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GENERALIZED M-SERIES AND MULTIVARIABLE H-FUNCTION ASSOCIATED WITH PATHWAY TRANSFORM

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ABSTRACT

The aim of this paper is to obtain a pathway transform or $\mathcal{P}$- transform of the product of generalized M-series and multivariable H-function with general arguments. As this transform is generalization of many integral transforms and Generalized M-series and multivariable H-function are general in nature. These results provide a number of new results on specializing of the parameters. In the later text we have also given some interesting special cases related to the main results.

Keywords: *Pathway transform, Generalized M-series, Multivariable H-function, Mittag Leffler function, Wright function, Hyper geometric function, H-function, Lauricella function.*

Mathematics subject classification: *44A20, 33C20, 33C60, 33E12, 33C65.*

1. INTRODUCTION

Kumar and Kilbas [11] introduced the $\mathcal{P}$ - transform or pathway transform as follows:

$$(\mathcal{P}_{\eta}^{\sigma,\alpha,\tau} f)(x) = \int_0^{\infty} D_{\sigma,\alpha}^{\eta,\tau}(x\,t)\, f(t)\, dt, \quad x > 0, \tag{1.1}$$

where $D_{\sigma,\alpha}^{\eta,\tau}(x)$ denotes the Kernel-function.

$$D_{\sigma,\alpha}^{\eta,\tau}(x) = \int_0^{\left[\frac{1}{a(1-\tau)}\right]^{\frac{1}{\sigma}}} y^{\eta-1}\left[1 - a(1-\tau)y^{\sigma}\right]^{\frac{1}{1-\tau}} e^{-x\,y^{-\alpha}}\, dy, \quad x > 0, \tag{1.2}$$

with $\eta \in C$, $\alpha > 0$, $\sigma > 0$, $a > 0$, $\tau < 1$, then (1.2) is called type -1 $\mathcal{P}$-transform. If we take

$$D_{\sigma,\alpha}^{\eta,\tau}(x) = \int_0^{\infty} y^{\eta-1}\left[1 + a(\tau-1)y^{\sigma}\right]^{-\frac{1}{\tau-1}} e^{-xy^{-\alpha}} dy, \quad x > 0, \tag{1.3}$$

for $\eta \in C$, $\alpha > 0$, $a > 0$, $\sigma \in R$, $\tau > 1$, then (1.3) is called type -2 $\mathcal{P}$- transform.

Both the types of $\mathcal{P}$-transform are defined in the space $\mathbb{L}_{\eta,r}(0,\infty)$, consisting the Lebesgue measurable complex valued functions f for which

$$\|f\|_{\eta,r} = \left\{\int_0^{\infty} |t^{\eta} f(t)|^r \frac{dt}{t}\right\}^{\frac{1}{r}} < \infty, \quad 1 \leq r < \infty, \eta \in \mathbb{R}. \tag{1.4}$$

The pathway model of Mathai [6], Mathai and Haubold [7] is considered to obtain $\mathcal{P}$-transforms. When $\alpha = 1, a = 1$ and $\tau \to 1$, the $\mathcal{P}$-transforms reduce to the Krätzel transform, given by Krätzel [12] in the form:

$$K_{\eta}^{(\sigma)} f(x) = \int_0^{\infty} Z_{\sigma}^{\eta}(xt)\, f(t) dt, \qquad x > 0, \tag{1.5}$$

where $Z_{\sigma}^{\eta}(x)$ is the Kernel function of the Krätzel transform and it is defined by

$$Z_{\sigma}^{\eta}(x) = \int_0^{\infty} y^{\eta-1}\, e^{-y^{\sigma} - x\,y^{-1}}\, dy \tag{1.6}$$

Krätzel transform and its generalized forms were studied by many mathematicians. Glaeske *et al.* [13] studied generalized Krätzel transform and established its composition formulae with fractional calculus(FC) operators on the spaces of $\mathcal{F}_{p,\mu}$ and $\mathcal{F}'_{p,\mu}$. Bonilla et al. [9, 10] considered the Krätzel transform in the space $\mathcal{F}_{p,\mu}$ and $\mathcal{F}'_{p,\mu}$. Kilbas *et al.* [1] proposed the asymptotic representation for the modified Krätzel function. Kilbas *et al.* [2] considered the Krätzel function in (1.6) for all values of σ and transform it in terms of Fox's H-function. For $\alpha = 1$, $a = 1$, $\sigma = 1$ and $\tau \to 1$, $\mathcal{P}$-transform of type-1 and type-2 reduces to the Meijer transform. when $\alpha = 1$, $a = 1$, $\sigma = 1$ and $\tau \to 1$ along with x replaced by $t^2/4$ in (1.2) and (1.3), reduces to modified Bessel function of third kind or Mc - Donald function [5. sect. 7. 2.2].

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We can observe that,

$$\lim_{\tau\to 1} D_{1,1}^{\eta,\tau}(t^2/4) = 2\left(\frac{t}{2}\right)^{\eta} K_{-\eta}(t), \tag{1.7}$$

where $K_{-\eta}(t)$ denotes the modified Bessel function of third kind or Mc-Donald function. Composition formulae of (1.1) with fractional operators are given by Kilbas and Kumar [3]. Ghiya [19] established some formulae of $\mathcal{P}$-transform with the product of H-function and a general class of polynomial.

Sharma and Jain [18] defined Generalized M -Series as given:

$$\underset{p',\,q'}{\overset{\alpha',\,\beta'}{M}}(z) = \underset{p',\,q'}{\overset{\alpha',\,\beta'}{M}}\left(a_1,\ldots,a_{p'};\ b_1,\ldots,b_{q'};z\right) = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_{p'})_n}{(b_1)_n \cdots (b_{q'})_n} \frac{z^n}{\Gamma(\alpha' n+\beta')} \tag{1.8}$$

where $z,\ \alpha',\ \beta' \in C$, $R(\alpha')>0$; $(a_i)_n$ $(i = 1,\ldots, p')$ and $(b_j)_n$ $(j = 1,\ldots,q')$ are the Pochhammer symbols. The series (1.8) is defined when none of the parameters $(b_j)_n$ $(j= 1,\ldots,q')$ is a negative integer or zero; if any numerator parameter a_i is a negative integer or zero then the series terminates to a polynomial in z. The series in (1.8) is convergent for all z if $p' \le q'$, it is convergent for $|z| < \delta = \alpha'^{\alpha'}$ when $p'= q' + 1$ and divergent if $p' > q'+ 1$. When $p'= q'+ 1$ and $|z| = \delta$, the series is convergent on conditions depending on the parameters.

The multivariable H-function was introduced by Srivastava, and Panda [16] and defined as

$$H_{p,q\,:\,p_1,q_1;\ldots;p_r,q_r}^{o,\,n\,:\,m_1,n_1;\ldots;m_r,n_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\left|\begin{matrix} \left(a_j;\mu_j',\ldots,\mu_j^{(r)}\right)_{1,p}: & \left(c_j',\chi_j'\right)_{1,p_1};\ldots; & \left(c_j^{(r)},\chi_j^{(r)}\right)_{1,p_r} \\ \left(b_j;\psi_j',\ldots,\psi_j^{(r)}\right)_{1,q}: & \left(d_j',\phi_j'\right)_{1,q_1};\ldots; & \left(d_j^{(r)},\phi_j^{(r)}\right)_{1,q_r} \end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r} \mu_1(\xi_1),\ldots,\mu_r(\xi_r)\,\theta(\xi_1,\ldots,\xi_r)\, z_1^{\xi_1},\ldots,z_r^{\xi_r}\,d\xi_1,\ldots,d\xi_r, \tag{1.9}$$

where $\omega = \sqrt{-1}$,

$$\mu_i(\xi_i) = \frac{\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)} - \phi_j^{(i)}\xi_i\right)\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}+\chi_j^{(i)}\xi_i\right)}{\prod_{j=m_i+1}^{q_i}\Gamma\left(1-d_j^{(i)}+\phi_j^{(i)}\xi_i\right)\prod_{j=n_i+1}^{p_i}\Gamma\left(c_j^{(i)}-\chi_j^{(i)}\xi_i\right)}, \quad \forall\, i \in \{1,\ldots,r\} \tag{1.10}$$

$$\theta(\xi_1,\ldots,\xi_r) = \frac{\prod_{j=1}^{n}\Gamma\left(1-a_j+\sum_{i=1}^{r}\mu_j^{(i)}\xi_i\right)}{\prod_{j=n+1}^{p}\Gamma\left(a_j-\sum_{i=1}^{r}\mu_j^{(i)}\xi_i\right)\prod_{j=1}^{q}\Gamma\left(1-b_j+\sum_{i=1}^{r}\psi_j^{(i)}\xi_i\right)}, \tag{1.11}$$

$$|\arg(z_i)| < \frac{1}{2}\Omega_i\pi,$$

Where

$$\Omega_i = \sum_{j=1}^{n}\mu_j^{(i)} - \sum_{j=n+1}^{p}\mu_j^{(i)} - \sum_{j=1}^{q}\psi_j^{(i)} + \sum_{j=1}^{ni}\chi_j^{(i)} - \sum_{j=ni+1}^{pi}\chi_j^{(i)} + \sum_{j=1}^{mi}\phi_j^{(i)} - \sum_{j=mi+1}^{qi}\phi_j^{(i)} > 0. \tag{1.12}$$

Further detailed account of the multivariable H-function can be seen in the book by Srivastava *et al.* [14]. Throughout this paper it is assumed that this function satisfies the conditions given in this book.

2. MAIN RESULTS

Theorem 1: Let $f \in L_{\eta,r}(0,\infty)$, $z, z_1,\ldots,z_r$, $\eta \in C$, $\alpha > 0$, $h, h_1,\ldots,h_r > 0$, $\tau < 1$ in the type -1 $\mathcal{P}$-transform for $\sigma > 0$, then

$$\mathcal{P}_{\eta}^{\sigma,\alpha,\tau}\left[\underset{p',q'}{\overset{\alpha',\beta'}{M}}\left(zx^h\right)H_{p,q\,:\,p_1,q_1;\ldots;p_r,q_r}^{o,\,n\,:\,m_1,n_1;\ldots;m_r,n_r}\left[z_1x^{h_1},\ldots,z_rx^{h_r}\right]\right]$$

$$= \sum_{n=0}^{\infty}\frac{(a_1)_n\cdots(a_{p'})_n}{(b_1)_n\cdots(b_{q'})_n}\,\frac{z^n\,\Gamma\left(1+\frac{1}{1-\tau}\right)}{\sigma\, x^{nh+1}\,[a(1-\tau)]^{\frac{\eta+\alpha(nh+1)}{\sigma}}\,\Gamma(\alpha' n+\beta')}$$

$$H_{p+2,q+1\,:\,p_1,q_1;\ldots;p_r,q_r}^{o,\,n+2\,:\,m_1,n_1;\ldots;m_r,n_r}\left[\begin{matrix}\frac{z_1}{x^{h_1}[a(1-\tau)]^{\frac{\alpha h_1}{\sigma}}} \\ \vdots \\ \frac{z_r}{x^{h_r}[a(1-\tau)]^{\frac{\alpha h_r}{\sigma}}}\end{matrix}\left|\begin{matrix}(a_j;\mu_j',\ldots,\mu_j^{(r)})_{1,p}, & [-nh;h_1,\ldots,h_r], \\ (b_j;\psi_j',\ldots,\psi_j^{(r)})_{1,q}, & \left[\frac{1}{\tau-1}-\frac{\eta+\alpha(nh+1)}{\sigma};\frac{\alpha h_1}{\sigma},\ldots,\frac{\alpha h_r}{\sigma}\right]\end{matrix}\right.\right.$$

$$\left.\left.\begin{matrix}\left[1-\frac{\eta+\alpha(nh+1)}{\sigma},\frac{\alpha h_1}{\sigma},\ldots,\frac{\alpha h_r}{\sigma}\right]: & \left(c_j',\chi_j'\right)_{1,p_1};\ldots;\left(c_j^{(r)},\chi_j^{(r)}\right)_{1,p_r} \\ - \qquad : & \left(d_j',\phi_j'\right)_{1,q_1};\ldots;\left(d_j^{(r)},\phi_j^{(r)}\right)_{1,q_r}\end{matrix}\right.\right], \tag{2.1}$$

provided that $R\left(1+\frac{1}{1-\tau}\right)>0, \quad R(\alpha' n+\beta')>0$.

Proof: Considering the definition of type-1 $\mathcal{P}$- transform as given in (1.1) by virtue of (1.8) & (1.9), we have

$$\mathcal{P}_{\eta}^{\sigma,\alpha,\tau}\left[\overset{\alpha',\beta'}{\underset{p',q'}{M}}(zx^{h})\, H_{p,q\,:\,p_1,q_1;\ldots;p_r,q_r}^{o,\,n\,:\,m_1,n_1;\ldots;m_r,n_r}\left[z_1x^{h_1},\ldots,z_rx^{h_r}\right]\right]$$

$$=\int_0^{\infty}\int_0^{\left[\frac{1}{a(1-\tau)}\right]^{\frac{1}{\sigma}}} y^{\eta-1}\left[1-a(1-\tau)y^{\sigma}\right]^{\frac{1}{1-\tau}} e^{-xty^{-\alpha}} \sum_{n=0}^{\infty}\frac{(a_1)_n\ldots(a_{p'})_n}{(b_1)_n\ldots(b_{q'})_n}\frac{z^n t^{nh}}{\Gamma(\alpha' n+\beta')}$$

$$\times\frac{1}{(2\pi\omega)^r}\int_{L_1}\ldots\int_{L_r}\mu_1(\xi_1)\ldots\mu_r(\xi_r)\,\theta(\xi_1,\ldots,\xi_r)\,z_1^{\xi_1}t^{h_1\xi_1},\ \ldots z_r^{\xi_r}t^{h_r\xi_r}\,dy\,dt\ \ d\xi_1\ldots d\xi_r,$$

Interchanging the order of integrations and summations, evaluating the inner integral using gamma function, we get

$$=\sum_{n=0}^{\infty}\frac{(a_1)_n\ldots(a_{p'})_n}{(b_1)_n\ldots(b_{q'})_n}\ \frac{z^n}{\Gamma(\alpha' n+\beta')}\ \frac{1}{(2\pi\omega)^r}\int_{L_1}\ldots\int_{L_r}\mu_1(\xi_1)\ldots\mu_r(\xi_r)\,\theta(\xi_1,\ldots,\xi_r)\,z_1^{\xi_1},\ \ldots,z_r^{\xi_r}$$

$$\frac{\Gamma(1+nh+h_1\xi_1+\ldots+h_r\xi_r)}{x^{\,nh+h_1\xi_1+\ldots+h_r\xi_r+1}}\int_0^{\left[\frac{1}{a(1-\tau)}\right]^{\frac{1}{\sigma}}} y^{\eta+\alpha(nh+h_1\xi_1+\ldots+h_r\xi_r+1)-1}\left[1-a(1-\tau)y^{\sigma}\right]^{\frac{1}{1-\tau}}\,dy\ \ d\xi_1\ldots d\xi_r,$$

On solving the y-integral with the help of beta function and expressing the result in terms of multivariable H-function as given in (1.9), we obtain (2.1).

Theorem 2: Let $f\in L_{\eta,r}(0,\infty)$, $z, z_1,\ldots,z_r$, $\eta\in C$, $\alpha>0$, $h, h_1>0$, $\tau>1$ be such that $\sigma\in R$ and $\sigma\neq 0$ in the type -2 $\mathcal{P}$-transform, then

$$\mathcal{P}_{\eta}^{\sigma,\alpha,\tau}\left[\overset{\alpha',\beta'}{\underset{p',q'}{M}}(zx^{h})\, H_{p,q\,:\,p_1,q_1;\ldots;p_i,q_i;\ldots;p_r,q_r}^{o,\,n\,:\,m_1,n_1;\ldots;m_i,n_i;\ldots;m_r,n_r}\left[z_1,\ldots,z_ix^{h_1},\ldots,z_r\right]\right]$$

$$=\sum_{n=0}^{\infty}\frac{(a_1)_n\ldots(a_{p'})_n}{(b_1)_n\ldots(b_{q'})_n}\ \frac{z^n}{\sigma\, x^{nh+1}\,\left[a(\tau-1)\right]^{\frac{\eta+\alpha(nh+1)}{\sigma}}\ \ \Gamma(\alpha' n+\beta')\,\Gamma\left(\frac{1}{\tau-1}\right)}$$

$$H_{p,q\,:\,p_1,q_1;\ldots;p_i+2,q_i+1;\ldots;p_r,q_r}^{o,\,n\,:\,m_1,n_1;\ldots;m_i+1,n_i+2;\ldots;m_r,n_r}\left[z_1,\ldots,\frac{z_i}{x^{h_1}\left[a(\tau-1)\right]^{\frac{\alpha h_1}{\sigma}}},\ldots,z_r\,\middle|\,\begin{matrix}(a_j;\mu_j',\ldots,\mu_j^{(r)})_{1,p}:\\ (b_j;\psi_j',\ldots,\psi_j^{(r)})_{1,q}:\end{matrix}\right.$$

$$\left.\begin{matrix}(c_j',\chi_j')_{1,p_1};\ldots;(c_j^{i},\chi_j^{i})_{1,p_i},[-nh,h_1],\left[1-\frac{\eta+\alpha(nh+1)}{\sigma},\frac{\alpha h_1}{\sigma}\right];\ldots;(c_j^{(r)},\chi_j^{(r)})_{1,p_r}\\ (d_j',\phi_j')_{1,q_1};\ldots;(d_j^{i},\phi_j^{i})_{1,q_i},\left[\frac{1}{\tau-1}-\frac{\eta+\alpha(nh+1)}{\sigma},\frac{\alpha h_1}{\sigma}\right];\ldots;(d_j^{(r)},\phi_j^{(r)})_{1,q_r}\end{matrix}\right], \tag{2.2}$$

provided that $R\left(\frac{1}{\tau-1}\right)$, $R(\alpha' n+\beta')>0$.

Proof: Following the definition of type-2 $\mathcal{P}$-transform, with help of (1.8) and (1.9), we have,

$$\mathcal{P}_{\eta}^{\sigma,\alpha,\tau}\left[\overset{\alpha',\beta'}{\underset{p',q'}{M}}(zx^{h})\, H_{p,q\,:\,p_1,q_1;\ldots;p_i,q_i;\ldots;p_r,q_r}^{o,\,n\,:\,m_1,n_1;\ldots;m_i,n_i;\ldots;m_r,n_r}\left[z_1,\ldots,z_ix^{h_1},\ldots,z_r\right]\right]$$

$$=\int_0^{\infty}\int_0^{\infty} y^{\eta-1}\left[1+a(\tau-1)y^{\sigma}\right]^{-\frac{1}{\tau-1}} e^{-xty^{-\alpha}}\left(\sum_{n=0}^{\infty}\frac{(a_1)_n\ldots(a_{p'})_n}{(b_1)_n\ldots(b_{q'})_n}\ \frac{z^n t^{nh}}{\Gamma(\alpha' n+\beta')}\right)$$

$$\frac{1}{(2\pi\omega)^r}\int_{L_1}\ldots\int_{L_r}\mu_1(\xi_1)\ldots\mu_r(\xi_r)\,\theta(\xi_1,\ldots,\xi_r)\,z_1^{\xi_1}\ \ldots z_i^{\xi_i}t^{h_1\xi_i}\ldots z_r^{\xi_r}\,dt\,dy\ \ d\xi_1\ldots d\xi_r,$$

interchanging the order of integrations and summations, using the integral representation of gamma function, we get

$$=\sum_{n=0}^{\infty}\frac{(a_1)_n\ldots(a_{p'})_n}{(b_1)_n\ldots(b_{q'})_n}\ \frac{z^n}{\Gamma(\alpha' n+\beta')}\ \frac{1}{(2\pi\omega)^r}\int_{L_1}\ldots\int_{L_r}\mu_1(\xi_1)\ldots\mu_r(\xi_r)\,\theta(\xi_1,\ldots,\xi_r)\,z_1^{\xi_1},\ \ldots,z_i^{\xi_i},\ldots,z_r^{\xi_r}$$

$$\frac{\Gamma(1+nh+h_1\xi_i)}{x^{\,nh+h_1\xi_i+1}}\int_0^{\infty} y^{\eta+\alpha(nh+h_1\xi_i+1)-1}\left[1+a(\tau-1)y^{\sigma}\right]^{-\frac{1}{\tau-1}}\,dy\ \ d\xi_1\ldots d\xi_r,$$

On solving the inner integral and then rearranging the terms, we obtain right hand side of (2.2).

3. SPECIAL CASES

(I) For taking a=1, α=1, τ→1 in result (2.1) and (2.2), we get,

(a) $$\lim_{\tau\to 1}\mathcal{P}_{\eta}^{\sigma,\alpha,\tau}\left[\overset{\alpha',\beta'}{\underset{p',q'}{M}}\left(zx^{h}\right) H^{o,\ n:\, m_1,\, n_1;\ldots;\, m_r,\, n_r}_{p,\, q\ :\, p_1,\, q_1;\ldots;\, p_r,\, q_r}\left[z_1x^{h_1},\ldots,z_rx^{h_r}\right]\right]$$

$$= \sum_{n=0}^{\infty}\frac{(a_1)_n\ldots(a_{p'})_n}{(b_1)_n\ldots(b_{q'})_n}\ \frac{z^n}{\Gamma(\alpha' n+\beta')\sigma\, x^{nh+1}}$$

$$H^{o,\ n+2\,:\, m_1,\, n_1;\ldots;\, m_r,\, n_r}_{p+2,\, q\ :\, p_1,\, q_1;\ldots;\, p_r,\, q_r}\left[\begin{matrix}\frac{z_1}{x^{h_1}}\\ \vdots\\ \frac{z_r}{x^{h_r}}\end{matrix}\ \middle|\ \begin{matrix}(a_j;\mu_j',\ldots,\mu_j^{(r)})_{1,p}, & [-nh;h_1,\ldots,h_r], & \left[1-\frac{\eta+(nh+1)}{\sigma};\frac{h_1}{\sigma},\ldots,\frac{h_r}{\sigma}\right] & : & \left(c_j',\chi_j'\right)_{1,p_1};\ldots;\left(c_j^{(r)},\chi_j^{(r)}\right)_{1,p_r}\\ (b_j;\psi_j',\ldots,\psi_j^{(r)})_{1,q}, & - & - & : & \left(d_j',\phi_j'\right)_{1,q_1};\ldots;\left(d_j^{(r)},\phi_j^{(r)}\right)_{1,q_r}\end{matrix}\right], \tag{3.1}$$

where $R(\alpha' n+\beta')>0$

(b) $$\lim_{\tau\to 1}\mathcal{P}_{\eta}^{\sigma,1,\tau}\left[\overset{\alpha',\beta'}{\underset{p',q'}{M}}\left(zx^{h}\right) H^{o,\ n\ :\, m_1,\, n_1;\ldots;\, m_i,\, n_i;\ldots;\, m_r,\, n_r}_{p,\, q\ :\, p_1,\, q_1;\ldots;\, p_i,\, q_i;\ldots;\, p_r,\ q_r}\left[z_1,\ldots,z_ix^{h_1},\ldots,z_r\right]\right]$$

$$= \sum_{n=0}^{\infty}\frac{(a_1)_n\ldots(a_{p'})_n}{(b_1)_n\ldots(b_{q'})_n}\ \frac{z^n}{\Gamma(\alpha' n+\beta')}\ \frac{1}{\sigma\, x^{\,nh+1}}$$

$$H^{o,\, n\ :\, m_1,\, n_1;\ldots;m_i,\, n_i+2;\ldots;m_r,\, n_r}_{p,\, q\ :\ p_1,\, q_1;\ldots;\, p_i+2,\, q_i;\ldots;p_r,\, q_r}\left[z_1,\ldots,\frac{z_i}{x^{h_1}[a(\tau-1)]^{\frac{\alpha h_1}{\sigma}}},\ldots,z_r\ \middle|\ \begin{matrix}(a_j;\mu_j',\ldots,\mu_j^{(r)})_{1,p}: & \left(c_j',\chi_j'\right)_{1,p_1};\ldots;\left(c_j^{i},\chi_j^{i}\right)_{1,p_i},[-nh,h_1],\left[1-\frac{\eta+\alpha(nh+1)}{\sigma},\frac{\alpha h_1}{\sigma}\right];\ldots;\left(c_j^{(r)},\chi_j^{(r)}\right)_{1,p_r}\\ (b_j;\psi_j',\ldots,\psi_j^{(r)})_{1,q}: & \left(d_j',\phi_j'\right)_{1,q_1};\ldots;\left(d_j^{i},\phi_j^{i}\right)_{1,q_i};\ldots;\left(d_j^{(r)},\phi_j^{(r)}\right)_{1,q_r}\end{matrix}\right], \tag{3.2}$$

where $R(\alpha' n+\beta')>0$

(II) For $p'=q'=1$, $a=\gamma$, b=1 in (2.1) and (2.2), Generalized M-series reduces to the Generalized Mittag Leffler function [4], by setting $\overset{\alpha',\beta'}{\underset{1,1}{M}}(x)=\sum_{n=0}^{\infty}\frac{(\gamma)_n\ x^n}{(1)_n}\frac{1}{\Gamma(\alpha' n+\beta')}=E^{\gamma}_{(\alpha',\beta')}(x)$, we get

(c) $$\mathcal{P}_{\eta}^{\sigma,\alpha,\tau}\left[E^{\gamma}_{(\alpha',\beta')}(zx^h)\, H^{o,\ n:\, m_1,\, n_1;\ldots;\, m_r,\, n_r}_{p,\, q\ :\, p_1,\, q_1;\ldots;\, p_r,\, q_r}\left[z_1x^{h_1},\ldots,z_rx^{h_r}\right]\right]$$

$$=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\alpha' n+\beta')(1)_n}\ \frac{z^n\ \Gamma\left(1+\frac{1}{1-\tau}\right)}{\sigma x^{nh+1}\,[a(1-\tau)]^{\frac{\eta+\alpha(nh+1)}{\sigma}}}$$

$$H^{o,\ n+2\,:\, m_1,\, n_1;\ldots;\, m_r,\, n_r}_{p+2,\, q+1\ :\, p_1,\, q_1;\ldots;\, p_r,\, q_r}\left[\begin{matrix}\frac{z_1}{x^{h_1}[a(1-\tau)]^{\frac{\alpha h_1}{\sigma}}}\\ \vdots\\ \frac{z_r}{x^{h_r}[a(1-\tau)]^{\frac{\alpha h_r}{\sigma}}}\end{matrix}\ \middle|\ \begin{matrix}(a_j;\mu_j',\ldots,\mu_j^{(r)})_{1,p}, & [-nh;h_1,\ldots,h_r], & \left[1-\frac{\eta+\alpha(nh+1)}{\sigma},\frac{\alpha h_1}{\sigma},\ldots,\frac{\alpha h_r}{\sigma}\right]: & \left(c_j',\chi_j'\right)_{1,p_1};\ldots;\left(c_j^{(r)},\chi_j^{(r)}\right)_{1,p_r}\\ (b_j;\psi_j',\ldots,\psi_j^{(r)})_{1,q}, & \left[\frac{1}{\tau-1}-\frac{\eta+\alpha(nh+1)}{\sigma};\frac{\alpha h_1}{\sigma},\ldots,\frac{\alpha h_r}{\sigma}\right], & - & : & \left(d_j',\phi_j'\right)_{1,q_1};\ldots;\left(d_j^{(r)},\phi_j^{(r)}\right)_{1,q_r}\end{matrix}\right], \tag{3.3}$$

where $R\left(1+\frac{1}{1-\tau}\right)>0,\quad R(\alpha' n+\beta')>0.$

(d) $$\mathcal{P}_{\eta}^{\sigma,\alpha,\tau}\left[E^{\gamma}_{(\alpha',\beta')}(zx^h)\, H^{o,\, n\ :\, m_1,\, n_1;\ldots;\, m_i,\, n_i;\ldots;\, m_r,\, n_r}_{p,\, q\ :\, p_1,\, q_1;\ldots;\, p_i,\, q_i;\ldots;\, p_r,\ q_r}\left[z_1,\ldots,z_ix^{h_1},\ldots,z_r\right]\right]$$

$$=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\alpha' n+\beta')(1)_n}\ \frac{z^n}{\sigma x^{nh+1}\,[a(\tau-1)]^{\frac{\eta+\alpha(nh+1)}{\sigma}}}\ \frac{1}{\Gamma\left(\frac{1}{\tau-1}\right)}$$

$$H^{o,\, n\ :\, m_1,\, n_1;\ldots;\, m_i+1,\, n_i+2;\ldots;\, m_r,\, n_r}_{p,\, q\ :\, p_1,\, q_1;\ldots;\, p_i+2,\, q_i+1;\ldots;\, p_r,\ q_r}\left[z_1,\ldots,\frac{z_i}{x^{h_1}[a(\tau-1)]^{\frac{\alpha h_1}{\sigma}}},\ldots,z_r\ \middle|\ \begin{matrix}(a_j;\mu_j',\ldots,\mu_j^{(r)})_{1,p}:\\ (b_j;\psi_j',\ldots,\psi_j^{(r)})_{1,q}:\end{matrix}\right.$$

$$\left.\begin{matrix} (c'_j,\chi'_j)_{1,p_1};\dots;(c^{i}_j,\chi^{i}_j)_{1,p_i},[-nh,h_1],\left[1-\frac{\eta+\alpha(nh+1)}{\sigma},\frac{\alpha h_1}{\sigma}\right];\dots;(c^{(r)}_j,\chi^{(r)}_j)_{1,p_r} \\ (d'_j,\phi'_j)_{1,q_1};\dots;(d^{i}_j,\phi^{i}_j)_{1,q_i},\left[\frac{1}{\tau-1}-\frac{\eta+\alpha(nh+1)}{\sigma},\frac{\alpha h_1}{\sigma}\right];\dots;(d^{(r)}_j,\phi^{(r)}_j)_{1,q_r} \end{matrix}\right], \tag{3.4}$$

where $R(\alpha'n+\beta'), R\left(\frac{1}{\tau-1}\right)>0.$

(III) For $p'=0,\ q'=1,\ b_1=1$ in (2.1) and (2.2), the Generalized M-series reduces to the Wright function [17 p. 37 (1.156)], by setting

$$\underset{0,1}{\overset{\alpha',\beta'}{M}}[x]=\phi(\alpha',\beta',x)={}_0\psi_1\left[\overline{(\beta',\alpha')}\,\middle|\ x\right]=\sum_{n=0}^{\infty}\frac{x^n}{(1)_n}\frac{1}{\Gamma(\alpha'n+\beta')},\ \text{we obtain}$$

(e) $$\mathcal{P}_{\eta}^{\sigma,\alpha,\tau}\left[{}_0\psi_1\left[\overline{(\beta',\alpha')}\,\middle|\,zx^h\right]\ H^{o,\,n\,:\,m_1,n_1;\dots;m_r,n_r}_{p,\,q\,:\,p_1,q_1;\dots;p_r,q_r}\left[z_1x^{h_1},\dots,z_rx^{h_r}\right]\right]$$

$$=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha'n+\beta')(1)_n}\ \frac{\Gamma\left(1+\frac{1}{1-\tau}\right)}{\sigma x^{nh+1}\,[a(1-\tau)]^{\frac{\eta+\alpha(nh+1)}{\sigma}}}$$

$$H^{o,\ n+2\,:\,m_1,n_1;\dots;m_r,n_r}_{p+2,\,q+1\,:\,p_1,q_1;\dots;p_r,q_r}\left[\begin{matrix}\dfrac{z_1}{x^{h_1}[a(1-\tau)]^{\frac{\alpha h_1}{\sigma}}}\\ \vdots \\ \dfrac{z_r}{x^{h_r}[a(1-\tau)]^{\frac{\alpha h_r}{\sigma}}}\end{matrix}\ \middle|\ \begin{matrix}(a_j;\mu_j',\dots,\mu_j^{(r)})_{1,p}, & [-nh;h_1,\dots,h_r], \\ (b_j;\psi_j',\dots,\psi_j^{(r)})_{1,q}, & \left[\frac{1}{\tau-1}-\frac{\eta+\alpha(nh+1)}{\sigma};\frac{\alpha h_1}{\sigma},\dots,\frac{\alpha h_r}{\sigma}\right],\end{matrix}\right.$$

$$\left.\begin{matrix}\left[1-\frac{\eta+\alpha(nh+1)}{\sigma},\frac{\alpha h_1}{\sigma},\dots,\frac{\alpha h_r}{\sigma}\right]: & (c'_j,\chi'_j)_{1,p_1};\dots;(c^{(r)}_j,\chi^{(r)}_j)_{1,p_r}, \\ : & (d'_j,\phi'_j)_{1,q_1};\dots;(d^{(r)}_j,\phi^{(r)}_j)_{1,q_r},\end{matrix}\right], \tag{3.5}$$

where $R\left(1+\frac{1}{1-\tau}\right),\ \ R(\alpha'n+\beta')>0.$

(f) $$\mathcal{P}_{\eta}^{\sigma,\alpha,\tau}\left[{}_0\psi_1\left[\overline{(\beta',\alpha')}\,\middle|\ zx^h\right]\ H^{o,\,n\,:\,m_1,n_1;\dots;m_i,n_i;\dots;m_r,n_r}_{p,\,q\,:\,p_1,q_1;\dots;p_i,q_i;\dots;p_r,q_r}\left[z_1,\dots,z_ix^{h_1},\dots,z_r\right]\right]$$

$$=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha'n+\beta')(1)_n}\ \frac{1}{\sigma x^{nh+1}\,[a(\tau-1)]^{\frac{\eta+\alpha(nh+1)}{\sigma}}}\ \frac{1}{\Gamma\left(\frac{1}{\tau-1}\right)}$$

$$H^{o,\,n\,:\,m_1,n_1;\dots;m_i+1,n_i+2;\dots;m_r,n_r}_{p,\,q\,:\,p_1,q_1;\dots;p_i+2,q_i+1;\dots;p_r,q_r}\left[z_1,\dots,\frac{z_i}{x^{h_1}[a(\tau-1)]^{\frac{\alpha h_1}{\sigma}}},\dots,z_r\ \middle|\ \begin{matrix}(a_j;\mu_j',\dots,\mu_j^{(r)})_{1,p}: \\ (b_j;\psi_j',\dots,\psi_j^{(r)})_{1,q}:\end{matrix}\right.$$

$$\left.\begin{matrix} (c'_j,\chi'_j)_{1,p_1};\dots;(c^{i}_j,\chi^{i}_j)_{1,p_i},[-nh,h_1],\left[1-\frac{\eta+\alpha(nh+1)}{\sigma},\frac{\alpha h_1}{\sigma}\right];\dots;(c^{(r)}_j,\chi^{(r)}_j)_{1,p_r} \\ (d'_j,\phi'_j)_{1,q_1};\dots;(d^{i}_j,\phi^{i}_j)_{1,q_i},\left[\frac{1}{\tau-1}-\frac{\eta+\alpha(nh+1)}{\sigma},\frac{\alpha h_1}{\sigma}\right];\dots;(d^{(r)}_j,\phi^{(r)}_j)_{1,q_r} \end{matrix}\right], \tag{3.6}$$

where $R(\alpha'n+\beta'), R\left(\frac{1}{\tau-1}\right)>0.$

(IV) For $\alpha'=\beta'=1$ with arbitrary p' and q' in (2.1) and (2.2), the Generalized M-series reduces to Hypergeometric function [8], by setting

$$\underset{p',q'}{\overset{1,1}{M}}[x]={}_{p'}F_{q'}\left[(a_j)_1^{p'};(b_j)_1^{q'};x\right]=\sum_{n=0}^{\infty}\frac{(a_1)_n\cdots(a_{p'})_n}{(b_1)_n\cdots(b_{q'})_n}\frac{x^n}{n!}\ ,\ \text{we obtain}$$

(g) $$\mathcal{P}_{\eta}^{\sigma,\alpha,\tau}\left[{}_{p'}F_{q'}\left[(a_j)_1^{p'};(b_j)_1^{q'};zx^h\right]\ H^{o,\,n\,:\,m_1,n_1;\dots;m_r,n_r}_{p,\,q\,:\,p_1,q_1;\dots;p_r,q_r}\left[z_1x^{h_1},\dots,z_rx^{h_r}\right]\right]$$

$$=\sum_{n=0}^{\infty}\frac{(a_1)_n\cdots(a_{p'})_n}{(b_1)_n\cdots(b_{q'})_n}\ \frac{z^n}{n!}\ \frac{\Gamma\left(1+\frac{1}{1-\tau}\right)}{\sigma x^{nh+1}\,[a(1-\tau)]^{\frac{\eta+\alpha(nh+1)}{\sigma}}}$$

$$H^{o,\ n+2\,:\,m_1,n_1;\dots;m_r,n_r}_{p+2,\,q+1\,:\,p_1,q_1;\dots;p_r,q_r}\left[\begin{matrix}\dfrac{z_1}{x^{h_1}[a(1-\tau)]^{\frac{\alpha h_1}{\sigma}}}\\ \vdots \\ \dfrac{z_r}{x^{h_r}[a(1-\tau)]^{\frac{\alpha h_r}{\sigma}}}\end{matrix}\ \middle|\ \begin{matrix}(a_j;\mu_j',\dots,\mu_j^{(r)})_{1,p}, & [-nh;h_1,\dots,h_r], \\ (b_j;\psi_j',\dots,\psi_j^{(r)})_{1,q}, & \left[\frac{1}{\tau-1}-\frac{\eta+\alpha(nh+1)}{\sigma};\frac{\alpha h_1}{\sigma},\dots,\frac{\alpha h_r}{\sigma}\right],\end{matrix}\right.$$

$$\left[1-\frac{\eta+\alpha(nh+1)}{\sigma};\frac{\alpha h_1}{\sigma},\ldots,\frac{\alpha h_r}{\sigma}\right]: \begin{matrix}(c_j',\chi_j')_{1,p_1};\ldots;(c_j^{(r)},\chi_j^{(r)})_{1,p_r} \\ -\quad : (d_j',\phi_j')_{1,q_1};\ldots;(d_j^{(r)},\phi_j^{(r)})_{1,q_r}\end{matrix}\Bigg], \tag{3.7}$$

where $R\left(1+\frac{1}{1-\tau}\right)>0$.

(h) $$\mathcal{P}_\eta^{\sigma,\alpha,\tau}\left[{}_{p'}F_{q'}\left[(a_j)_1^{p'};(b_j)_1^{q'};zx^h\right]H^{0,n\,:\,m_1,n_1;\ldots;m_i,n_i;\ldots;m_r,n_r}_{p,q\,:\,p_1,q_1;\ldots;p_i,q_i;\ldots;p_r,q_r}\left[z_1,\ldots,z_ix^{h_1},\ldots,z_r\right]\right]$$

$$=\sum_{n=0}^{\infty}\frac{(a_1)_n\ldots(a_{p'})_n}{(b_1)_n\ldots(b_{q'})_n}\frac{z^n}{n!}\frac{1}{\sigma x^{nh+1}[a(\tau-1)]^{\frac{\eta+\alpha(nh+1)}{\sigma}}\Gamma\left(\frac{1}{\tau-1}\right)}$$

$$H^{0,n\,:\,m_1,n_1;\ldots;m_i+1,n_i+2;\ldots;m_r,n_r}_{p,q\,:\,p_1,q_1;\ldots;p_i+2,q_i+1;\ldots;p_r,q_r}\left[z_1,\ldots,\frac{z_i}{x^{h_1}[a(\tau-1)]^{\frac{\alpha h_1}{\sigma}}},\ldots,z_r\,\middle|\,\begin{matrix}(a_j;\mu_j',\ldots,\mu_j^{(r)})_{1,p}: \\ (b_j;\psi_j',\ldots,\psi_j^{(r)})_{1,q}:\end{matrix}\right.$$

$$\left.\begin{matrix}(c_j',\chi_j')_{1,p_1};\ldots;(c_j^{i},\chi_j^{i})_{1,p_i},[-nh,h_1],\left[1-\frac{\eta+\alpha(nh+1)}{\sigma},\frac{\alpha h_1}{\sigma}\right];\ldots;(c_j^{(r)},\chi_j^{(r)})_{1,p_r} \\ (d_j',\phi_j')_{1,q_1};\ldots;(d_j^{i},\phi_j^{i})_{1,q_i},\left[\frac{1}{\tau-1}-\frac{\eta+\alpha(nh+1)}{\sigma},\frac{\alpha h_1}{\sigma}\right];\ldots;(d_j^{(r)},\phi_j^{(r)})_{1,q_r}\end{matrix}\right], \tag{3.8}$$

where $R\left(\frac{1}{\tau-1}\right)>0$.

(V) For $n=p$, $m_i=1$, $n_i=p_i$, $q_i=q_i+1$, $\forall i=1,\cdots,r$ in (2.1) and (2.2), the multivariable H-function reduces to the Lauricella function [15], we have

(i) $$\mathcal{P}_\eta^{\sigma,\alpha,\tau}\left[M^{\alpha',\beta'}_{p',q'}(zx^h)F^{p:\,p_1;\cdots;p_r}_{q:\,q_1;\cdots;q_r}\left[\begin{matrix}\left[(1-a_j:\mu_j^{(1)},\cdots,\mu_j^{(r)})\right]_{1,p}:\left[(1-c_j^{(1)},\chi_j^{(1)})\right]_{1,p_1};\cdots;\left[(1-c_j^{(r)};\chi_j^{(r)})\right]_{1,p_r}; \\ \left[(1-b_j:\psi_j^{(1)},\cdots,\psi_j^{(r)})\right]_{1,q}:\left[(1-d_j^{(1)},\phi_j^{(1)})\right]_{1,q_1};\cdots;\left[(1-d_j^{(r)};\phi_j^{(r)});\right]_{1,q_r};\end{matrix}\;-z_1x^{h_1},\cdots,-z_rx^{h_r}\right]\right]$$

$$=\sum_{n=0}^{\infty}\frac{(a_1)_n\ldots(a_{p'})_n}{(b_1)_n\ldots(b_{q'})_n}\frac{z^n\,\Gamma\left(1+\frac{1}{1-\tau}\right)}{\sigma x^{nh+1}[a(1-\tau)]^{\frac{\eta+\alpha(nh+1)}{\sigma}}\Gamma(\alpha'n+\beta')}\frac{\Gamma(1-nh)\,\Gamma\left(\frac{\eta+\alpha(nh+1)}{\sigma}\right)}{\Gamma\left(1-\frac{1}{\tau-1}+\frac{\eta+\alpha(nh+1)}{\sigma}\right)}$$

$$F^{p+2:\,p_1;\cdots;p_r}_{q+1:\,q_1;\cdots;q_r}\left[\begin{matrix}[1-nh;h_1,\ldots,h_r], & \left[\frac{\eta+\alpha(nh+1)}{\sigma},\frac{\alpha h_1}{\sigma},\ldots,\frac{\alpha h_r}{\sigma}\right], \\ \left[1-\frac{1}{\tau-1}+\frac{\eta+\alpha(nh+1)}{\sigma};\frac{\alpha h_1}{\sigma},\ldots,\frac{\alpha h_r}{\sigma}\right], & -\end{matrix}\right.$$

$$\left.\begin{matrix}\left[(1-a_j:\mu_j^{(1)},\cdots,\mu_j^{(r)})\right]_{1,p}:\left[(1-c_j^{(1)},\chi_j^{(1)})\right]_{1,p_1};\cdots;\left[(1-c_j^{(r)};\chi_j^{(r)})\right]_{1,p_r}; \\ \left[(1-b_j:\psi_j^{(1)},\cdots,\psi_j^{(r)})\right]_{1,q}:\left[(1-d_j^{(1)},\phi_j^{(1)})\right]_{1,q_1};\cdots;\left[(1-d_j^{(r)};\phi_j^{(r)});\right]_{1,q_r};\end{matrix}\right.$$

$$\left.-\frac{z_1}{x^{h_1}[a(1-\tau)]^{\frac{\alpha h_1}{\sigma}}},\cdots,-\frac{z_r}{x^{h_r}[a(1-\tau)]^{\frac{\alpha h_r}{\sigma}}}\right], \tag{3.9}$$

where $R\left(1+\frac{1}{1-\tau}\right)$, $R(\alpha'n+\beta')$, $R(1-nh)$, $R\left(\frac{\eta+\alpha(nh+1)}{\sigma}\right)$, $R\left(1-\frac{1}{\tau-1}+\frac{\eta+\alpha(nh+1)}{\sigma}\right)>0$.

(j) $$\mathcal{P}_\eta^{\sigma,\alpha,\tau}\left[M^{\alpha',\beta'}_{p',q'}(zx^h)F^{p:\,p_1;\cdots;p_i;\cdots;p_r}_{q:\,q_1;\cdots;q_i;\cdots;q_r}\left[\begin{matrix}\left[(1-a_j:\mu_j^{(1)},\cdots,\mu_j^{(r)})\right]_{1,p}:\left[(1-c_j^{(1)},\chi_j^{(1)})\right]_{1,p_1};\cdots;\left[(1-c_j^{(r)};\chi_j^{(r)})\right]_{1,p_r}; \\ \left[(1-b_j:\psi_j^{(1)},\cdots,\psi_j^{(r)})\right]_{1,q}:\left[(1-d_j^{(1)},\phi_j^{(1)})\right]_{1,q_1};\cdots;\left[(1-d_j^{(r)};\phi_j^{(r)});\right]_{1,q_r};\end{matrix}\;-z_1,\ldots,-z_ix^{h_1},\ldots,-z_r\right]\right]$$

$$=\sum_{n=0}^{\infty}\frac{(a_1)_n\ldots(a_{p'})_n}{(b_1)_n\ldots(b_{q'})_n}\frac{z^n}{\sigma x^{nh+1}[a(\tau-1)]^{\frac{\eta+\alpha(nh+1)}{\sigma}}}\frac{\Gamma(1+nh)\,\Gamma\left(\frac{\eta+\alpha(nh+1)}{\sigma}\right)\Gamma\left(1-\frac{1}{\tau-1}+\frac{\eta+\alpha(nh+1)}{\sigma}\right)}{\Gamma(\alpha'n+\beta')\,\Gamma\left(\frac{1}{\tau-1}\right)}$$

$$F^{p:\,p_1;\cdots;p_i+2;\cdots;p_r}_{q:\,q_1;\cdots;q_i+1;\cdots;q_r}\left[\begin{matrix}\left[(1-a_j:\mu_j^{(1)},\cdots,\mu_j^{(r)})\right]_{1,p}:\left[(1-c_j^{(1)},\chi_j^{(1)})\right]_{1,p_1};\cdots;(1-c_j^{i},\chi_j^{i})_{1,p_i}, \\ \left[(1-b_j:\psi_j^{(1)},\cdots,\psi_j^{(r)})\right]_{1,q}:\left[(1-d_j^{(1)},\phi_j^{(1)})\right]_{1,q_1};\cdots;(1-d_j^{i},\phi_j^{i})_{1,q_i},\end{matrix}\right.$$

$$\left.\begin{matrix} [1+nh, h_1], \left[\frac{\eta+\alpha\,(nh+1)}{\sigma}, \frac{\alpha h_1}{\sigma}\right] ; \cdots ; \left[\left(1- c_j^{(r)} ; \chi_j^{(r)}\right)\right]_{1,p_r} ; \\ \left[1-\frac{1}{\tau-1}+\frac{\eta+\alpha\,(nh+1)}{\sigma}, \frac{\alpha h_1}{\sigma}\right]; \ldots ; \; ; \cdots ; \left[\left(1- d_j^{(r)}; \phi_j^{(r)}\right) ; \right]_{1,q_r}; \end{matrix} \; -z_1, \ldots, -\frac{z_i}{x^{h_1}[a(\tau-1)]^{\frac{\alpha h_1}{\sigma}}}, \ldots, -z_r \right], \quad (3.10)$$

where $R\left(\frac{1}{\tau-1}\right)$, $R(\alpha' n+\beta')$, $R\left(\left(1-\frac{1}{\tau-1}+\frac{\eta+\alpha\,(nh+1)}{\sigma}\right)\right)$, $R(1+nh)$, $R\left(\frac{\eta+\alpha(nh+1)}{\sigma}\right)>0$.

4. CONCLUSION

In this paper, we have proposed the images of the product of M-series and multivariable H-function under pathway transform. We have found that number of special cases can be obtained of our main results, which are related with M-series, multivariable H-function and pathway transform.

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