BLOOD CIRCULATORY SYSTEM WITH RECOVERY PERIOD DEPENDING ON DEPOSIT OF LOW DENSITY LIPOPROTEINS WITH TREATMENT AT TIME (L) OR AT TIME LDL EXCEEDS (A)

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ABSTRACT

The human Blood Circulatory System (BCS) in which the deposit of Low Density Lipoproteins (LDL) during prophylactic period and recovering period after the deposit is treated. After the bulk deposit or build up of low density lipoproteins medical attention starts. It is assumed that the time to prophylactic treatment has general distribution and deposit of LDL is proportional to time. Expected Recovery period is obtained.

Key Words: BCS, Joint transform, LDL deposit.

INTRODUCTION

Cholesterol is a waxy fat like substance that is found in all cells of the body. Our body needs some cholesterol to make hormones, vitamin D and substances that help us to digest foods. Our body itself makes all the cholesterol it needs, however cholesterol also is found in some of foods we eat. Cholesterol travels through blood stream in small packages called lipoproteins. These packages are made of fat (lipid) on the inside and proteins on the outside. Two kinds of lipoproteins carry cholesterol throughout the body namely, LDL and HDL. Healthy levels of both types of lipoproteins are important. LDL cholesterol is sometimes called "bad cholesterol". A high LDL level leads to a buildup of cholesterol in arteries. The higher is the level of LDL cholesterol in blood, the greater is the chance of getting heart disease. In this paper we consider a model in which the prophylactic time has general distribution and the deposit of LDL is proportional to time. After the bulk deposit or buildup of LDL, medical attention starts. Two distinct distributions for the recovery periods depending on the quantity of deposit of LDL are within or exceeding a random critical quantity are considered. The Laplace joint transform is obtained to derive the expected quantity of LDL deposit and recovery time. Numerical examples are also presented. The plaque constricts the lumen of the blood vessel thereby increasing the sheer force of blood flow. Frink R J [1] and Moreno P R [2]. As the plaque continues to grow, the increased sheer force may cause rupture of the plaque, possibly resulting in the formation of thrombus (blood clot), ischemic stroke, and heart attack. Frink R J [1]. Mathematical modeling of the atherosclerotic plaque formation has been studied by Calvez V, Ebde A, Meunier N, Raoult A [3]. Wenrui Hao and Avner Friedman [4] developed a mathematical model of the formation of a plaque, using a system of partial differential equations. For studies on man power models and inventory systems using similar transform techniques applied here one may refer K.Usha, A.C.Tamil Selvi, R. Ramanarayanan [5] and K.Usha, N.Nithiya Priya and R.Ramanarayanan [6].

MODEL: General Prophylactic Time and LDL Deposit Proportional to Time
Model assumptions are,
1. The time to prophylactic treatment '**L**' of a person is a random variable with Cdf **H(u)** and pdf **h(u)**.
2. The deposit quantity of LDL in the artery is proportional to time and the proportionality constant is '**c**'.
3. The recovering period **R** has general distribution with Cdf **G₁(v)** and pdf **g₁(v)** when deposit of LDL is less than the critical threshold size '**A**' and it has Cdf **G₂(v)** and pdf **g₂(v)** when the deposit of LDL is more than the critical quantity '**A**' to speed up the medical attention.
4. Critical threshold size '**A**' has exponential distribution with parameter μ .
5. The treatment starts after the completion of prophylactic time or when the critical threshold size is exceeded whichever occurs first.

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Let T denote the time at which the treatment is provided. Here T is the time at which the prophylactic period is over or the time at which the LDL exceeds the threshold 'A' whichever occurs first. We may derive the joint distribution of **T** and **R** as follows.

The joint probability density function of **T** and **R** is,

$$\frac{\partial}{\partial u}(\frac{\partial}{\partial v}P(T \leq u, R \leq v)) = f(u, v)$$
$$= h(u) \, e^{-\mu cu} g_1(v) + \bar{H}(u) \, \mu c e^{-\mu cu} \, g_2(v) \tag{1}$$

Let us define the double Laplace transform function as follows.

$$E(e^{-sT}e^{-tR}) = \int_0^\infty \int_0^\infty e^{-su} e^{-tv} f(u,v) du dv$$
$$= \int_0^\infty \int_0^\infty e^{-su} e^{-tv} (h(u) \, e^{-\mu cu} g_1(v) + \bar{H}(u) \, \mu c e^{-\mu cu} \, g_2(v)) du dv \tag{2}$$

This on simplification gives,

$$E(e^{-sT}e^{-tR}) = h^*(s + c\mu)g_1^*(t) + c\mu \left[\frac{1-h^*(s+c\mu)}{s+c\mu} \right] g_2^*(t) \tag{3}$$

Here * indicates Laplace transform.

We may note $E(e^{-sT}) = \left[\frac{sh^*(s+c\mu)+c\mu}{s+c\mu} \right]$ \hfill (4)

and using differentiation we note that $E(T) = \left[\frac{1-h^*(c\mu)}{c\mu} \right]$. \hfill (5)

$$E(e^{-tR}) = h^*(c\mu)g_1^*(t) + (1 - h^*(c\mu))g_2^*(t)$$

Using differentiations and setting $p = h^*(c\mu)$ and q = 1 - p
$$E(R) = p \, E(G_1) + q \, E(G_2)$$

If h(u) has exponential distribution with parameter 'a' then, $p = \frac{a}{\mu c + a}$

Hence, $E(R) = (\frac{a}{\mu c + a}) E(G_1) + (\frac{\mu c}{\mu c + a}) E(G_2)$

NUMERICAL ILLUSTRATION

We assume the fixed values for $\mu = 25$, a = 100 mg/dl G_1 and G_2 have exponential distributions with parameters 0.1 and = 0.5 respectively then, E(G_1) = 10 and E(G_2) = 2. We provide the different values for the parameter of the deposit proportionality of LDL in the artery (c) as 10, 20, 30, 40 and 50.

The Effect of variation of E(R)

C	10	20	30	40	50
E(R)	4.2857	3.3333	2.9412	2.7272	2.5926
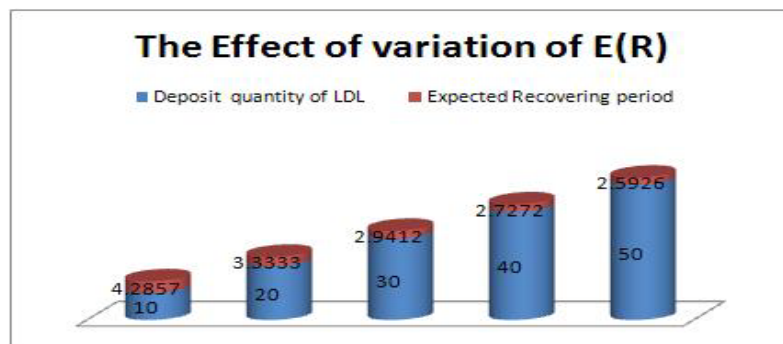
CONCLUSION

From the result the increase of the deposit quantity of LDL in the artery reduces the expected prophylactic treatment time. The higher is the level of LDL cholesterol in blood the greater is the risk of heart disease. Using the above model one may decide the requisite medical attention.

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