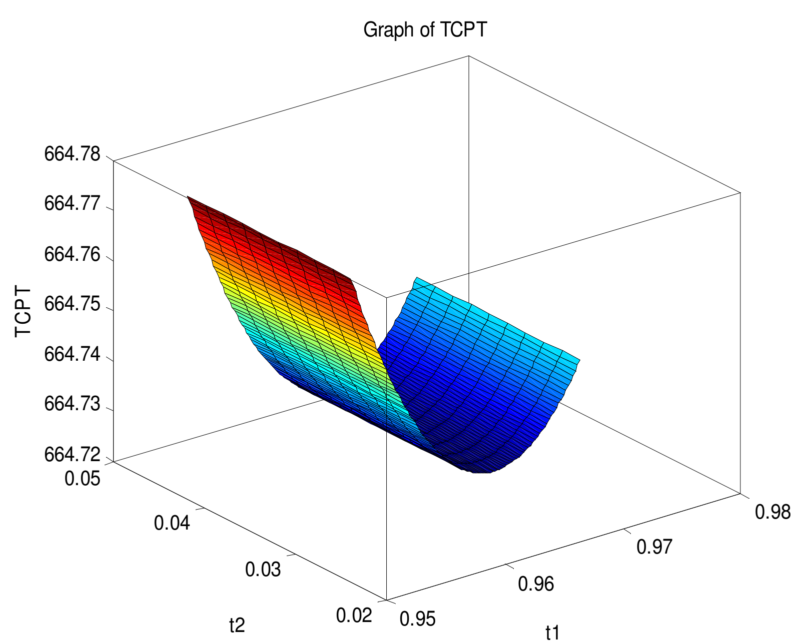
In the above table it can be observed that, increase in backlogging parameter increases

- Total cost per time unit of the inventory system.
- Inventory time period and ordering quantity.

Graphical Interpretation:

The three dimensional total cost per time unit graph is shown in Figure 2 by plotting t_1 in the range of [0.952864, 0.972287] and t_2 in the range of [0.027713, 0.047136]. Figure 2 indicates that total cost per time unit is strictly convex.

Variation in backlogging parameter 'δ'



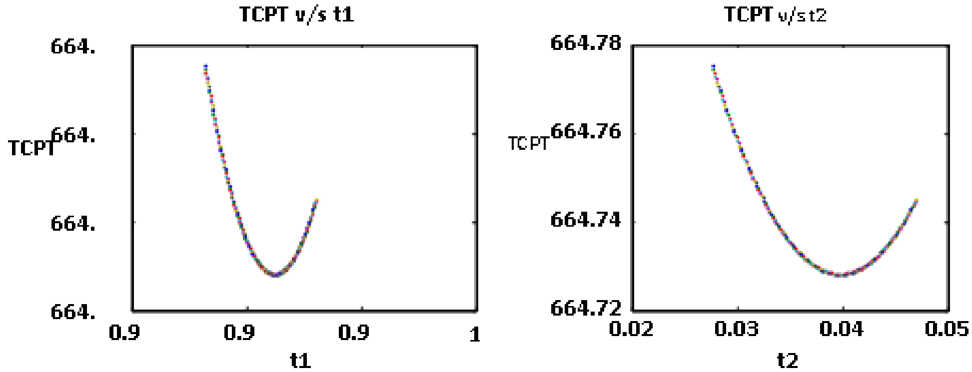


Figure: 2 Total cost per time unit

Effect of deterioration parameter (θ):

Initially the deterioration parameter has been taken as 0.05. The following table is obtained with the variation in deterioration parameter from 0.025 to 0.075.

Table: 2 Variation in deterioration parameter 'θ'

Parameter value (θ)	% Change	t_1 (Year)	Q (Units)	TCPT ($)
0.0250	-50	0.972903	50.381321	661.49
0.0375	-25	0.968957	50.571777	663.11
0.0500	0	0.965018	50.759033	664.73
0.0625	+25	0.961083	50.943089	666.33
0.0750	+50	0.957151	51.123951	668.92

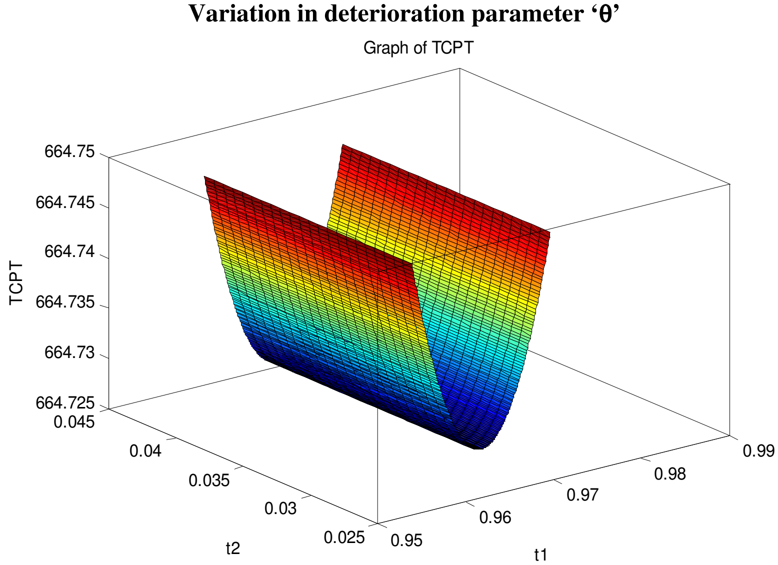
In the above table it can be observed that, increase in deterioration rate

- Increases total cost per time unit of an inventory system and ordering quantity.
- Decrease in inventory time period.

Graphical Interpretation:

The three dimensional total cost per time unit graph is shown in Figure 3 by plotting t_1 in the range of [0.957151, 0.972903] and t_2 in the range of [0.027099, 0.04285]. Figure 3 indicates that total cost per time unit is strictly convex.

Variation in deterioration parameter 'θ'



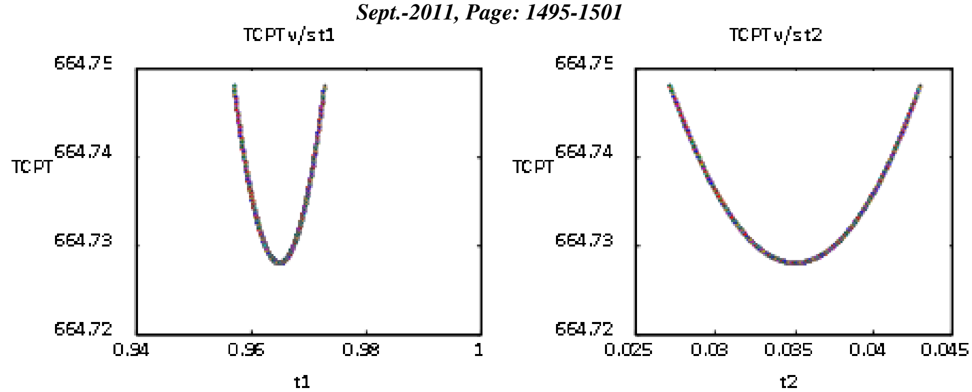


Figure: 3 Total cost per time unit

CONCLUSION

In this paper, we have developed a deterministic inventory model with power pattern demand and constant deterioration rate. Shortages have been allowed and completely backlogged in this model. The optimal order quantity has been computed by minimizing the total inventory cost. Analytical solutions of the model are illustrated with the help of suitable numerical examples. Sensitivity analysis made is also carried out and it can be inferred that the variation in backlogging parameter and deterioration parameter increases the total cost.

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