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INTUITIONISTIC LEFT OPERATOR SEMIGROUP OF AN ORDERED Γ-SEMIGROUPS

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ABSTRACT

In this paper we obtain operator ordered semigroup of an ordered -Γ-semigroups have been made to work by obtaining various relationship between the intuitionistic fuzzy ordered filters of an ordered Γ-semigroups and that of its left operator semigroups. Also we obtain some theorem related to such left operator semigroups

Keywords: *Ordered Γ- semigroup, intuitionistic fuzzy ordered filters, left operator Semigroups.*

1. INTRODUCTION

The concept of a fuzzy set given by L.A. Zadeh in his clasis paper of 1965 [13] has been used by many authors to generalize some of the basic notions of algebra. Fuzzy semigroups have been first considered by N. Kuroki [8], and fuzzy ordered groupoids and ordered semigrous, by Kehayopulu and Tsingelis [6] [7]. The notion of a Γ-semigroup was introduced by Sen [10]. Many classical notions of semigroups have been extended to Γ-semigroups. The concept of intuitionistic fuzzy set was introduced by K. T. Atanassov [2][3][4]. In [11], M. Shabir and A. Khan introduced fuzzy filters in ordered semigroups. In [12] Sujith kumar, Pavel pal, Samith kumar, Majumder and Parimal Das, operator semigroups intheir paper Atanassov's intuitionistic fuzzy ideals of a po- Γ- semigroups. In this paper we obtain left operator ordered semigroup of an ordered -Γ-semigroups and also study about the relation between left operator ordered semigroup and intuitionistic fuzzy ordered filters of an ordered Γ-semigroups. Also we obtain some theorem related to such operator semigroups.

2. PRELIMINARIES

Definition 2.1: Let S be a Γ-semigroup. Let us define a relation ρ on S×Γ as follows: (x, α) ρ (y, β) iff xαs = yβs for all s∈S and γxα = γyβ for all γ∈Γ. Then ρ is an equivalence relation. Let [x,α] denote the equivalence class containing (x,α).Let L={[x,α] : x∈S,α∈Γ}. Then L is a semigroup with respect to the multiplication defined by [x,α][y, β]=[xαy,β]. This semigroup L is called left operator semigroup of the Γ-semigroup S.

Definition 2.2: Let S be a Γ-semigroup. Let us define a relation ρ on S×Γ as follows: (x,α) ρ (y,β) iff xαs = yβs for all s∈S and γxα = γyβ for all γ∈Γ. Then ρ is an equivalence relation. Let [x,α] denote the equivalence class containing (x,α). Let L={[x,α] : x∈S, α∈Γ}. Then L is a semigroup with respect to the multiplication defined by [x,α][y,β]=[xαy,β]. This semigroup L is called left operator semigroup of the Γ-semigroup S.

Definition 2.3: Let (S,Γ,≤) be an ordered Γ-semigroup we define a relation ≤ on L by [a,α] ≤ [b,β] iff aαs ≤ bβs for all s∈S and γaα ≤ γbβ for all γ∈Γ with respect to this relation L becomes ordered Γ-semigroup.

Definition 2.4: If there exists an element [a,α] ∈ L such that aαs = s for all s∈S then [a,α] is called the left unity of S.

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Definition 2.5: For an intuitionistic fuzzy subset $A = <\mu_A,\nu_A>$ of L, define an intuitionistic fuzzy subset $A^+ = <\mu_A^+,\nu_A^+>$ of S by $\mu_A^+(x) = \wedge_{\alpha\in\Gamma} \mu_A([x,\alpha])$ and $\nu_A^+(x) = \vee_{\alpha\in\Gamma} \nu_A([x,\alpha])$, where $x\in S$. For an intuitionistic fuzzy subset $B = <\mu_B,\nu_B>$ of S, define an intuitionistic fuzzy subset $B^{+'} =< \mu_{B^{+'}}, \nu_{B^{+'}} >$ by $\mu_{B^{+'}}([a,\alpha]) = \wedge_{s\in S} \mu_B(a\alpha s)$ and $\nu_{B^{+'}}([a,\alpha]) = \vee_{s\in S} \nu_B(a\alpha s)$ where $[a,\alpha]\in L$

3. LEFT OPERATOR SEMIGROUP OF AN ORDERED Γ-SEMIGROUPS

Theorem 3.1: If $\{A_i/i\in I\}$ is a collection of intuitionistic fuzzy subsets of L. Then

$$(\bigcap_{i\in I} \mu_{Ai}^+) = (\bigcap_{i\in I} \mu_{A_i})^+ \text{ and } (\bigcup_{i\in I} \nu_{Ai}^+) = (\bigcup_{i\in I} \nu_{A_i})^+$$

Proof: Let $x\in S$. Now

$$\begin{aligned}(\bigcap_{i\in I} \mu_{A_i})^+(x) &= \wedge_{\alpha\in\Gamma} [(\bigcap_{i\in I} \mu_{Ai})([x,\alpha])]\\ &= \wedge_{\alpha\in\Gamma} [\wedge_{i\in I} (\mu_{A_i}[x,\alpha])]\\ &= \wedge_{i\in I} [\wedge_{\alpha\in\Gamma} [\mu_{A_i}[x,\alpha])]\\ &= \wedge_{i\in I} [\mu_{Ai}^+(x)] = (\bigcap_{i\in I} \mu_{Ai}^+)(x) \text{ for each } x\in S.\end{aligned}$$

Hence $(\bigcap_{i\in I} \mu_{Ai}^+) = (\bigcap_{i\in I} \mu_{A_i})^+$

Also
$$\begin{aligned}(\bigcup_{i\in I} \nu_{A_i})^+(x) &= \vee_{\alpha\in\Gamma} [(\bigcup_{i\in I} \nu_{A_i})([x,\alpha])]\\ &= \vee_{\alpha\in\Gamma} [\vee_{i\in I} (\nu_{A_i}[x,\alpha])]\\ &= \vee_{i\in I} [\vee_{\alpha\in\Gamma} [\nu_{A_i}([x,\alpha])]]\\ &= \vee_{i\in I} [\nu_{Ai}^+(x)] = (\bigcup_{i\in I} \nu_{Ai}^+)(x) \text{ for each } x\in S\end{aligned}$$

Hence $(\bigcup_{i\in I} \nu_{Ai}^+) = (\bigcup_{i\in I} \nu_{A_i})^+$

Theorem 3.2: If $A = <\mu_A,\nu_A>$ is an intuitionistic fuzzy ordered filter of L, then the intuitionistic fuzzy set $A^+ = <\mu_A^+,\nu_A^+>$ is an intuitionistic fuzzy ordered filter of S.

Proof: Let A be an intuitionistic fuzzy ordered filter of L. Then $\mu_A(1_L) = 1$ & $\nu_A(1_L) = 0$.

Also $\mu_A^+(1_S) = \wedge_{\alpha\in\Gamma} \mu_A[1_S,\alpha] = \wedge_{\alpha\in\Gamma} \mu_A(1_S) = 1.$

$\nu_A^+(1_S) = \vee_{\alpha\in\Gamma} \nu_A[1_S,\alpha] = \wedge_{\alpha\in\Gamma} \nu_A(1_S) = 0.$

So A^+ is non-empty.

$$\begin{aligned}\mu_A^+[a\alpha b] &= \wedge_{\gamma\in\Gamma} \mu_A([a\alpha b,\gamma]) = \wedge_{\gamma\in\Gamma} \mu_A([a,\alpha]\,[b,\gamma])\\ &\le \wedge_{\gamma\in\Gamma} \mu_A([a,\alpha])\\ &= \mu_A([a,\alpha])\\ &\le \wedge_{\gamma\in\Gamma} \mu_A([a,\gamma]) = \mu_A^+(a)\end{aligned}$$

Also $\mu_A^+(a\alpha b) = \wedge_{\gamma\in\Gamma} \mu_A([a\alpha b,\gamma]) = \wedge_{\gamma\in\Gamma} \mu_A([a,\alpha]\,[b,\gamma]).$

$\le \wedge_{\gamma\in\Gamma} \mu_A([b,\gamma]) = \mu_A^+(b).$

and L is the left operator of the ordered Γ-semigroup S.

$\mu_A^+(a\alpha b) = \min\{\mu_A^+(a), \mu_A^+(b)\}$

Similarly $\nu_A^+(a\alpha b) = \vee_{\gamma\in\Gamma} \nu_A([a\alpha b,\gamma])$.

$= \vee_{\gamma\in\Gamma} \nu_A([a,\alpha][b,\gamma])$.

$\geq \vee_{\gamma\in\Gamma} \nu_A([a,\alpha])$

$= \nu_A([a,\alpha]) \geq \vee_{\gamma\in\Gamma} \nu_A([a,\gamma] = \nu_A^+(a)$.

Also $\nu_A^+(a\alpha b) = \vee_{\gamma\in\Gamma} \nu_A([a\alpha b,\gamma])$

$= \vee_{\gamma\in\Gamma} \nu_A([a,\alpha][b,\gamma])$

$\geq \vee_{\gamma\in\Gamma} \nu_A([b,\gamma]) = \nu_A^+(b)$.

$\nu_A^+(a\alpha b) = \max\{\nu_A^+(a), \nu_A^+(b)\}$.

Let a, b∈S be such that a ≤ b. Then $[a,\alpha] \leq [b,\alpha]$, for all $\alpha\in\Gamma$.

Since A is intuitionistic fuzzy ordered filter of L, $\mu_A[a,\alpha] \leq \mu_A([b,\alpha])$, for all $\alpha\in\Gamma$.

This implies $\inf_{\alpha\in\Gamma} \mu_A([a,\alpha]) \leq \inf_{\alpha\in\Gamma} \mu_A([b,\alpha])$.Therefore $\mu_A^+(a) \leq \mu_A^+(b)$.

Now $\nu_A[a,\alpha] \geq \nu_A([b,\alpha])$, for all $\alpha\in\Gamma$.This implies $\mathit{Sup}_{\alpha\in\Gamma} \nu_A([a,\alpha]) \geq \mathit{Sup}_{\alpha\in\Gamma} \nu_A([b,\alpha])$.

Therefore $\nu_A^+(a) \geq \nu_A^+(b)$. Hence $A^+ = <\mu_A^+,\nu_A^+>$ is an intuitionistic fuzzy ordered filter of S.

Theorem 3.3: If $A= <\mu_A,\nu_A>$ is an intuitionistic fuzzy ordered filter of S, then the intuitionistic set $A^{+'} = <\mu_A^{+'},\nu_A^{+'}>$ is an intuitionistic fuzzy ordered filter of L.

Proof: Let A be an intuitionistic fuzzy ordered filter of S. Then $\mu_A(1_S) = 1$ and $\nu_A(1_S) = 0$. Therefore now,

$\mu_{A^{+'}}([1_S,\gamma]) = \wedge_{s\in S} \mu_A(1_S,\gamma_S) = \mu_A(1_S) = 1$ and

$\nu_{A^{+'}}([1_S,\gamma]) = \vee_{s\in S} \nu_A(1_S,\gamma_S) = \nu_A(1_S)=0$.So $A^{+'}$ is non-empty

Let $[a,\alpha], [b,\beta] \in L$

$\mu_{A^{+'}}([a,\alpha], [b,\beta]) = \mu_{A^{+'}}[a\alpha b,\beta] = \wedge_{s\in S} \mu_A(a\alpha b\beta s)$

$\leq \wedge_{s\in S} \mu_A(b\beta s) = \wedge_{s\in S} \mu_A[b,\beta]$

$\mu_{A^{+'}}([a,\alpha], [b,\beta]) = \mu_{A^{+'}}[a\alpha b,\beta] = \wedge_{s\in S} \mu_A(a\alpha b\beta s)$

$= \wedge_{s\in S} (\min\{\mu_A(a), \mu_A(b\beta s)\})$

$= \wedge_{s\in S} (\min\{\mu_A(a), \min\{\mu_A(b), \mu_A(s)\})$

$\leq \wedge_{s\in S} (\min\{\mu_A(a), \mu_A(s)\})$

$= \wedge_{s\in S} \mu_A(a\alpha s) = \wedge_{s\in S} \mu_A[a,\alpha]$.

$\nu_{A^{+'}}([a,\alpha], [b,\beta]) = \nu_{A^{+'}}[a\alpha b,\beta] = \vee_{s\in S} \nu_A(a\alpha b\beta s)$

$\geq \vee_{s\in S} \nu_A(b\beta s) = \wedge_{s\in S} \nu_A[b,\beta]$

$$\nu_{A^{+'}}([a,\alpha],[b,\beta]) = \nu_{A^{+'}}[a\alpha b,\beta] = \vee_{s\in S} \nu_A(a\alpha b\beta s)$$
$$= \vee_{s\in S} (\min\{\nu_A(a), \nu_A(b\beta s)\})$$
$$= \vee_{s\in S} (\min\{\nu_A(a), \min\{\nu_A(b), \nu_A(s)\}\})$$
$$\geq \vee_{s\in S} (\min\{\nu_A(a), \nu_A(s)\})$$
$$= \vee_{s\in S} \nu_A(a\alpha s) = \vee_{s\in S} \nu_A[a,\alpha]$$

Hence $A^{+'}$ is an intuitionistic fuzzy ordered filter of S

Theorem 3.4: Let S be an ordered Γ- semigroup with unities and L be its left operator semigroup. Then there enists an ordered Γ- is isomorphism via inclision preserving $A\to A^{+'}$ between the set of all intuitionistic fuzzy ordered filters of S and the set of all intuitionistic fuzzy ordered filters of L, where $A=<\mu_{A,}\nu_A>$ is an intuitionistic fuzzy ordered filter of S.

Proof: Let A be an intuitionistic fuzzy ordered filter of S. Then clearly A^{+1} is also an intuitionistic fuzzy ordered filter of S. Let $a\in S$. Then

$$(\mu_{A^{+1}})^+(a) = \wedge_{\gamma\in\Gamma} [\mu_{A^{+1}}([a,\gamma])] = \wedge_{\gamma\in\Gamma}\left[\wedge_{s\in S}[\mu_A(a\gamma S)\right] = \wedge_{\gamma\in\Gamma}\left[\wedge_{s\in S}[\mu_A(a)\right] = \mu_A(a).$$

$$(\nu_{A^{+1}})^+(a) = \vee_{\gamma\in\Gamma} [\nu_{A^{+1}}([a,\gamma])] = \vee_{\gamma\in\Gamma}\left[\vee_{S\in S}[\nu_A(a\gamma S)\right] \geq \vee_{\gamma\in\Gamma}\left[\vee_{S\in S}\nu_A(a)\right] = \nu_A(a)$$

Hence $(A^{+'}) \subseteq A$

Let $x\in S$ and let [e, s] be the left unity of R such that $e\delta y=y$ for all $y\in s$

$$\mu_A(x) = \mu_A(e\delta x) \leq \wedge_{s\in S}\mu_A(e\delta s) \leq \wedge_{s\in S}\mu_A(x\delta s) = (\mu_{A^{+1}})(x) = \inf_{\gamma\varepsilon\Gamma}(\mu_{A^{+1}})(x) = (\mu_{A^{+1}})^+(x)$$

Also $\nu_A(x)=\nu_A(e\delta x) \geq \vee_{s\varepsilon S}\mu_A(e\delta s) \geq \vee_{s\varepsilon S}\mu_A(x\delta s) = (\nu_A^{+'})(x) = \inf_{\gamma\varepsilon\Gamma}(\nu_{A^{+1}})(x) = (\nu_{A^{+1}})^+(x).$

Hence $A \subseteq (_A^{+'})^+$ Let $b\in s$, $\alpha\in\Gamma$ and $[b,\alpha]\in L$.

$$(\mu_{A^{+1}})^{+'}(b,\alpha) = \wedge_{s\in S}\mu_A^+(e\alpha s) = \wedge_{s\in S}\left[\wedge_{\gamma\in\Gamma}\mu_A[b\alpha s,\gamma]\right] = \wedge_{s\in S}\left[\wedge_{\gamma\in\Gamma}\mu_A[b,\alpha][s,\gamma]\right]$$
$$\leq \wedge_{s\in S}\left[\wedge_{\gamma\in\Gamma}\mu_A[[b,\quad\alpha]]\right] = \mu_A([b,\alpha])$$

$$(\nu_{A^+})^{+'}(b,\alpha) = \vee_{s\in S}\nu_A^+(b\alpha s) = \vee_{S\varepsilon S}\left[\vee_{\gamma\in\Gamma}\nu_A([b\alpha s,\gamma])\right] = \vee_{s\in S}\left[\vee_{\gamma\in\Gamma}\nu_A([b,\alpha][s,\gamma])\right]$$
$$\geq \vee_{s\in S}\left[\vee_{\gamma\in\Gamma}\nu_A([b,\alpha])\right] = \nu_A[b,\alpha]$$

$\therefore (A^+)^{+'} \subseteq A$

Also Let $[a,\beta]\in L$. Let $[e,\delta]$ be the left unity of L Such that $[e,\delta][a,\beta] = [a,\beta]$

$$\mu_A([a,\beta]) = \mu_A([e,\delta][a,\beta]) \leq \wedge_{s\in S}[\mu_A([S,\delta][a,\beta])] \leq \wedge_{s\in S}\left[\wedge_{\gamma\in\Gamma}[\mu_A([S,\gamma][a,\gamma])]\right]$$
$$= (\mu_A^+)^{+'}([a,\beta])$$

Also

$$\nu_A([a,\beta]) = \nu_A([e,\delta][a,\beta]) \geq \bigvee_{s \in S} \nu_A([S,\delta][a,\beta]) \geq \bigvee_{s \in S}\left[\bigvee_{\gamma \in \Gamma} \nu_A([S,\gamma][a,\beta])\right]$$
$$= (\nu_A{}^{+})^{+'}([a,\beta])$$

Hence $A \subseteq (A^{+})^{+'}$

Hence the correspondence $A \mapsto A^{+}$ is bijection Now let A_1, A_2 be intuitionistic fuzzy ordered filter of S such that $A_1 \subseteq A_2$ Then

$$\mu_A{}^{+'}([a,\alpha]) = \left[\bigwedge_{s \in S} \mu_{A_1}(a\alpha s)\right] \leq \left[\bigwedge_{s \in S} \mu_{A_2}(a\alpha s)\right] = \mu_{A2}{}^{+'}([a,\alpha]) \text{ for all } [a,\alpha] \in \Gamma$$

$$\nu_{A1}{}^{+1}([a,\alpha]) = \left[\bigvee_{s \in S} \nu_{A_1}(a\alpha s)\right] \geq \left[\bigvee_{s \in S} \nu_{A_2}(a\alpha s)\right] = \nu_{A2}{}^{+'}([a,\alpha]) \text{ for all} [a,\alpha] \in \Gamma$$

Therefore $A_1{}^{+'} \subseteq A_2{}^{+'}$. Now,

$$(\mu_{A1}{}^{+'})^{+}(a) = \bigwedge_{\alpha \in \Gamma} \mu_{A_1}{}^{+'}([a,\alpha]) = \bigwedge_{\alpha \in \Gamma}\left[\bigwedge_{s \in S} \mu_{A_1}(a\alpha s)\right] \leq \bigwedge_{\alpha \varepsilon \Gamma}\left[\bigwedge_{s \in S} \mu_{A_2}(a\alpha s)\right]$$
$$= \bigwedge_{\alpha \in \Gamma} \mu_{A_2}{}^{+'}([a,\alpha]) = (\mu_{A2}{}^{+'})^{+}(a) \text{ for all } a \in L$$

$$(\nu_{A1}{}^{+'})^{+}(a) = \bigvee_{\alpha \in \Gamma} \nu_{A_1}{}^{+1}([a,\alpha]) = \bigvee_{\alpha \varepsilon \Gamma}\left[\bigvee_{s \in S} \nu_{A_1}(a\alpha s)\right] \geq \bigvee_{\alpha \varepsilon \Gamma}\left[\bigvee_{s \in S} \nu_{A_2}(a\alpha s)\right]$$
$$= \bigvee_{\alpha \in \Gamma} \nu_{A_2}{}^{+1}([a,\alpha]) = (\nu_{A2}{}^{+'})(a) \text{ for all } a \in L$$

So $(A_1{}^{+'})^{+} \subseteq (A_2{}^{+'})^{+}$

Also $A_1 \subseteq A_2$

$$\mu_{A1}{}^{+}(a) = \bigwedge_{\gamma \varepsilon \Gamma} \mu_{A_1}([a,\gamma]) \leq \bigwedge_{\gamma \varepsilon \Gamma} \mu_{A_2}([a,\gamma]) = \mu_{A2}{}^{+}(a) \text{ for all } a \in L$$

$$\nu_{A1}{}^{+}(a) = \bigvee_{\gamma \varepsilon \Gamma} \nu_{A_1}([a,\gamma]) \geq \bigvee_{\gamma \varepsilon \Gamma} \nu_{A_2}([a,\gamma]) = \nu_{A2}{}^{+}(a) \text{ for all } a \in L$$

Therefore $A_1{}^{+} \subseteq A_2{}^{+}$.Hence $(A_1{}^{+})^{+'} \subseteq (A_2{}^{+})^{+'}$

So the mapping $A \to A^{+'}$ is an ordered Γ- isomorphism

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