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SPLIT AND STRONG SPLIT EDGE DETOUR DOMINATION NUMBER OF GRAPHS

A. MAHALAKSHMI*[1], K. PALANI[2] AND S. SOMASUNDARAM[3]

[1]Assistant Professor Department of Mathematics,
Sri Sarada College for Women, Tirunelveli - 627011, Tamil Nadu, India.

[2]Assistant Professor Department of Mathematics,
A.P.C.Mahalaxmi College for Women, Thoothukudi – 628002, Tamil Nadu, India.

[3]Professor, Department of Mathematics,
Manonmaniam Sundaranar University, Tirunelveli - 627012, Tamil Nadu, India.

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ABSTRACT

In this paper, we introduce the new concept split and strong split edge detour domination number of a graph and obtain the split and strong split edge detour domination number for some well known graphs.

Keywords: *Domination, Edge detour, Edge detour domination, Split and Strong split edge detour domination.*

1. INTRODUCTION

The concept of domination was introduced by Ore and Berge [5]. Let G be a finite, undirected connected graph with neither loops nor multiple edges. A subset D of V(G) is a dominating set of G if every vertex in V-D is adjacent to at least one vertex in D. The minimum cardinality among all dominating sets of G is called the domination number $\gamma(G)$ of G. Throughout this paper, only connected graphs with at least two vertices are considered. For basic definitions and terminologies, refer Harary [1]. For vertices u and v in a connected graph G, the detour distance D(u,v) is the length of longest u-v path in G. A u-v path of length D(u,v) is called a u-v detour. A subset S of V is called a detour set if every vertex in G lie on a detour joining a pair of vertices of S. The detour number dn(G) of G is the minimum order of a detour set and any detour set of order dn(G) is called a detour basis of G. These concepts were studied in [3]. A subset S of V is called an edge detour set of G if every edge in G lie on a detour joining a pair of vertices of S. The edge detour number $dn_1(G)$ of G is the minimum order of its edge detour sets and any edge detour set of order dn_1 is an edge detour basis. A graph G is called an edge detour graph if it has an edge detour set. Edge detour graphs were introduced and studied in [7]. A split (G,D)-set S of a graph G is said to be a split (G,D)-set of G if the subgraph induced by V – S is disconnected. A (G,D)-set S of a graph G is said to be a strong split (G,D)-set of G if the subgraph induced by V – S is totally disconnected. Split and Strong split (G, D)-set were introduced in [8]. An edge detour dominating set is a subset S of V(G) which is both a dominating and an edge detour set of G. An edge detour dominating set of G is minimal if no proper subset of S is an edge detour dominating set of G. An edge detour dominating set S is said to be minimum if there is no other edge detour dominating set S’ such that |S’|<|S|. The smallest cardinality of an edge detour dominating set of G is called the edge detour domination number of G. It is denoted $\gamma_{eD}(G)$. Any edge detour dominating set of G of cardinality $\gamma_{eD}(G)$ is called a γ_{eD}-set of G. Edge detour domination number of a graph were introduced by A.Mahalakshmi, K.Palani, and S.Somasundaram [5].

The following theorems are from [9].

Theorem 1.1: Every end vertex of an edge detour graph G belongs to every edge detour set of G. Also, if the set of all end vertices of G is an edge detour set, then S is the unique edge detour basis for G.

Theorem 1.2: If G is an edge detour graph of order p>3 such that {u,v} is an edge detour basis of G, then u and v are not adjacent.

Corresponding Author: A. Mahalakshmi[1],*
[1]Assistant Professor Department of Mathematics,
Sri Sarada College for Women, Tirunelveli - 627011, Tamil Nadu, India.

Theorem 1.3: If T is a tree with k end vertices, then $dn_1(T) = k$.

The following theorems are from [5].

Theorem 1.4: K_p is an edge detour dominating graph and $\gamma_{eD}(K_p) = 3$ for p>3.

Theorem 1.5: $\gamma_{eD}(K_{1,n}) = n$.

Theorem 1.6: $$\gamma_{eD}(P_n) = \begin{cases} \left\lceil \frac{n-4}{3} \right\rceil + 2 \text{ if } n \geq 5 \\ 2 \text{ if } n = 2, 3 \text{ or } 4. \end{cases}$$

The following concepts are from [1].

Theorem 1.7: A vertex and an edge are said to cover each other if they are incident. A set of vertices which covers all the edges of a graph G is called a vertex cover for G, while a set of edges which covers all the vertices is an edge cover. The smallest number of vertices in any vertex cover for G is called its vertex covering number and is denoted by $\alpha_0(G)$ or α_0.

Theorem 1.8: A set of vertices in a graph G is independent if no two vertices are adjacent in G. The largest number of vertices in such set is called the vertex independence number of G and it is denoted by $\beta_0(G)$ or β_0.

Theorem 1.9: If G is a graph with p vertices, then $\alpha_0(G) + \beta_0(G) = p$.

2. SPLIT EDGE DETOUR DOMINATION NUMBER OF GRAPHS

Definition 2.1: An edge detour dominating set S of a graph G is said to be a split edge detour dominating set of G if the subgraph induced by V-S is disconnected.

Example 2.2: For the Figure 2.1, $\{v_1, v_4\}$ is a minimum split edge detour dominating set of G. Therefore, $\gamma^s_{eD}(G) = 2$.

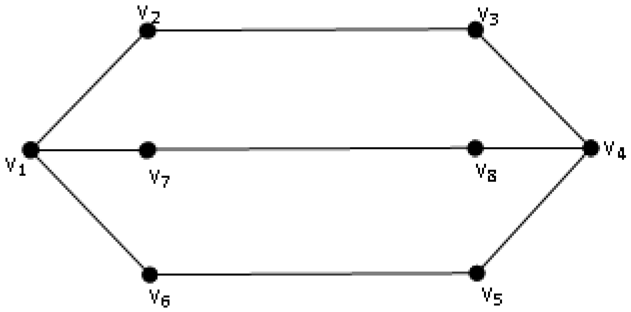

Figure 2.1

Let ζ ' denote the collection of all graphs having atleast one split edge detour dominating set.

Definition 2.3: Let $G \in \zeta$ 'then, the minimum cardinality of all split edge detour dominating set of G is called the split edge detour dominating number of G. It is denoted by $\gamma^s_{eD}(G)$. A split edge detour dominating set of minimum cardinality $\gamma^s_{eD}(G)$ is called γ^s_{eD} – set of G.

Observation 2.4:
1. A complete graph has no split edge detour dominating set, as the vertex set is the unique edge detour dominating set of G.
2. In general, all graphs need not have split edge detour dominating sets.
3. Every split edge detour dominating set is a edge detour dominating set of G. Therefore, $\gamma^s_{eD}(G) \geq \gamma_{eD}(G) \geq \max\{\gamma(G), dn_1(G)\}$.
4. Every split edge detour dominating set is a split dominating set of G. Hence, $\gamma^s_{eD}(G) \geq \gamma_s(G)$.
5. $2 \leq \gamma_{eD}(G) \leq \gamma^s_{eD}(G) \leq p$.

Proposition 2.5: Let $G \in \zeta'$. Then, an edge detour dominating set S of G is a split edge detour dominating set of G if and only if there exists two vertices $w_1, w_2 \in$ V-S such that every w_1, w_2 path contains a vertex of S.

Proof: Suppose S is a split edge detour dominating set of G. Then, V-S is not connected and has at least two components. Choose $w_1, w_2 \in$ V-S such that they are in two different components of V-S. Therefore, every w_1-w_2 path contains a vertex of S. Converse is obvious.

Proposition 2.6: For n>3, $\gamma^s_{eD}(P_n) = \gamma_{eD}(P_n)$.

Proof: Let n>3. Clearly, for every minimum edge detour dominating set S of P_n, V-S is disconnected. Therefore, every γ_{eD} - set of P_n is a split edge detour dominating set of P_n and $\gamma^s_{eD}(P_n) \leq \gamma_{eD}(P_n)$. By Observation 2.4(4), $\gamma^s_{eD}(P_n) = \gamma_{eD}(P_n)$.

Proposition 2.7: For n>3, $\gamma^s_{eD}(C_n) = \gamma_{eD}(C_n)$.

Proof: Let n>3. Clearly, for every minimum edge detour dominating set S of C_n, V-S is disconnected. Therefore, every γ_{eD} - set of C_n is a split edge detour dominating set of C_n and $\gamma^s_{eD}(C_n) \leq \gamma_{eD}(C_n)$. By observation 2.4(4), $\gamma^s_{eD}(C_n) = \gamma_{eD}(C_n)$.

Proposition 2.8: Let m, n ≥ 2. Then, $\gamma_e{}^s{}_D(K_{m,n}) = \min\{m, n\}$.

Proof: Let U, W be the partition of $V(K_{m,n})$ with |U| = m and |W| = n. Clearly, U and W are split edge detour dominating sets of $K_{m,n}$. Let S be a split edge detour dominating set of $K_{m,n}$. Then, S is not a proper subset of U or W, otherwise V-S is connected. If U or W is a proper subset of S, then S is not a minimal split edge detour dominating set of G. Therefore, either U or W is the only minimal split edge detour dominating set of G. Hence, $\gamma_e{}^s{}_D(K_{m,n}) = \min\{m, n\}$.

3. STRONG SPLIT EDGE DETOUR DOMINATION NUMBER OF GRAPHS

Definition 3.1: An edge detour dominating set S of a graph G is said to be a strong split edge detour dominating set of G if the subgraph induced by V-S is totally disconnected.

Example 3.2:

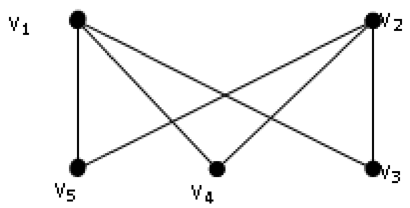

Figure-3.1

Here, S = $\{v_1, v_2\}$ is a strong split edge detour dominating set of G.

Observation 3.3:
1. A complete graph has no strong split edge detour dominating set.
2. All graphs need not have strong split edge detour dominating set.

Let ζ'' denote the collection of all graphs having atleast one strong split edge detour dominating set.

Definition 3.4: Let $G \in \zeta''$. Then, the minimum cardinality of all strong split edge detour dominating sets of G is called the strong split edge detour domination number of G. It is denoted by $\gamma^{ss}_{eD}(G)$. A strong split edge detour dominating set of $\gamma^{ss}_{eD}(G)$ is called γ^{ss}_{eD} – set of G.

Observation 3.5: Let $G \in \zeta''$.
1. Every strong split edge detour dominating set is a split edge detour dominating set of G and an edge detour dominating set of G.
   Therefore, $\gamma^{ss}_{eD}(G) \geq \gamma^s_{eD}(G) \geq \gamma_{eD}(G) \geq \max\{\gamma(G), dn_1(G)\}$.

2. Every strong split edge detour dominating set is a strong split dominating set of G and a split dominating set of G. Hence, $\gamma^{ss}_{eD}(G) \geq \gamma^{s}_{eD}(G) \geq \gamma_s(G)$.
3. $2 \leq \gamma_{eD}(G) \leq \gamma^{s}_{eD}(G)$.

Proposition 3.6: Let $G \in \zeta$ ". Then, $\gamma^{ss}_{eD}(G) \geq \alpha_0(G)$.

Proof: Let S be a $\gamma^{ss}_{eD}(G)$ set of G. Then, V-S is totally disconnected and hence independent. Therefore, $|V\text{-}S| \leq \beta_0(G)$. That is, $p-\gamma^{ss}_{eD}(G) \leq \beta_0(G) = p - \alpha_0(G)$. Hence, $\gamma^{ss}_{eD}(G) \geq \alpha_0(G)$.

Proposition 3.7: Let $G \in \zeta$ ". Then, an edge detour dominating set S of G is a strong split edge detour dominating set of G if and only if for $w_1, w_2 \in$ V-S, every w_1-w_2 path contains a vertex of S.

Proof: Suppose S is a strong split edge detour dominating set of G. Then, V-S is an independent set. Hence, every path joining two vertices of V-S contains a vertex of S. Conversely, let S be an edge detour dominating set of G. If two vertices $u,v \in$ V-S are adjacent, then the edge uv is a u-v path in the subgraph induced by V-S and it contains no vertex of S. This is a contradiction to our assumption and hence V-S is totally disconnected. Hence, S is a strong split edge detour dominating set of G.

Proposition 3.8: For n>3, $\gamma^{ss}_{eD}(P_n) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil \text{if } n \text{ is odd} \\ \frac{n}{2} + 1 \text{if } n \text{ is even} \end{cases}$

Proof: Let n>3 and $P_n = (v_1, v_2, \ldots, v_n)$.

Case-1: n is odd.

Let $S = \{v_1, v_3, \ldots, v_n\}$. If $w = v_i \in$ V-S, then $v_{i-1}, v_{i+1} \in S$ and w lies in the $v_{i-1}v_i v_{i+1}$ edge detour joining v_{i-1} and v_{i+1}.

Further, it is dominated by both v_{i-1} and v_{i+1}. Therefore, S is an edge detour dominating set of P_n. By construction of S, no two vertices of V-S are adjacent. Therefore, S is a strong split edge detour dominating set of P_n and so

$$\gamma^{ss}_{eD}(P_n) \leq |S| = \left\lceil \frac{n}{2} \right\rceil \quad (1).$$

Further, as every edge detour dominating set contains the end vertices of the path, the maximum cardinality of V-S, where S is a strong split edge detour dominating set of P_n is $\left\lfloor \frac{n}{2} \right\rfloor$. Therefore, $|V\text{-}S| \leq \left\lfloor \frac{n}{2} \right\rfloor$.

That is, $|S| \geq |V| - \left\lfloor \frac{n}{2} \right\rfloor = \left\lceil \frac{n}{2} \right\rceil$, as n is odd. Therefore,

$$\gamma^{ss}_{eD}(P_n) \geq \left\lceil \frac{n}{2} \right\rceil \quad (2)$$

From, (1) and (2) $\gamma^{ss}_{eD}(P_n) = \left\lceil \frac{n}{2} \right\rceil$.

Case-2: n is even.

Let $S = \{v_1, v_3, \ldots, v_{n-1}, v_n\}$. Since $v_1, v_n \in S$, if $w = v_i \in$ V-S, then $v_{i-1}, v_{i+1} \in S$ and w lies in the $v_{i-1}v_iv_{i+1}$ edge detour joining v_{i-1} and v_{i+1}. Further, it is dominated by both v_{i-1} and v_{i+1}. Therefore, S is an edge detour dominating set of P_n. By the construction of S, no two vertices of V-S are adjacent. Therefore, S is a strong split edge detour dominating set of Pn and

$$\gamma^{ss}_{eD}(P_n) \leq |S| = \frac{n}{2} + 1. \quad (3)$$

Further, as every edge detour dominating set contains the end vertices of the path, the maximum cardinality for V-S, where S is a strong split edge detour dominating set of P_n, is $\frac{n}{2}$ - 1.

Therefore, $|V\text{-}S| \leq \frac{n}{2} - 1$. That is, $|S| \geq \frac{n}{2} + 1$.

$$\text{Then, } \gamma^{ss}_{eD}(P_n) \geq |S| = \frac{n}{2} + 1. \quad (4)$$

Hence, from (3) and (4), if n is even, then $\gamma^{ss}_{eD}(P_n) = \frac{n}{2} + 1$.

Theorem 3.9: For n>3, $\gamma^{ss}_{eD}(C_n) = \left\lceil \frac{n}{2} \right\rceil$.

Proof: Let n>3 and $C_n=(v_1,v_2,\ldots,v_n,v_1)$

Case-1: n is odd. Let S = $\{v_1, v_3,\ldots,v_n\}$ is a strong split edge detour dominating set of C_n. If $w = v_i \in V\text{-}S$, then $v_{i-1},v_{i+1} \in S$ and w lies in the $v_{i-1}v_iv_{i+1}$ edge detour joining v_{i-1} and v_{i+1}. Further, it is dominated by both v_{i-1} and v_{i+1}. Therefore, S is an edge detour dominating set of C_n. By construction of S, no two vertices of V-S are adjacent. Therefore, S is a strong split edge detour dominating set of C_n and so

$$\gamma^{ss}_{eD}(C_n) \leq |S| = \left\lceil \frac{n}{2} \right\rceil \quad (1)$$

Further, if S is a strong split edge detour dominating set of Cn, then V-S is independent and the maximum cardinality for V-S is $\left\lfloor \frac{n}{2} \right\rfloor$.Therefore,

$$\gamma^{ss}_{eD}(C_n) \geq \left\lceil \frac{n}{2} \right\rceil \quad (2)$$

From (1) and (2), if n is odd, then $\gamma^{ss}_{eD}(Cn) = \left\lceil \frac{n}{2} \right\rceil$.

Case-2: n is even.

Clearly, S = $\{v_1, v_2, \ldots, v_{n-1}, v_n\}$is a strong split edge detour dominating set of C_n and

$$\gamma^{ss}_{eD}(C_n) \leq |S| = \frac{n}{2}. \quad (3)$$

Further, if S is a strong split edge detour dominating set of C_n, then V-S is independent and the maximum cardinality for V-S is $\frac{n}{2}$.

$$\text{Therefore, } \gamma^{ss}_{eD}(C_n) \geq \frac{n}{2} \quad (4)$$

From (3) and (4), $\gamma^{ss}_{eD}(C_n) = \frac{n}{2}$.

Hence, for n > 3, $\gamma^{ss}_{eD}(C_n) = \frac{n}{2}$.

Theorem 3.10: For m, n≥2, $\gamma^{ss}_{eD}(K_{m,n})$ = min{m, n}.

Proof: Let U, W be the partition of $V(K_{m,n})$ with |U| = m and | W | = n. Obviously, U and W are strong split edge detour dominating sets of $K_{m,n}$. As, V-S is independent, S not contain part of U as well as part of W. If U or W is a proper subset of S, then S is not a minimal strong split edge detour dominating sets of $K_{m,n}$. Therefore, U and w are the only minimal strong split edge detour dominating sets of $K_{m,n}$. Hence, $\gamma^{ss}_{eD}(K_{m,n})$ = min {|U|, |W|} = min{m, n}.

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