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ABOUT SOFT $I\alpha\psi$ CLOSED SETS IN SOFT IDEAL TOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to introduce a new concept of soft ψ closed, soft $\alpha\psi$ closed, soft $I_{\alpha\psi}$ closed set in soft ideal topological spaces and investigate some of their properties.

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1. INTRODUCTION

Molodtsov[12] wasintroduced soft set theory in 1999. In 2011, shabir and Naz[18] applied soft set theory to topological spaces. They introduced concept like, soft topological space, soft interior, soft closure and soft subspaces and studied some of their properties. The notion of soft ideal was introduced by sahin and kucuk [16] in 2012. Akdag and Erol [3] introduced the concept of soft I open sets and investigated some of their properties in 2014.

The purpose of this paper is to define a new concept of soft ψ closed, soft $\alpha\psi$ closed, soft $I_{\alpha\psi}$ closed set in soft ideal topological spaces and study some of their properties.

2. PRELIMINARIES

Throughout this paper X will be a non empty initial universal set and A will be a set of parameters. Let $P(X)$ denote power set of X and $S(X)$ denote the set of all soft sets over X.

Definition 2.1: [12] A pair (F, A) is called a soft set over (X, A), where $F: A \rightarrow P(X)$.

Definition 2.2: [20] The complement of a soft set (F,A) is defined as $(F,A)^C = (F^C,A)$, where $F^C(\alpha) = (F(\alpha))^C = X - F(\alpha)$, for all $\alpha \in A$. Clearly, we have $(\widetilde{\varphi})^C = \widetilde{X}$ and $(\widetilde{X})^C = \widetilde{\varphi}$.

Definition 2.3: [18]Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X *if*,
(a) $\widetilde{\varphi}, \widetilde{X} \in \tau$
(b) the intersection of any two soft sets in τ belongs to τ.
(c) the union of any number of soft sets in τ belongs to τ.

Let (X, τ, A) is called a soft topological space over X. The members of τ are said to be τ-soft open sets.

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Proposition 2.4: [7] Let (X,τ,A) be a soft topological space and $(F,A),(G,A)$ be two soft sets. Then
(a) $int(int(F,A)) = int(F,A)$.
(b) $(F,A)\tilde{\subseteq}(G,A)$ implies $int(F,A)\tilde{\subseteq}int(G,A)$.
(c) $cl(cl(F,A)) = cl(F,A)$.
(d) $(F,A)\tilde{\subseteq}(G,A)$ implies $cl(F,A)\tilde{\subseteq}cl(G,A)$

Definition 2.5: [5] A soft ideal $\boldsymbol{I}$ is a nonempty collection of soft sets over X if
a) $(F,A)\tilde{\in}\boldsymbol{I}, (G,A)\tilde{\subset}(F,A)$ implies $(G,A)\tilde{\in}\boldsymbol{I}$
b) $(F,A)\tilde{\in}\boldsymbol{I}, (G,A)\tilde{\in}\boldsymbol{I}$ implies $(F,A)\tilde{\cup}(G,A) \in \boldsymbol{I}$

Let a soft topological space (X,τ,A) with a soft ideal $\boldsymbol{I}$ called soft ideal topological space and denoted by (X,τ,A,I).

Definition 2.6: [9] Let $(X,\tau,A,\boldsymbol{I})$ be a soft ideal topological space, then $(F,A)^*(\boldsymbol{I},\tau) = \tilde{\cap}(x_e \in: U_{x_e}\tilde{\cap}(F,A) \not\tilde{\in} \boldsymbol{I}$, $\forall x_e \in\tau$ is called the soft local function of (F,A).

Definition 2.7: [5] Let (X, τ, A, I) be a soft ideal topological space, then the soft closure operator is defined by $cl^*(F,A) = (F,A)\tilde{\cup}(F,A)^*$.

Lemma 2.8: [5] Let (X,τ,A,I) be a soft ideal topological space and $(F,A),(G,A)$ be two soft sets. Then
a) $(F,A)\tilde{\subset}(G,A) \Rightarrow (F,A)^*\tilde{\subset}(G,A)^*$ and $((F,A)\tilde{\cup}(G,A))^* = (F,A)^*\tilde{\cup}\ (G,A)^*$.
b) $(F,A)^*\tilde{\subset}$ cl(F,A) and $((F,A)^*)^*\tilde{\subset}(F,A)^*$.
c) (F,A) is soft open and $(F,A)\tilde{\cap}(G,A)\tilde{\in}\boldsymbol{I} \Rightarrow (F,A)\tilde{\cap}(G,A)^* = \tilde{\varphi}$.
d) $(F, A)^*$ is soft closed.
e) If (F,A) is soft closed then $(F,A)^*\tilde{\subset}(F,A)$.

Proposition 2.9: [5] Let (X,τ,A,I) be a soft ideal topological space and $(F,A),(G,A)$ be two soft sets. Then
(a) $cl^*(\widetilde{\varphi}) = \tilde{\varphi}$ and $cl^*(\widetilde{X}) = \tilde{X}$.
(b) $(F,A)\tilde{\subset}$ $cl^*(F,A)$ and $cl^*(cl^*(F,A)) = cl^*(F,A)$.
(c) If $(F,A)\tilde{\subset}(G,A)$ then $cl^*(F,A)\tilde{\subset}cl^*(G,A)$.
(d) $cl^*(F,A)\tilde{\cup}$ $cl^*(G,A) = cl^*((F,A)\tilde{\cup}(G,A))$.

Definition 2.10: A subset (F, A) of a soft topological space (X, τ, A) is called
(i) A soft α open set [10] if $(F, A) \subseteq$ $int(cl(int(F, A)))$.
(ii) A soft semi open set [2] if $(F, A) \subseteq cl(int(F, A))$.
(iii) A soft τ^* closed [19] if $(F,A)^*\tilde{\subseteq}(F,A)$.
(iv) A soft * dense in itself [19] if $(F,A)\tilde{\subseteq}(F,A)^*$.
(v) A soft generalized α closed [15] set if $\alpha cl(F, A) \subseteq (U, A)$ *whenever* $(F, A) \subseteq (U, A)$ and (U,A) is soft α open.

Definition 2.11: [17] A subset (F, A) of a soft ideal topological space (X, τ, A, I) is called a soft $I_{\pi g}$ closed set if $(F, A)^* \subseteq (U, A)$ whenever $(F, A) \subseteq (U, A)$ and (U, A) is soft π open.

3. SOFT $I_{\alpha\psi}$ CLOSEDSETS

Definition 3.1: A soft topological space (X, τ, A) is said to be a soft ψ closed if $scl(F, A) \subseteq (U, A)$ whenever $(F, A) \subseteq (U, A)$ and (U, A) is soft sg open.

Definition 3.2: A soft topological space (X, τ, A) is said to be a soft $\alpha\psi$ closed if $\psi cl(F,A) \subseteq (U,A)$ whenever $(F, A) \subseteq (U,A)$ and (U,A) is soft α open.

Definition 3.3: A soft ideal topological space (X, τ, A, I) is said to be a soft $I_{\alpha\psi}$ closed if $(F, A)^* \subseteq (U, A)$ whenever $(F,A) \subseteq (U,A)$ and (U,A) is soft $\alpha\psi$ open.

Theorem 3.4: Let (X, τ, A) be a soft topological space. Then the following are hold:
(a) Every soft α closed set is soft $\alpha\psi$ closed set.
(b) Every soft semi closed set is soft $\alpha\psi$ closed set.
(c) Every soft ψ closed set is soft $\alpha\psi$ closed set.
(d) Every soft $g\alpha$ closed set is soft $\alpha\psi$ closed set.

Proof:
(a) Let (F, A) be a soft α closed set, then $(F, A) = \alpha cl(F,A)$. Let $(F, A) \subseteq (U, A)$, (U, A) is soft α open. Since (F, A) be a soft α closed, $\psi cl(F,A) \subseteq \alpha cl(F,A) \subseteq (U,A)$. This shows is soft $\alpha\psi$ closed set.
(b) Let (F, A) be a soft semi closed set, then $(F,A) = scl(F,A)$. Let $(F, A) \subseteq (U, A)$, (U, A) is soft α open. Since (F, A) be a soft semi closed, $\psi cl(F, \subseteq A) \subseteq scl(F,A) \subseteq (U,A)$. This shows that (F, A) is soft $\alpha\psi$ closed set.
(c) Let (U, A) is soft α open set such that (F,A) ⊆ (U,A). Every soft α open set is soft sg open set. Then ψcl(F,A) ⊆ scl(F,A) ⊆ (U,A). This shows that (F,A) is soft αψ closed set.
(d) Let (U, A) is soft α *open set such that* $(F,A) \subseteq (U,A)$. *Then* $\psi\, cl(F,A) \subseteq \alpha cl(F,A) \subseteq (U,A)$. *This shows that* (F, A) *is soft* $\alpha\psi$ *closed set.*

The converse of the above theorem need not be true as shown by the following examples.

Example 3.5:
Let $X = \{a, b\}, A = \{e_1, e_2\}$, Here

$(F,A)_1 = \{(e_1,\varphi),(e_2,\varphi)\}$ $\quad (F,A)_2 = \{(e_1,\varphi),(e_2,\{a\})\}$
$(F,A)_3 = \{(e_1,\varphi),(e_2,\{b\})\}$ $\quad (F,A)_4 = \{(e_1,\varphi),(e_2,\{a,b\})\}$
$(F,A)_5 = \{(e_1,\{a\}),(e_2,\varphi)\}$ $\quad (F,A)_6 = \{(e_1,\{a\}),(e_2,\{a\})\}$
$(F,A)_7 = \{(e_1,\{a\}),(e_2,\{b\})\}$ $\quad (F,A)_8 = \{(e_1,\{a\}),(e_2,\{a,b\})\}$
$(F,A)_9 = \{(e_1,\{b\}),(e_2,\varphi)\}$ $\quad (F,A)_{10} = \{(e_1,\{b\}),(e_2,\{a\})\}$
$(F,A)_{11} = \{(e_1,\{b\}),(e_2,\{b\})\}$ $\quad (F,A)_{12} = \{(e_1,\{b\}),(e_2,\{a,b\})\}$
$(F,A)_{13} = \{(e_1,\{a,b\}),(e_2,\varphi)\}$ $\quad (F,A)_{14} = \{(e_1,\{a,b\}),(e_2,\{a\})\}$
$(F,A)_{15} = \{(e_1,\{a,b\}),(e_2,\{b\})\}$ $\quad (F,A)_{16} = \{(e_1,\{a,b\}),(e_2,\{a,b\})\}$

(a) Let $\tau = \{\phi, X, (F,A)_1, (F,A)_2, (F,A)_3, (F,A)_4, (F,A)_{16}\}$. Clearly $(F,A)_3$ is soft $\alpha\psi$ closed set but not soft α closed set.
(b) Let $\tau = \{\phi, X, (F,A)_1, (F,A)_2, (F,A)_3, (F,A)_4, (F,A)_{16}\}$. Clearly $(F,A)_8$ is soft $\alpha\psi$ closed set but not soft ψ closed set.
(c) Let $\tau = \{\phi, X, (F,A)_1, (F,A)_2, (F,A)_3, (F,A)_4, (F,A)_{16}\}$. Clearly $(F,A)_6$ is soft $\alpha\psi$ closed set but not soft semi closed set.
(d) Let $\tau = \{\phi, X, (F,A)_1, (F,A)_2, (F,A)_3, (F,A)_4, (F,A)_{16}\}$. Clearly $(F,A)_5$ is soft $\alpha\psi$ closed set but not soft $g\alpha$ closed set.

Theorem 3.6: Let (X, τ, A, I) be a soft ideal topological space. Then every soft $I_{\alpha\psi}$ closed set is soft $I_{\pi g}$ closed set.

Proof: Let (U, A) is soft π open set such that $(F, A) \subseteq (U, A)$. Every soft π open set is soft open and every soft open set is soft $\alpha\psi$ open set. Here (F, A) is soft $I_{\alpha\psi}$ closed set then $(F, A)^* \subseteq (U, A)$. Therefore (F, A) is soft $I_{\pi g}$ closed set. The converse of the above theorem need not be true as shown by the following examples.

Example 3.7: Let $\tau = \{\phi, X, (F,A)_1, (F,A)_2, (F,A)_3, (F,A)_4, (F,A)_{16}\}$ and $I = \{\varphi, \{(e_1, \varphi), (e_2, \{a\})\}\}$. Clearly $(F, A)_5$ is soft $I_{\pi g}$ closed set but not soft $I_{\alpha\psi}$ closed set.

Theorem 3.8: Union of two closed sets is soft soft $I_{\alpha\psi}$ closed set.

Proof: Let (F, A) and (G, A) be two closed sets and hence soft $I_{\alpha\psi}$ closed sets in (X, A). Let (U, A) be a soft $\alpha\psi$ open set such that $(F, A) \subseteq (G, A) \subseteq (U, A)$. Then $(F, A) \subseteq (U, A)$ and $(G, A) \subseteq (U, A)$. Since (F, A) and (G, A) are soft $I_{\alpha\psi}$–closed sets. We have $(F, A)^* \subseteq (U, A)$ and $(G, A)^* \subseteq (U, A)$. Hence $(F, A)^* \subseteq (G, A)^* = ((F, A) \subseteq (G, A))^* \subseteq (U, A)$. Therefore $(F, A) \subseteq (G, A)$ is soft $I_{\alpha\psi}$ -closed sets.

Theorem 3.9: If (X,τ,A,I) be a soft ideal topological space and $(F,A) \subseteq (X,A)$. Then the following are equivalent:

(i) (F, A) is soft $I_{\alpha\psi}$ closed set.

(ii) $cl^*(F,A) \subseteq (U,A)$, whenever $(F,A) \subseteq (U,A)$ and (U,A) is soft $\alpha\psi$ open in (X, A).

(iii) For all $x_e \in cl^*(F,A), \alpha\psi cl(\{x_e\}) \cap (F,A) \neq \varphi$.

(iv) $cl^*(F,A) - (F,A)$ contains no non empty soft $\alpha\psi$ closed set.

(v) $(F,A)^* - (F,A)$ contains no non empty soft $\alpha\psi$ closed set.

Proof:

(i)⇒(ii): If (F, A) is soft $I_{\alpha\psi}$ closed set, then $(F,A)^* \subseteq (U,A)$, whenever $A \subseteq (U,A)$ and (U, A) is soft $\alpha\psi$ open in (X, A) and so $cl^*(F,A) = (F,A) \cup (F,A)^* \subseteq (U,A)$ whenever $(F,A) \subseteq (U,A)$ and (U,A) is $\alpha\psi$ open in (X,A).

(ii)⇒(iii): Suppose $x_e \in cl^*(F,A)$. If $\alpha\psi cl(\{x_e\}) \cap (F,A) = \varphi$, then $(F,A) \subseteq (X,A) - \alpha\psi cl(\{x_e\})$, by (ii) $cl^*(F,A) \subseteq (X,F) - \alpha\psi cl(\{x_e\})$, a contradiction. Since $x_e \in cl^*(F,A)$.

(iii)⇒(iv): Suppose $(G, A) \subseteq cl^*(F, A) - (F,A)$, (G,A) is soft $\alpha\psi$ closed and $x_e \in (G,A)$, since $(G,A) \subseteq (X, A) - (F,A)$ and (G,A) is soft $\alpha\psi$ closed, then $(F,A) \subseteq (X,A) - (G,A)$, $\alpha\psi cl(\{x_e\}) \cap (F,A) = \varphi$, since $x_e \in cl^*(F,A)$ by (iii) $\alpha\psi cl(\{x_e\} \cap (F, A) \neq \varphi$. Therefore $cl^*(F, A) - (F, A)$ contains no non empty soft $\alpha\psi$ closed set.

(iv)⇒(v): Since $cl^*(F, A) - (F, A) = ((F, A) \cup (F, A)^*) - (F, A) = ((F, A) \cup (F,A)^*) \cap (F,A)^C = ((F,A) \cap (F,A)^C) \cup ((F,A)^* \cap (F,A)^C) = ((F,A)^* \cap (F,A)^C) = ((F, A)^* - (F, A))$. Therefore $(F, A)^* - (F, A)$ contains no non empty soft $\alpha\psi$ closed set.

(v)⇒(i): Let $(F,A) \subseteq (U,A)$ where (U,A) is soft $\alpha\psi$ open set. Therefore $(X,A) - (U,A) \subseteq (X,A) - (F,A)$ and so $(F,A)^* \cap ((X,A) - (U,A)) \subseteq (F,A)^* \cap ((X,A) - (F,A)) = (F,A)^* - (F,A)$.

Therefore $(F, A)^* \cap (X, A) - (U, A) = (F, A)^* - (F, A)$. Since $(F, A)^*$ is always closed set, so $(F, A)^*$ is soft $\alpha\psi$ closed set and so $(F, A)^* \cap (X, A) - (U, A)$ is soft $\alpha\psi$ closed set contained in $(F, A)^* - (F, A)$. Therefore $(F, A)^* \cap (X, A) - (U, A) = \varphi$ and hence $(F, A)^* \subseteq (U, A)$. Therefore (F, A) is soft $I_{\alpha\psi}$ closed set.

Theorem 3.10: Let (X, τ, A, I) be a soft ideal topological space. For every $(F, A) \in I$ is soft $I_{\alpha\psi}$ closed set.

Proof: Let $(F, A) \subseteq (U, A)$ where (U, A) is soft $\alpha\psi$ open set. Since $(F, A)^* = \varphi$ for every $(F,A) \in I$, then $cl^*(F,A) = (F,A)^* \cup (F,A) = (F,A) \subseteq (U,A)$. Therefore by Theorem3.9 (F, A) is soft $I_{\alpha\psi}$ closed set.

Theorem 3.11: If (X, τ, A, I) be a soft ideal topological space. Then $(F, A)^*$ is always soft $I_{\alpha\psi}$ closed set for every subset (F, A) of (X, A).

Proof: Let $(F,A)^* \subseteq (U,A)$ where (U,A) is soft $\alpha\psi$ open set. Since $((F,A)^*)^* \subseteq (F,A)^*$, we have $((F,A)^*)^* \subseteq (U,A)$ whenever $(F,A)^* \subseteq (U,A)$ and (U,A) is soft $\alpha\psi$ open set. Hence $(F, A)^*$ is soft $I_{\alpha\psi}$ closed set.

Theorem 3.12: Let (X, τ, A, I) be a soft ideal topological space.Then every soft $I_{\alpha\psi}$ closed, soft $\alpha\psi$ open set is soft τ^* closed set.

Proof: Since (F, A) is soft $I_{\alpha\psi}$ closed. If (F, A) is soft $\alpha\psi$ open set and $(F, A) \subseteq (F, A)$. Then $(F, A)^* \subseteq (F, A)$. Hence (F, A) is soft τ^* closed set.

Corollary 3.13: Let (X, τ, A, I) be a soft ideal topological space and (F, A) be a soft $I_{\alpha\psi}$ closed set, then the following are equivalent:

(i) (F, A) is soft τ^* closed set.

(ii) $cl^*(F,A) - (F,A)$ is a soft $\alpha\psi$ closed set.

(iii) $(F,A)^* - (F,A)$ is a soft $\alpha\psi$ closed set.

Proof:
(i) ⇒(ii): If (F, A) is soft τ * closed, then (F, A)* ⊆ (F, A), and so cl*(F, A)− (F, A) = ((F, A) ∪ (F, A)*)− (F, A) = φ. Hence cl*(F, A)− (F, A) is a soft αψ closed set.

(ii)⇒(iii): Since cl*(F, A)− (F, A) = (F, A)* − (F, A) and so (F, A)*− (F, A) is soft αψ closed set.

(ii)⇒(i): If $(F,A)^*-(F,A)$ is soft $\alpha\psi$ closed set. Since (F,A) is soft $I_{\alpha\psi}$ closed set by Theorem 3.9, $(F, A)^*-(F, A)=\varphi$ and so (F, A) is soft τ^* closed set.

Theorem 3.14: Let (X, τ, A, I) be a soft ideal topological space and (F, A) is soft *dense in itself, then (F, A) is soft $\alpha\psi$ closed.

Proof: Suppose (F, A) is soft * dense in itself, soft $I_{\alpha\psi}$ closed subset (X, A). Let $(F, A) \subseteq (U, A)$ where (U, A) is soft α open. Every soft α open set is soft $\alpha\psi$ open. Then by Theorem 3.9 $cl^*(F, A) \subset (U, A)$ whenever $(F, A) \subseteq (U, A)$ and (U, A) is soft $\alpha\psi$ open. Since (F, A) is soft * dense in itself then $cl(F, A) = cl^*(F, A)$, every soft closed set is soft ψ closed. Therefore $\psi cl(F, A) \subseteq (U, A)$ whenever $(F, A) \subseteq (U, A)$ and (U, A) is soft α open. Hence (F, A) is soft $\alpha\psi$ closed.

Theorem 3.15: Let (X, τ, A, I) be a soft ideal topological space and $(F, A) \subseteq (X, A)$. Then (F, A) is soft $I_{\alpha\psi}$ closed iff $(F, A) = (G, A) - (N, A)$ where (G,A) is soft τ^* closed and (N, A) contains no non empty soft $\alpha\psi$ closed set.

Proof: If (F, A) is soft $I_{\alpha\psi}$ closed then by Theorem 3.7(v), $(N, A) = (F, A)^* - (F,A)$ contains no non empty soft $\alpha\psi$ closed set. If $(G,A) = cl^*(F,A)$, then (G,A) is soft τ^* closed such that, $(G,A)-(N,A) = ((F,A)\cup (F, A)^*)-((F,A)^*-(F,A)) = ((F,A)\cup(F,A)^*)\cap((F,A)^*\cap(F,A)^C)^C = ((F,A)\cup(F,A)^*)\cap(((F,A)^*)^C\cup(F,A)) = ((F,A)\cup(F,A)^*)\cap((F,A)\cup((F,A)^*)^C) = (F,A)\cup((F,A)^*\cap((F,A)^*)^C) = (F,A)$.

Conversely, suppose $(F, A) = (G, A) - (N, A)$ where (G, A) is soft τ^*closed and (N, A) contains no non empty soft $\alpha\psi$ closed set. Let (U, A) be a soft $\alpha\psi$ open set such that $(F, A) \subseteq (U, A)$. Then $(G, A) - (N, A) \subseteq (U, A)$ which implies that $(G, A) \cap ((X, A) - (U, A)) \subseteq (N, A)$. Now $(F, A) \subseteq (G, A)$ and $(G, A)^* \subseteq (G, A)$ then $(F,A)^* \subseteq (G,A)^*$ and so $(F,A)^* \cap((X,A) -(U,A))\subseteq(G,A)^* \cap((X, A) -(U, A)) \subseteq(G, A) \cap((X, A) -(U, A))\subseteq(N, A)$. By hypothesis, since $(F, A)^* \cap((X, A) -(U, A)) = \varphi$ and so $(F, A)^*\subseteq(U, A)$. Hence (F, A) is soft $I_{\alpha\psi}$ closed.

Theorem 3.16: Let (X, τ, A, I) be a soft ideal topological space. If (F, A) and (G, A) are subsets of (X, A) such that $(F, A) \subseteq (G, A) \subseteq cl^*(F, A)$ and (F, A) is soft $I_{\alpha\psi}$ closed then (G, A) is soft $I_{\alpha\psi}$ closed.

Proof: Since (F,A) is soft $I_{\alpha\psi}$ closed then by Theorem 3.9(iv), $cl^*(F,A)-(F,A)$ contains no non empty soft $\alpha\psi$ closed set. Since $cl^*((G,A))-(G,A) \subseteq cl^*(F,A)-(F,A)$ and so $cl^*((G,A))-(G,A)$ contains no non empty soft $\alpha\psi$ closed set. Hence (G, A) is soft $I_{\alpha\psi}$ closed.

Corollary 3.17: Let (X, τ, A, I) be a soft ideal topological space. If (F, A) and (G, A) are subsets of (X, A) such that $(F, A) \subseteq (G, A) \subseteq (F, A)^*$ and (F, A) is soft $I_{\alpha\psi}$ closed then (F, A) and (G, A) are soft $\alpha\psi$ closed.

Proof: Let (F, A) and (G, A) be subsets of (X, A) such that $(F, A) \subseteq (G, A) \subseteq (F, A)^*$ which implies that $(F, A) \subseteq (G, A) \subseteq (F, A)^* \subseteq cl^*(F, A)$ and (F, A) is soft $I_{\alpha\psi}$ closed by Theorem 3.16, (G, A) is soft $I_{\alpha\psi}$ closed. Since $(F, A) \subseteq (G, A) \subseteq (F, A)^*$ then $(F, A)^* = (G, A)^*$ and so (F, A) and (G, A) are soft * dense in itself by Theorem 3.14, (F, A) and (G, A) are soft $\alpha\psi$ closed.

Theorem 3.18: Let (X, τ, A, I) be a soft ideal topological space and $(F, A) \subseteq (X, A)$. Then (F, A) is soft $I_{\alpha\psi}$ open iff $(G, A) \subseteq int^*(F, A)$ whenever (G, A) is soft $\alpha\psi$ closed and $(G, A) \subseteq (F,A)$.

Proof: Suppose (F, A) is soft $I_{\alpha\psi}$ open, If (G, A) is soft $\alpha\psi$ closed and $(G,A)\subseteq(G, A)$ then $(X, A) -(F, A) \subseteq(X, A) -(G, A)$ and so $cl^*(X -A) \subseteq (X, A) -(G, A)$ by Theorem 3.9, therefore $(G, A) \subseteq (X, A) - cl^*((X, A)-(F, A)) = int^*(F, A)$. Hence $(G, A) \subseteq int^*(F, A)$.

Conversely, Let (U,A) be soft $\alpha\psi$ open set such that $(X,A)-(G,A) \subseteq(U,A)$. Then $(X,A) -(U,A) \subseteq(F,A)$ and so $(X,A) -(U,A) \subseteq int^*(F,A)$. Therefore $cl^*((X, A) - (F, A)) \subseteq (U, A)$ by Theorem 3.9, $(X, A) -(G, A)$ is soft $I_{\alpha\psi}$ closed. Hence (F, A) is soft $I_{\alpha\psi}$ open.

Theorem 3.19: Let (X, τ, A, I) be a soft ideal topological space and $(F, A) \subseteq (X, A)$. If (F, A) is soft $I_{\alpha\psi}$ open and $int^*(F, A) \subseteq (G, A) \subseteq (F, A)$, then (G, A) is soft $I_{\alpha\psi}$ open.

Proof: Since (F, A) is soft $I_{\alpha\psi}$ open, then $(X, A) - (F, A)$ soft $I_{\alpha\psi}$ closed by Theorem 3.9 $cl^*((X,A) - (F,A)) - ((X,A) - (F,A))$ contains no non empty soft $\alpha\psi$ closed set. Since $int^*(F, A) \subseteq int^*(G, A)$ which implies that $cl^*((X, A) - (G,A)) \subseteq cl^*((X,A) - (F,A))$ and so $cl^*((X,A) - (G,A)) - ((X,A) - (G,A)) \subseteq cl^*((X, A) - (F, A)) - ((X, A) - (F, A))$. Hence (G, A) is soft $I_{\alpha\psi}$ open.

Theorem 3.20: Let (X, τ, A, I) be a soft ideal topological space and $A \subseteq X$. Then the following are equivalent:

(i) (F, A) is soft $I_{\alpha\psi}$ closed set.

(ii) $(F,A) \cup ((X,A) - (F,A)^*)$ is soft $I_{\alpha\psi}$ closed set.

(iii) $(F,A)^* - (F,A)$ is soft $I_{\alpha\psi}$ closed set.

Proof:

(i) ⇒ (ii): Suppose (F, A) is soft $I_{\alpha\psi}$ closed set, if (U, A) is soft $\alpha\psi$ open set such that $(F, A) \cup ((X, A) - (F, A)^*) \subseteq (U, A)$ then $(X, A) - (U,A) \subseteq (X, A) - ((F, A) \cup ((X, A) - (F, A)^*)) = (X, A) \cap ((F, A) \cup ((F, A)^*)^C) = (F, A)^* \cap (F, A)^C = (F, A)^* - (F, A)$. Since (F, A) is soft $I_{\alpha\psi}$ closed set by Theorem 3.9(v) it follows that $(X, A) - (U, A) = \varphi$ and so $(X, A) = (U, A)$. Therefore $(F,A) \cup ((X,A) - (F,A)^*) \subseteq (U,A)$ which implies that $(F,A) \cup ((X, A) - (F,A)^*) \subseteq (X, A)$ and so $((F, A) \cup ((X, A) - (F, A)^*))^* \subseteq (X, A)^* \subseteq (X, A) = (U, A)$. Hence $(F, A) \cup ((X, A) - (F, A)^*)$ is soft $I_{\alpha\psi}$ closed set.

(ii) ⇒ (i): Suppose $(F, A) \cup ((X, A) - (F, A)^*)$ is soft $I_{\alpha\psi}$ closed set. If (G, A) is soft $\alpha\psi$ closed set such that $(G,A) \subseteq (F,A)^* - (F,A)$ then $(G,A) \subseteq (F,A)^*$ and $(G,A) \subseteq (X,A) - (F,A)$ which implies that $(X,A) - (F,A)^* \subseteq (X,A) - (G,A)$ and $(F,A) \subseteq (X,A) - (G,A)$. Therefore $(F,A) \cup ((X,A) - (F,A)^*) \subseteq (F,A) \cup ((X, A) - (G, A)) = (X, A)$ (G, A) and $(X, A) - (G, A)$ is soft $\alpha\psi$ open. Since $((F, A) \cup ((X, A) - (F, A)^*))^* \subseteq (X, A) - (G, A)$ which implies that $(F, A)^* \cup ((X, A) - (F, A)^*)^* \subseteq (X, A) - (G, A)$ and so $(F, A)^* \subseteq (X, A) - (G, A)$ which implies that, $(G,A) \subseteq (X,A) - (F,A)^*$. Since $(G,A) \subseteq (F,A)^*$, it follows that $(G, A) = \varphi$. Hence (F, A) is soft $I_{\alpha\psi}$ closed set.

(ii) ⇒ (iii): Since $(X, A) - ((F, A)^* - (F, A)) = (X, A) \cap ((F, A)^* \cap (F, A)^C)^C = (X,A) \cap (((F,A)^*)^C \cup (F,A)) = ((X,A) \cap ((F,A)^*)^C) \cup ((X,A) \cap (F,A)) = (F,A) \cup ((X,A) - (F,A)^*)$.

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