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ON THE SCATTERING OF LOVE WAVES IN THE PRESENCE OF LIQUID LAYER AT COASTAL REGIONS

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ABSTRACT

In this paper we investigate Love waves scattering at the corner of a rigid quarter space in the presence of a liquid layer of depth h on the elastic solid. The model fits with Love wave scattering at coastal regions. A shallow ocean ($x \geq 0,\ \ 0 \leq z \leq h$) of depth h on the elastic solid earth ($z \geq h$) and the quarter space ($x \leq 0, z \geq 0$) is composed of hard materials like rocks at coastal regions. We use the Wiener-Hopf technique and Fourier transformation in integrals whose evaluation along appropriate contours in the complex plane yields the equations of reflected, transmitted and scattered waves. At distant points, the scattered wave retains the behavior of a cylindrical wave but decays exponentially.

The variation of the amplitude versus wave number and wavelength of scattered Love waves is also discussed graphically. The amplitude of scattered waves dies out rapidly with small increase in wave number. The graphical behavior of the results has also been shown for a particular case to improve the importance of study.

Keywords: *Love Waves, Integral Transform, Rigid Quarter Space. Scattered Waves, Wiener – Hopf Technique, Reflected Wave, Surface Layer.*

INTRODUCTION

In Seismology, the study of scattering of seismic waves by structural discontinuities has acquired importance because of the existence of such discontinuities in the Earth's crust. Love waves are responsible for damage to human lives and buildings on the surface of earth during an earthquake. During earthquake, scattering of seismic waves due to discontinuities in the surface leads to large amplification and variation in ground motion. When seismic waves appear on the surface of earth they lose their energies around the irregularities in surface of earth. The paper represents a mechanism by which the energy of surface wave is lessened by partial conversion into the reflected, transmitted and scattered waves due to the inhomogeneities and irregularities in the earth's crust.

A number of authors have used a variety of analytical, approximate and numerical methods to find the exact solutions for various problems concerning the passage of seismic surface waves through surface layer in the presence of mountain valleys, ditches and trenches in the crust surface. The problem of scattering of Love waves due to an infinitely long rigid strip in a surface layer over a solid half-space is studied by Deshwal[4]. Deshwal and Mann [5] studied a problem of scattering of seismic waves at the foot of mountain. Shuvalov *et al.* [12] analyzed the dispersion of Love shear horizontal (SH) waves in coated vertically periodic substrates based on the properties of the scalar SH impedances of substrate and coated layer. Singh and Deshwal [13] studied the scattering of seismic surface wave due to mountain of finite depth at the coastal region. Chattaraj and Samal [3] have studied the propagation of Love waves in fiber-reinforced layer lying over a gravitating anisotropic porous half-space. Bin *et al.* [1] investigated the scattering of Love waves by an interface crack between a piezoelectric layer and an elastic substrate by using the integral transform and singular integral equation techniques. Saito [10] formulates Love-wave excitation in terms of the interaction between a propagating ocean wave and the sea-bottom topography using far-field approximation. Zaman [15] studied

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the diffraction of SH-waves in an infinite elastic plate under a variety of boundary conditions using Wiener-Hopf techniques. Gupta *et al.* [7] have studied the propagation of Love waves in a non-homogenous substratum over an initially stressed heterogeneous half-space and derived the dispersion equation of phase velocity. Based on the shear spring model, the propagation of Love wave in two-layered piezoelectric composite plates under the influence of interfacial defect is investigated by Wang [14].

This study is based on a paper by Sato [11]who analyzed the problem of Love wave propagation in case of a surface layer on a solid half space using the Weiner-Hopf technique. The scattering of Love waves in the presence of liquid layer $(0 \le z \le h, x \ge 0)$ of depth h superimposed on the solid earth $(z \ge h)$ has been discussed. The reflected, transmitted and scattered waves have been obtained by Fourier transformations [9] and Wiener-Hopf technique [8].

FORMULATION OF THE PROBLEM

The problem is two dimensional and plane of propagation is the zx-plane. The x-axis lies along the free surface and z-axis has been taken vertically downward. A solid layer of thickness H ($-H \le z \le 0, \quad -\infty < x < \infty$) lies over a solid half space ($z \ge 0, \quad -\infty < x < \infty$).The region ($(x \ge 0, \quad z \ge h)$) is an isotropic and homogeneous elastic solid while the region $(z \ge 0, \quad x \le 0)$ is rigid such that no displacement takes place across it. There is a shallow ocean in the form of a liquid layer of depth h on the elastic solid as shown in figure 1. A time harmonic Love waves strike at the interface $z = h$ from the region $x \ge 0$, some are transmitted to the liquid layer and some of the transmitted waves strike the corner of the rigid quarter space and give rise to the scattered waves. The Velocities and rigidities of the shear waves in the crustal layer and solid half space are U_2, α_2 and U_1, α_1 respectively.

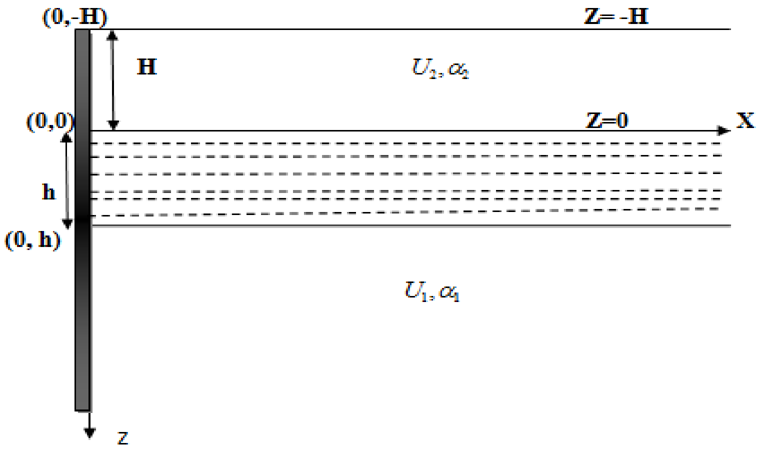

Figure-1: Geometry of the problem

The incident Love wave are given by

$$v_{0,1} = A\cos\varphi_{2N} He^{-(\varphi_{1N}z + ik_{1N}x)}, \quad z \ge 0, \tag{1}$$

$$v_{0,2} = A\cos\varphi_{2N}(z+H)e^{-ik_{1N}x}, \quad -H \le z \le 0, \tag{2}$$

where,

$$\varphi_{1N} = \sqrt{k_{1N}^2 - k_1^2}\ , \ \varphi_{2N} = \sqrt{k_2^2 - k_{1N}^2}, \quad , \ |k_1| < |k_{1N}| < |k_2|$$

and k_{1N} is a root of the equation

$$\tan\varphi_{2N} H = \mu\frac{\varphi_{1N}}{\varphi_{2N}}, \ \mu = \frac{\alpha_1}{\alpha_2}, \tag{3}$$

In two dimensions, the wave equation is

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} + \frac{\varepsilon}{c^2}\frac{\partial\phi}{\partial t}, \tag{4}$$

where c is the velocity of propagation and $\varepsilon > 0$ is the damping constant. If the displacement be harmonic in time, then

$$\phi(x,z,t) = v(x,z)e^{-i\omega t} \tag{5}$$

and equation (4) reduce to

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}+k^2\, v=0. \tag{6}$$

The wave equation (6) in the present study can be written as

$$(\nabla^2+k_j^2)\ v_j=0, \quad j=1,2 \tag{7}$$

where $k_j=\frac{(\omega^2+i\varepsilon\omega)^{1/2}}{U_j}=k_j^{'}+ik_j^{''}$

The total displacement is given by

$$v=v_{0,1}+v_1\,, \qquad z\geq h\,, \qquad -\infty<x<\infty, \tag{8}$$

$$=v_{0,1}+v_3\,, \quad 0\leq z\leq h\,, \qquad x\geq 0, \tag{9}$$

$$=v_{0,2}+v_2\,, \quad -H\leq z\leq 0\,, \quad -\infty<x<\infty, \tag{10}$$

Boundary Conditions

The boundary conditions are

(i) $$\frac{\partial v_3}{\partial z}=0, \qquad z=h, \quad x\geq 0 \tag{11}$$

(ii) $$v_{0,1}+v_3=0, \qquad 0\leq z\leq h, x=0 \tag{12}$$

(iii) $$v_{0,1}+v_1=0, \qquad z=h, x\geq 0 \tag{13}$$

(iv) $$v_1=v_3, \quad \frac{\partial v_1}{\partial z}=\frac{\partial v_3}{\partial z}, \quad z=h, \quad x\geq 0, \tag{14}$$

(v) $$\alpha_1\frac{\partial}{\partial z}\left(v_{0.1}+v_3\right)=\kappa\left(v_{0.1}+v_3\right), \quad z=h, \quad x\geq 0 \tag{15}$$

(vi) $$v_1=v_2\,, \quad \alpha_1\frac{\partial v_1}{\partial z}=\alpha_2\frac{\partial v_2}{\partial z}\,, \qquad z=0, \quad -\infty<x<\infty. \tag{16}$$

Using boundary condition (13) and (12) in equation (1), we obtain

$$v_3=-A\cos\varphi_{2N}He^{-(\varphi_{1N}h+ik_{1N}x)}, \quad z=h, \quad x\geq 0\,, \tag{17}$$

$$v_1=-A\cos\varphi_{2N}He^{-\varphi_{1N}z}, \quad 0\leq z\leq h, \quad x=0\,, \tag{18}$$

Solution of the Problem

Taking Fourier transform of equation (7), we obtain

$$\int_{-\infty}^{\infty}(\frac{\partial^2 v_j}{\partial x^2}+\frac{\partial^2 v_j}{\partial z^2}+k_j^2 v_j)e^{ipx}dx=0, \quad j=1,2$$

$$\frac{d^2\overline{v}_j\,(p,z)}{dz^2}-\varphi_j^2\overline{v}_j\,(p,z)=0, \tag{19}$$

where $\overline{v}_j\,(p,z)$ represents Fourier transform of $v_j(x,z)$ defined as

$$\overline{v}_j(p,z)=\int_{-\infty}^{\infty}v_j(x,z)e^{ipx}dx, \quad p=\alpha+i\beta$$

$$=\int_{-\infty}^{0}v_j(x,z)e^{ipx}dx+\int_{0}^{\infty}v_j(x,z)e^{ipx}dx$$

$$=\overline{v_{j-}}(x,z)+\overline{v_{j+}}(x,z)$$

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and $\varphi_j = \pm\sqrt{p^2 - k_j^2}$.

If for a given z , as $|x| \to \infty$ and $M, \tau > 0$, $|v_j(x,z)| \sim Me^{-\tau|x|}$, then $\bar{v}_{j+}(p,z)$ is analytic in $\beta > -\tau$ and $\bar{v}_{j-}(p,z)$ is analytic in $\beta < \tau\ (= lm(k_j))$. By analytic continuation, $\bar{v}_j\ (p,z)$ and its derivatives are analytic in the strip $-\tau < \beta < \tau$ in the complex p - plane [3]. Choosing the sign of φ_j such that its real part is always positive for all p , we find the solutions to equation (19)

$$\bar{v}_1\ (p,z) = E(p)e^{-\varphi_1 z}, \quad z \geq 0, \tag{20}$$

$$\bar{v}_2(p,z) = F(p)e^{-\varphi_2 z} + G(p)e^{\varphi_2 z}, \quad -H \leq z \leq 0 . \tag{21}$$

Using condition (16) in equations (20) and (21), we find

$$F(p) = \frac{E(p)}{2\varphi_2}\left(\varphi_2 + \mu\varphi_1\right) \text{ and } G(p) = \frac{E(p)}{2\varphi_2}\left(\varphi_2 - \mu\varphi_1\right) \tag{22}$$

Now, eliminating $F(p)$ *and* $G(p)$ from (22), we get

$$\bar{v}_2(p,z) = E(p)\frac{[\varphi_2 \cosh\varphi_2 z - \mu\varphi_1 \sinh\varphi_2 z]}{\varphi_2} \tag{23}$$

Differentiating equation (23) with respect to z , we obtain

$$\bar{v}_2^{'}(p,z) = E(p)[\varphi_2 \sinh\varphi_2 z - \mu\varphi_1 \cosh\varphi_2 z] \tag{24}$$

Putting $z = h$ and denoting $\bar{v}_2^{'}(p,h)$ by $\bar{v}_2^{'}(p)$, we obtain

$$\bar{v}_2^{'}(p) = E(p)[\varphi_2 \sinh\varphi_2 h - \mu\varphi_1 \cosh\varphi_2 h] \tag{25}$$

Now eliminating E (p) from (23) and (25), we find

$$\bar{v}_2\ (p,z) = \frac{(\varphi_2 \cosh\varphi_2 z - \mu\varphi_1 \sinh\varphi_2 z)}{\varphi_2(\varphi_2 \sinh\varphi_2 h - \mu\varphi_1 \cosh\varphi_2 h)}\bar{v}_2^{'}(p) \tag{26}$$

Putting $z = h$ in (26) and denoting $\bar{v}_2\ (p,h)$ by $\bar{v}_2\ (p)$, we obtain

$$\bar{v}_2\ (p) = [\bar{v}_{2+}(p) + \bar{v}_{2-}(p)] = \frac{\varphi_2 \cosh\varphi_2 h - \mu\varphi_1 \sinh\varphi_2 h}{\varphi_2(\varphi_2 \sinh\varphi_2 h - \mu\varphi_1 \cosh\varphi_2 h)} \times [\bar{v}_{2+}^{'}(p) + \bar{v}_{2-}^{'}(p)] \tag{27}$$

Multiplying equation (7) by e^{ipx} and integrating from x = 0 to x = ∞ $(j = 3)$, we find

$$\frac{d^2}{dz^2}[\bar{v}_{3+}(p,z)] - \varphi_1^2\ \bar{v}_{3+}(p,z) = \left(\frac{\partial v_3}{\partial x}\right)_{x=0} - ip(v_3)_{x=0} \quad . \tag{28}$$

Replacing p to - p in equation (28), we get

$$\frac{d^2}{dz^2}[\bar{v}_{3+}(-p,z)] - \varphi_1^2\ \bar{v}_{3+}(-p,z) = \left(\frac{\partial v_3}{\partial x}\right)_{x=0} + ip(v_3)_{x=0} . \tag{29}$$

Now subtracting equation (29) from equation (28), we obtain

$$\frac{d^2}{dz^2}[\bar{v}_{3+}(p,z) - \bar{v}_{3+}(-p,z)] - \varphi_1^2[\bar{v}_{3+}(p,z) - \bar{v}_{3+}(-p,z)] = -2ip\left(v_3\right)_{x=0} \tag{30}$$

Using the boundary condition (17) in equation (27), we find

$$\frac{d^2}{dz^2}[\bar{v}_{3+}(p,z) - \bar{v}_{3+}(-p,z)] - \varphi_1^2[\bar{v}_{3+}(p,z) - \bar{v}_{3+}(-p,z)] = 2ip\ A\cos\ \varphi_{2N} He^{-\varphi_{1N} h} \tag{31}$$

The solution of equation (31) is

$$\bar{v}_{3+}(p,z)-\bar{v}_{3+}(-p,z)=J_1(p)e^{-\varphi_1 z}+J_2(p)e^{\varphi_1 z}-\frac{2ipA\cos\varphi_{2N}He^{-\varphi_{1N}h}}{p^2-k_{1N}^2} \tag{32}$$

Differentiating equation (29) with respect to z and putting h=0, we obtain

$$\bar{v}'_{3+}(p,-H)-\bar{v}'_{3+}(-p,-H)=\varphi_1(-J_1(p)e^{-\varphi_1 z}+J_2(p)e^{\varphi_1 z}) \tag{30}$$

Multiplying equation (15) by e^{ipx} and integrating from x = 0 to x = ∞ $(j=3)$, use z = h, we find

$$\alpha_1\frac{\partial}{\partial z}\int_0^\infty v_3(x,z)e^{ipx}dx=\kappa\int_0^\infty v_3(x,z)e^{ipx}dx+\kappa A\int_0^\infty e^{i(p-k_{1N})x}dx \tag{31}$$

$$\alpha_1\bar{v}'_{3+}(p,h)=\kappa\bar{v}_{3+}(p,h)+\frac{i\kappa A}{p-k_{1N}} \tag{32}$$

Replacing p to - p in equation (32) and adding it to the resulting equation, we get

$$\alpha_1(\bar{v}'_{3+}(p,h)+\bar{v}'_{3+}(-p,h))=\kappa(\bar{v}_{3+}(p,h)+\bar{v}_{3+}(-p,h))+\frac{2i\kappa A}{p^2-k_{1N}^2}k_{1N} \tag{33}$$

Using equation (33) in (30), we have

$$\frac{\kappa}{\alpha_1}(\bar{v}_{3+}(p,h)+\bar{v}_{3+}(-p,h)+\frac{2i\kappa A}{p^2-k_{1N}^2}k_{1N})=\varphi_1(-J_1(p)e^{-\varphi_1 z}+J_2(p)e^{\varphi_1 z}) \tag{34}$$

Putting $z=h$ in equation (29), we obtain

$$\bar{v}_{3+}(p,h)-\bar{v}_{3+}(-p,h)=J_1(p)e^{-\varphi_1 h}+J_2(p)e^{\varphi_1 h}-\frac{2ipA\cos\varphi_{2N}He^{-\varphi_{1N}h}}{p^2-k_{1N}^2} \tag{35}$$

Obtaining the values of J_1 and J_2 from equations (34) and (35)

$$J_1=\frac{e^{-\varphi_1 h}}{2\cosh\varphi_1\delta}\left[\begin{array}{l}\bar{v}_{3+}(p)+\bar{v}_{3+}(-p)+\frac{2iA\cosh\varphi_{1N}H}{p^2-k_{1N}^2}k_{1N}\\ -\frac{\kappa e^{\varphi_1 h}}{\alpha_1\varphi_1}\left[\bar{v}_{3+}(p,h)+\bar{v}_{3+}(-p,h)+\frac{2iA}{p^2-k_{1N}^2}k_{1N}\right]\end{array}\right] \tag{36}$$

$$J_2=\frac{e^{\varphi_1 h}}{2\cosh\varphi_1\delta}\left[\begin{array}{l}\bar{v}_{3+}(p)+\bar{v}_{3+}(-p)+\frac{2iA\cosh\varphi_{1N}H}{p^2-k_{1N}^2}k_{1N}\\ -\frac{\kappa e^{\varphi_1 h}}{\alpha_1\varphi_1}(\bar{v}_{3+}(p,h)+\bar{v}_{3+}(-p,h)+\frac{2iA}{p^2-k_{1N}^2}k_{1N})\end{array}\right]$$
$$+\frac{\kappa e^{\varphi_1 h}}{\alpha_1\varphi_1}\left[\bar{v}_{3+}(p,h)+\bar{v}_{3+}(-p,h)+\frac{2iA}{p^2-k_{1N}^2}k_{1N}\right] \tag{37}$$

Differentiating both sides of equation (29) with respect to z and putting z = h, we get

$$\bar{v}'_{3+}(p)+\bar{v}'_{3+}(-p)=-\varphi_1 J_1(p)e^{-\varphi_1 h}+\varphi_1 J_2(p)e^{\varphi_1 h}+\frac{2i\varphi_{1N}pA\cos\varphi_{2N}He^{-\varphi_{1N}h}}{p^2-k_{1N}^2}. \tag{38}$$

Putting the value of J_1 and J_2 from equations (36) and (37) in equation (38), we have

$$\bar{v}'_{3+}(p)+\bar{v}'_{3+}(-p)=-\frac{\varphi_1 e^{-\varphi_1 h}}{2\cosh\varphi_1 h}\begin{bmatrix}\bar{v}_{3+}(p)+\bar{v}_{3+}(-p)+\dfrac{2iA\cosh\varphi_{2N}h}{p^2-k_{1N}^2}k_{1N}\\ -\dfrac{\kappa e^{\varphi_1 h}}{\alpha_1\varphi_1}\left[\bar{v}_{3+}(p,h)+\bar{v}_{3+}(-p,h)+\dfrac{2iA}{p^2-k_{1N}^2}k_{1N}\right]\end{bmatrix}$$
$$+\frac{\varphi_2 e^{\varphi_1 h}}{2\cosh\varphi_1 h}\left[\bar{v}_{3+}(p)+\bar{v}_{3+}(-p)+\frac{2iA\cosh\varphi_{2N}h}{p^2-k_{1N}^2}k_{1N}\right]$$
$$-\frac{\kappa e^{2\varphi_1 h}}{2\alpha_1\cosh\varphi_1 h}\left(\bar{v}_{3+}(p,h)+\bar{v}_{3+}(-p,h)+\frac{2iA}{p^2-k_{1N}^2}k_{1N}\right) \tag{39}$$
$$+\frac{\kappa e^{\varphi_1 h}}{\alpha_1}\left[\bar{v}_{3+}(p,h)+\bar{v}_{3+}(-p,h)+\frac{2iA}{p^2-k_{1N}^2}k_{1N}\right]-\frac{2ip\varphi_{1N}k_{1N}A\varphi_{2N}\sin\varphi_{2N}h}{p^2-k_{1N}^2}.$$

Now taking $h\to 0$, we get

$$\bar{v}'_{3+}(p)+\bar{v}'_{3+}(-p)=\frac{\kappa}{2\alpha_1}\left[\bar{v}_{3+}(p)+\bar{v}_{3+}(-p)+\frac{2iA}{p^2-k_{1N}^2}k_{1N}\right]$$
$$+\frac{\kappa}{\alpha_1}\left[\bar{v}_{3+}(p)+\bar{v}_{3+}(-p)+\frac{2iA}{p^2-k_{1N}^2}k_{1N}\right] \tag{40}$$
$$-\frac{\kappa}{2\alpha_1}\left[\bar{v}_{3+}(p)+\bar{v}_{3+}(-p)+\frac{2iA}{p^2-k_{1N}^2}k_{1N}\right]$$

$$\bar{v}'_{3+}(p)+\bar{v}'_{3+}(-p)=\frac{\kappa}{\alpha_1}\left[\bar{v}_{3+}(p)+\bar{v}_{3+}(-p)+\frac{2iA}{p^2-k_{1N}^2}k_{1N}\right] \tag{41}$$

From the boundary condition (12), we obtain

$$\bar{v}_{1+}(p)=\bar{v}_{3+}(p)\quad and\quad \bar{v}'_{1+}(p)=\bar{v}'_{3+}(p). \tag{42}$$

From equation (42) and (41), we write

$$\bar{v}'_{1+}(p)+\bar{v}'_{1+}(-p)=\frac{\kappa}{\alpha_1}\left[\bar{v}_{2+}(p)+\bar{v}_{2+}(-p)+\frac{2iA}{p^2-k_{1N}^2}k_{1N}\right] \tag{43}$$

Now, Multiplying equation (13) by e^{ipx} and integrating from x = 0 to x = ∞, we obtain

$$\bar{v}'_{2-}(p)=-\frac{iA\varphi_{2N}\sin\varphi_{2N}\delta}{p-k_{1N}} \tag{44}$$

Eliminating $\bar{v}_{2+}(p)$ from equations (24) and (43), we obtain

$$\bar{v}'_{1+}(p)+\bar{v}'_{1+}(-p)=\frac{\kappa}{\alpha_2}\begin{bmatrix}-\dfrac{\varphi_1\cosh\varphi_2 h+\mu\varphi_1\sinh\varphi_2 h}{\varphi_2(\varphi_2\sinh\varphi_2 h+\mu\varphi_1\cosh\varphi_2 h)}\times(\bar{v}'_{1+}(p)-\bar{v}'_{1-}(-p))\\ -\bar{v}_{1-}(p)-\bar{v}_{1+}(-p)+\dfrac{2iA}{p^2-k_{1N}^2}k_{1N}\end{bmatrix} \tag{45}$$

$$\bar{v}'_{1+}(p)\left(1+\frac{\kappa g_1(p)}{\alpha_1\varphi_1 g_2(p)}\right)-\frac{iA\kappa}{\alpha_1(p+k_{1N})}=\frac{\kappa}{\alpha_1}\left[\bar{v}_{1+}(-p)-\bar{v}_{1-}(p)+\frac{iA}{p-k_{1N}}\right]-\frac{\kappa}{\alpha_1}\left(\frac{g_1(p)\bar{v}'_{2-}(p)}{\varphi_1 g_2(p)}\right)-\bar{v}'_{1+}(-p) \tag{46}$$

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where,

$$g_1(p) = \varphi_2 \cosh \varphi_2 h + \mu\varphi_1 \sinh \varphi_2 h \quad (47)$$

$$g_2(p) = \varphi_2 \sinh \varphi_2 h + \mu\varphi_1 \cosh \varphi_2 h \quad (48)$$

The equation (46) is the Wiener-Hopf type differential equation (Noble, 1958) whose solution will give $\bar{v}'_{2+}(p)$.

Solution of Wiener-Hopf equation

For solution of equation (46), we factorize $\dfrac{g_1(p)}{\varphi_2 g_2(p)}$ as

$$\frac{g_1(p)}{\varphi_2 g_2(p)} = M_+(p)M_-(p). \quad (49)$$

Where

$$M_+(p) = M_-(-p) = \frac{L_+(p)}{J_+(p)} \prod_{n=1}^{\infty} \frac{(p+p_{1n})}{(p+p_{2n})}$$

Using equation (49) in equation (46), we obtain

$$\bar{v}'_{1+}(p) + M_+(p)M_-(p)\frac{\kappa}{\alpha_1}\bar{v}'_{2+}(p) - \frac{iA\kappa}{\alpha_1(p+k_{1N})} = \frac{\kappa}{\alpha_1}\left[\bar{v}_{1+}(-p) - \bar{v}_{1-}(p) + \frac{iA}{p-k_{1N}}\right]$$
$$-\frac{\kappa}{\alpha_1}M_+(p)M_-(p)\bar{v}'_{1-}(p) - \bar{v}'_{1+}(-p) \quad (50)$$

$$\bar{v}'_{1+}(p) + M_-(p)\frac{\kappa}{\alpha_1}\left(M_+(p)\bar{v}'_{1+}(p) - M_+(p_m)\bar{v}'_{1+}(p_m)\right)$$
$$-\frac{iA\kappa}{\alpha_1(p+k_{1N})} + M_+(p)\frac{\kappa}{\alpha_1}\left(M_-(-p_m)\bar{v}'_{1-}(-p_m)\right) = \frac{\kappa}{\alpha_1}\left[\bar{v}_{1+}(-p) - \bar{v}_{1-}(p) + \frac{iA}{p-k_{1N}}\right]$$
$$-\frac{\kappa}{\alpha_1}M_+(p)\left(M_-(p)\bar{v}'_{1-}(p) - M_-(-p_m)\bar{v}'_{1-}(-p_m)\right) - \bar{v}'_{1+}(-p) - \frac{\kappa}{\alpha_1}M_-(p)\left(M_+(p_m)\bar{v}'_{1+}(p_m)\right) \quad (51)$$
$$= O_-(p)$$

where $p_m = k_2$.

In equation (51), $O_-(p)$ include the terms which are analytic in $\beta < \tau$ and left hand member of above equation is analytic in the region $\beta > -\tau$. Therefore by analytic continuation each member tends to zero in its region of analyticity as $|p| \to \infty$. Hence by Liouville's theorem, the entire function is identically zero. So equating to zero the left hand side of equation (51), we obtain

$$\bar{v}'_{1+}(p) = \frac{\kappa\varphi_1(\varphi_2 \sinh \varphi_2 h + \mu\varphi_1 \cosh \varphi_2 h)}{(\alpha_1\varphi_2^{\,2} + \kappa\mu\varphi_1)\sinh \varphi_2 h + (\mu\varphi_1\alpha_1 + \kappa)\cosh \varphi_2 h}\begin{bmatrix} M_+(p_m)\bar{v}'_{1+}(p_m)M_-(p) \\ +\dfrac{iA}{(p+k_{1N})} - M_+(p)\left(M_-(-p_m)\bar{v}'_{1-}(-p_m)\right)\end{bmatrix} \quad (52)$$

The displacement $v_1(x,z)$ is obtained by inversion of Fourier transform as given below

$$v_1(x,z) = \frac{1}{2\pi}\int_{-\infty+i\beta}^{\infty+i\beta} \bar{v}_1(p,z)e^{-ipx}dp$$
$$= \frac{1}{2\pi}\int_{-\infty+i\beta}^{\infty+i\beta} \frac{-1}{\varphi_1}\left[\frac{\varphi_2 \cosh \varphi_2 z - \mu\varphi_1 \sinh \varphi_2 z}{\varphi_2 \sinh \varphi_2 h + \mu\varphi_1 \cosh \varphi_2 h}\right] \times \left[\bar{v}'_{1^+}(p) + \bar{v}'_{1^-}(p)\right]e^{-ipx}dp \quad (53)$$

where $\bar{v}'_{1^+}(p)$ and $\bar{v}'_{1^+}(-p)$ are given in equations (52) and (44) respectively.

THE REFLECTED AND TRANSMITTED WAVES

The incident Love waves are reflected when these waves encounter with surface irregularities like surface impedance in the crustal layer of earth. The factor $e^{-ipx} = e^{-i\alpha x} e^{\beta x}$ vanishes at infinity in lower-half plane ($x > 0$) and in the upper-half plane ($x < 0$). For finding the reflected component, we evaluate the integral in equation (53) in upper half plane when $x < 0$. The contour of integration is shown in figure (2).

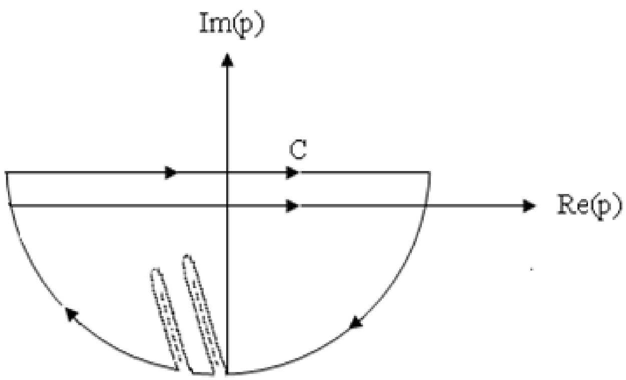
Im(φ_2 ***)<0*** k_1 ***Im(*** φ_1 ***)<0***

k_2

Figure-2: The contour of integration in the lower half of the complex p-plane

There is a pole at $p = k_{1N}$ and the corresponding wave is given as

$$v_{2,1}(x,z) = \frac{-A\,\kappa \cos\varphi_{2N}(z+H)e^{-ik_{1N}x}}{\alpha_2 \varphi_{2N} \sin\varphi_{2N}\delta + \kappa\cos\varphi_{2N}\delta}, \quad x<0,\ -H \le z \le -h, \tag{54}$$



This represents the reflected love wave in the upper half plane. If width of vertical discontinuity is very small i.e. $\delta \to 0$, then

$$v_{2,1}(x,z) = -A\,\kappa \cos\varphi_{2N}(z+H)e^{-ik_{1N}x} \tag{55}$$

which cancels the incident wave and the amplitude of reflected wave is same as the amplitude of incident wave. We now evaluate the integral of equation (53) in lower half plane when $x > 0$.

There is a pole at $p = -k_{1N}$ and the corresponding wave is given as

$$v_{2,2}(x,z) = -A\,\kappa \cos\varphi_{2N}(z+H)e^{ik_{1N}x} \tag{56}$$

Let p_{2n} be the roots of the equation

$$g_2(p) = \varphi_2 \sinh\varphi_2 h + \mu\varphi_1 \cosh\varphi_2 h = 0 \tag{57}$$

$$\text{i.e.} \quad \tan\varphi_{2n}^{'} h = \mu \frac{\varphi_{1n}^{'}}{\varphi_{2n}^{'}}, \tag{58}$$

where,

$$\varphi_{2n}^{'} = \sqrt{k_2^2 - p_{2n}^2}, \quad \varphi_{1n}^{'} = \sqrt{p_{2n}^2 - k_1^2} \tag{59}$$

Equation (58) is the frequency equation of the Love waves in the surface layer of thickness h. There is a pole at $p = -p_{2n}$, which contributes to

$$v_{2,3}(x,z) = C_n \cos\varphi_{2n}(z+h)e^{-ip_{2n}x}, \quad x<0,\ -h \le z \le 0, \tag{60}$$

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where,

$$C_n = \sum_{n=1}^{\infty}\left[\begin{array}{l} -\dfrac{i\kappa\varphi_{2N}\bar{v}'_{2+}(p_m)M_+(p_m)}{M_+(ip_n)\{(-\alpha_2\varphi_{2N}{}^2+\mu\kappa\varphi_{1N})\sin\varphi_{2N}h+(\kappa+\mu\alpha_2\varphi_{1N})\cos\varphi_{2N}h\}} \\ -\dfrac{A\varphi_{2N}\sin\varphi_{2N}\delta}{(p_{2n}-K_{1N})}\end{array}\right]\times\frac{1}{\left[\dfrac{d}{dp}g_2(p)\right]} \quad (61)$$

Equation (61) gives the transmiteed Love waves of n-th mode in the surface layer due to surface impedance. If there is no surface impedance i.e. $\kappa = 0$, we have

$$v_{2,4}(x,z) = \sum_{n=1}^{\infty}\left[-\frac{A\varphi_{2N}\sin\varphi_{2N}\delta}{(p_{2n}-K_{1N})}\right]\times\frac{\cos\varphi_{2n}(z+h)e^{-ip_{2n}x}}{\left[\dfrac{d}{dp}g_2(p)\right]_{p=p_{2n}}}$$

$$v_{2,4}(x,z) = i\sum_{n=1}^{\infty}\left[\bar{v}'_{2-}(p_{2n})\right]\times\frac{\cos\varphi_{2n}(z+h)e^{-ip_{2n}x}}{\left[\dfrac{d}{dp}g_2(p)\right]_{p=p_{2n}}}$$

$$v_{2,4}(x,z) = i\sum_{n=1}^{\infty}\frac{\bar{v}'_{2-}(p_{2n})\cos(z+h)e^{-ip_{2n}x}}{\varphi_{2n}h}\frac{p_{2n}}{\varphi_{2n}}\left(\frac{c_{2n}}{V_{2n}}-1\right) \quad (62)$$

Where $c_{2n} = \dfrac{\omega}{p_{2n}}$; phase velocity of Love waves of the n-th mode propagated in layered structure with surface having thickness h. V_{2n} is the group-velocity of love waves corresponding to the phase-velocity c_{2n}. If there is no surface impedance and the width of the vertical discontinuity is very small i.e. $\delta \to 0$, then from equation (62) no wave is transmitted. Again, assume $p = -p_{1n}$ be the roots of the equation

$$g_2(p) = \varphi_2 \sinh\varphi_2 h + \mu\varphi_1 \cosh\varphi_2 h = 0 \quad (63)$$

i.e. $\tan\varphi_{2n}h = \mu\dfrac{\varphi_{1n}}{\varphi_{2n}},$ (64)

where,

$$\varphi_{2n} = \sqrt{k_2^2 - p_{1n}^2}, \quad \varphi_{1n} = \sqrt{p_{1n}^2 - k_1^2} \quad (65)$$

The residues due to poles at $p = -p_{1n}$ are

$$v_{2,4}(x,z) = D_n \cos\varphi'_{2N}(z+h)e^{ip_{1n}x} \quad (66)$$

where,

$$D_n = \sum_{n=1}^{\infty}\left[\begin{array}{l} \dfrac{i\kappa\varphi_{2N}\bar{v}'_{2-}(p_m)M_-(-p_m)}{M_-(-p_{1n})\{(-\alpha_2\varphi'_{2N}{}^2+\mu\kappa\varphi_{1N})\sin\varphi'_{2N}h+\varphi'_{2N}(\kappa+\mu\alpha_2\varphi'_{1N})\cos\varphi'_{2N}h\}} \\ +\dfrac{A\varphi_{2N}\sin\varphi_{2N}\delta}{(p_{1n}+K_{1N})}\end{array}\right]\times\frac{1}{\left[\dfrac{d}{dp}g_2(p)\right]} \quad (67)$$

Equation (67) gives the reflected Love waves of n-th mode in the surface layer due to surface impedance. If there is no surface impedance i.e. $\kappa = 0$, we have

$$v_{2,4}(x,z) = \sum_{n=1}^{\infty}\left[-\frac{A\varphi_{2N}\sin\varphi_{2N}\delta}{(p_{1n}+K_{1N})}\right]\times\frac{\cos\varphi'_{2N}(z+h)e^{ip_{1n}x}}{\left[\dfrac{d}{dp}g_2(p)\right]_{p=-p_{1n}}}$$

$$v_{2,4}(x,z) = -i\sum_{n=1}^{\infty}\frac{\cos\varphi'_{2N}(z+h)e^{ip_{1n}x}\bar{v}'_{2-}(p_{2n})}{\left[\dfrac{d}{dp}g_2(p)\right]_{p=-p_{1n}}} \quad (68)$$

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The equation (68) represents the reflected Love waves of n[th] mode in the surface layer with thickness h. If $\kappa = 0$ i.e. there is no surface impedance and $\delta \to 0$, then no wave is reflected in half space at all.

THE SCATTERED WAVES

The incident Love waves are not only reflected but they are scattered also by the surface irregularity. There is a branch point $p = -k_1$ in the lower half-plane. We put $p = -k_1 - it$; t being small. The branch cut is obtained by taking $\mathrm{Re}(\varphi_1) = 0$. Now, $\varphi_1^2 = p^2 - k_1^2$ gives $\varphi_1 = \pm i\bar{\varphi}_1$ and $\varphi_2 = \bar{\varphi}_2$. The imaginary part of φ_1 has different signs on two sides of the branch cut. If width of the vertical discontinuity is very small i.e. $\delta \to 0$, then $\bar{v}'_{2-}(p) = 0$.

Now integrating equation (53) along two sides of branch cut, we get

$$v_{2,5}(x,z) = \frac{i}{2\pi}\int_0^\infty [\{\bar{v}_1(p,z)\}_{\varphi_1 = i\bar{\varphi}_1} - \{\bar{v}_1(p,z)\}_{\varphi_1 = -i\bar{\varphi}_1}] e^{-xk_1''} e^{-sx} ds \tag{69}$$

$$= -\frac{i\kappa}{2\pi}\int_0^\infty \left[\frac{\bar{\varphi}_2 \cosh\bar{\varphi}_2 z - i\mu \sinh\bar{\varphi}_2 z}{(\alpha_1\bar{\varphi}_2^2 \sinh\bar{\varphi}_2 h + \kappa\bar{\varphi}_2 \cosh\bar{\varphi}_2 h) + i\bar{\varphi}_1(\kappa\mu \sinh\bar{\varphi}_2 h + \mu\alpha_1\bar{\varphi}_2 \cosh\bar{\varphi}_2 h)} - \frac{\bar{\varphi}_2 \cosh\bar{\varphi}_2 z + i\mu \sinh\bar{\varphi}_2 z}{(\alpha_1\bar{\varphi}_2^2 \sinh\bar{\varphi}_2 h + \kappa\bar{\varphi}_2 \cosh\bar{\varphi}_2 h) - i\bar{\varphi}_1(\kappa\mu \sinh\bar{\varphi}_2 h + \mu\alpha_1\bar{\varphi}_2 \cosh\bar{\varphi}_2 h)} \right]$$
$$\times \left(\frac{iA}{k_1 + is - k_{1N}} + \bar{v}'_{1+}(p_m) M_+(p_m) M_-(-k_1 - is)\right) \times e^{-k_1'' x} e^{-sx} ds \tag{70}$$

$$= e^{-k_1'' x}\int_0^\infty s^{1/2} Q(s)\, e^{-sx} ds, \tag{71}$$

where,

$$Q(s) = -\frac{(2k_1'')^{1/2}\kappa\mu\left(\kappa\bar{\varphi}_2 \sinh\bar{\varphi}_2 h + \alpha_1\bar{\varphi}_2^2 \cosh\bar{\varphi}_2(z+h)\right)\zeta(s)}{\pi(\bar{\varphi}_2^2\left(\alpha_1\bar{\varphi}_2 \sinh\bar{\varphi}_2 h + \kappa\cosh\bar{\varphi}_2 h\right)^2 + \mu\bar{\varphi}_1^2\left(\kappa\sinh\bar{\varphi}_2 h + \alpha_1\bar{\varphi}_2 \sinh\bar{\varphi}_2 h\right)^2} \tag{72}$$

and $$\zeta(s) = \frac{iA}{k_1 + is - k_{1N}} + \bar{v}'_{1+}(p_m) M_+(p_m) M_-(-k_1 - is) \tag{73}$$

Now, the integral $\int_0^\infty s^{1/2} Q(s) e^{-sx} ds$ can be evaluated using the result of Ewing et al. (1957), we write

$$\int_0^\infty s^{1/2} Q(s) e^{-sx} ds = \frac{Q(0)\Gamma(3/2)}{x^{3/2}} + \frac{Q'(0)\Gamma(5/2)}{x^{5/2}} + \frac{Q''(0)\Gamma(7/2)}{x^{7/2}} + \dots\dots\dots \tag{74}$$

where $\Gamma(x)$ is Gamma function. Using equation (74) in equation (71)

$$e^{-k_1'' x}\int_0^\infty s^{1/2} Q(s) e^{-sx} ds = \left[\frac{Q(0)\Gamma(3/2)}{x^{3/2}} + \frac{Q'(0)\Gamma(5/2)}{x^{5/2}} + \frac{Q''(0)\Gamma(7/2)}{x^{7/2}} + \dots\dots\dots\dots\right] e^{-k_1'' x} \tag{75}$$

Neglecting first and higher order derivatives of $Q(s)$ at $s = 0$. Equation (75) becomes written as

$$v_{2,1}(x,z) = \frac{Q(0)\Gamma(3/2)}{x^{3/2}} e^{-k_1'' x} \tag{76}$$

where,

$$Q(0) = -\frac{(2k_1'')^{1/2}\kappa\mu\left(\kappa\varphi'_2 \sinh\varphi'_2(z+h) + \alpha_2\varphi_2'^2 \cosh\varphi'_2(z+h)\right)\zeta(0)}{\pi\varphi_2'^2\left(-\alpha_1\varphi'_2 \sinh\varphi'_2 h + \kappa\cosh\varphi'_2 h\right)^2} \tag{77}$$

and $\varphi_2^{''} = \sqrt{k_2^2 - k_1^2}$. Hence, we have

$$v_{2,5}(x,z) = A_1\left[\frac{\kappa\sinh\varphi_2^{'}(z+h) + \alpha_1\varphi_2^{'}\cos\varphi_2^{'}(z+h)}{x^{3/2}}\right]e^{-k_1^{''}x} \tag{78}$$

Where $$A_1 = -\frac{(2k_1^{''})^{1/2}\kappa\mu\Gamma(3/2)\left(\frac{iA}{k_1-k_{1N}} + \bar{v}_{1+}^{'}(p_m)M_+(p_m)M_-(-k_1)\right)}{\pi\varphi_2^{'2}\left(-\alpha_1\varphi_2^{'}\sinh\varphi_2^{'}h + \kappa\cosh\varphi_2^{'}h\right)^2} \tag{79}$$

The equation (78) represents the scattered waves in the lower half of the plane due to the presence of surface impedance in the crustal layer $-H \le z \le 0$. The amplitude of scattered waves is obtained by taking the modulus of equation (79).

NUMERICAL COMPUTATIONS AND DISCUSSION OF RESULTS

The incident Love waves are scattered as well as reflected due to a vertical discontinuous surface layer of earth in the presence of surface impedance. The scattered Love waves given in the equation (78) are of the form

$$A_1\left[\frac{\kappa\sinh\varphi_2^{'}(z+h) + \alpha_1\varphi_2^{'}\cos\varphi_2^{'}(z+h)}{x^{3/2}}\right]e^{-k_1^{''}x},$$

which shows that scattered waves decreases rapidly for large values of x. In above equation, the term $\sinh\varphi_2^{'}(z+h)$ represents the scattered waves due to surface impedance and $\alpha_1\varphi_2^{'}\cos\varphi_2^{'}(z+h)$ represents the scattered waves due to the corner of vertical discontinuity. The scattered waves propagate with the speed of waves in the half space and not with that of the waves in the layer. If there is no surface impedance $(\kappa = 0)$ and width of vertical discontinuity is very small $(\delta \to 0)$, scattered and reflected waves are not obtained. When h = 0, the reflected and transmitted waves propagate with a velocity equal to that of shear waves in the half space. The numerical computations are made by taking $\mu = 2,\ z = -H, \kappa = 0.15\ km,\ h = 0.35\ km,\quad H = 0.50\ km$ and also taking $k_2\delta$ to be small. The graph showing comparison of variation of amplitude versus the wave number of scattered waves by taking the material constant of different sizes have been shown in figure (3). From comparisons it is clear that amplitude of scattered waves depends upon the material constant to some extent.

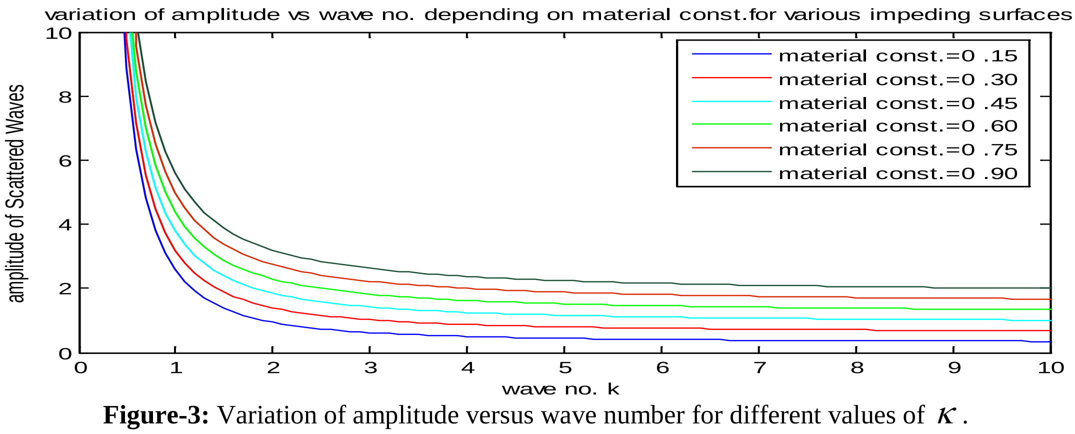

Figure-3: Variation of amplitude versus wave number for different values of κ.

CONCLUSIONS

Scattering of Love waves due to a vertical discontinuous surface layer of earth in the presence of surface impedance has been analysis. The impeding surface affects the propagation of love waves though the layered structure. The graphs show that amplitude of the scattered waves decreases sharply with the small increase in the wave number and also amplitude of reflected waves decreases as the phase velocity increases, which indicates that amplitude of scattered and reflected waves approaches to zero after long interval of time. This explains why the scattered Love waves are considered one of the most destructive seismic waves during earthquake with high intensity on the surface of earth in the presence of surface impedance. The type of material not only significantly affected the behavior of shear waves but thickness of impedance surface also.

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