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ON WEAKLY SEMI CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this research paper, a new class of closed sets called weakly semi closed sets (ws-closed sets) in topological spaces are introduced and studied. A subset A of a topological space (X,τ) is called ws-closed set if U contains semi closure of A whenever U contains A and U is w-open set in (X,τ). This new class of sets lies between the class of all semi-closed sets and generalised semi-pre regular closed sets in topological spaces. Also some of their properties have been investigated.

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Keywords: *Semi-closed sets, w-closed sets, semi pre-closed sets and ws-closed sets.*

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1. INTRODUCTION

In 1970 N. Levine [18], first introduced the concept of generalized closed sets were defined and investigated. In 2000 M. Sheik John [33], introduced and studied w-closed sets in topological space X. Throughout this paper X or (X,τ) represent non-empty topological space. Let A be subset of a topological space X. cl(A), int(A), scl(A), αcl(A) and spcl(A)) denote the closure of A, the interior of A, the semi-closure of A, the α-closure of A and the semi pre closure of A in X respectively.

2. PRELIMINARIES

Definition 2.1: A subset A of a topological space (X, τ) is called a

i. Regular open set [32] if A=int(cl(A)) and regular closed if A=cl(int(A))
ii. Semi-open set [19] if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.
iii. α-open set [20] if $A \subseteq int(cl(int(A)))$ and a α-closed set if $cl(int(cl(A))) \subseteq A$.
iv. Generalized semi pre closed set (gsp-closed) [8] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
v. w-closed set[33] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi -open in (X, τ).
vi. gspr-closed set[10] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular -open in (X, τ).
vii. αgp-closed set[11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre-open in (X, τ).
viii. *gα–closed set [41] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gα - open in (X, τ).
ix. $g^{\#}$s-closed set[40] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg -open in (X, τ).
x. rb-closed set[24] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is b-open in (X, τ).
xi. gξ* -closed set[17] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is #gα- open in (X, τ).

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3. BASIC PROPERTIES OF WS-CLOSED SETS IN TOPOLOGICAL SPACE

Definition 3.1: A subset *A* of a topological space (X,τ) is called weakly semi closed (ws-closed) set if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is w-open set in (X, τ). The family of all ws –closed sets X is denoted by WSC(X). the compliment of ws –closed set is called ws-open set in (X, τ). The family of all ws-open sets in X is denoted by WSO(X).

Example 3.2: Let X = {a, b, c, d}, τ = {X,ϕ, {a}, {b}, {a, b}, {a, b, c}}. Then
Closed sets in (X,τ) are X, ϕ, {d},{c, d},{a ,c},{a, c, d}, {b, c, d}.
Semi-closed sets in (X, τ) are X,ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, c, d}, {b, c, d}.
W-closed sets in (X,τ) are X, ϕ, {d}, {c, d}, {a, c, d}, {b, c, d}.
W-open sets in (X,τ) are X, ϕ, {a}, {b}, {a, b}, {a, b, c}.
ws-closed sets in (X, τ) are X,ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, d},{a, c, d}, {b, c, d}.
ws-open sets in (X, τ) are X,ϕ,{a},{b},{c},{a, c},{a, d},{b, c},{b, d}, {c, d},{a, b, c}, {a, b, d}, {a, c, d},{b, c, d}.

We prove that the class of ws-closed sets are properly lies between the class of all semi-closed sets and generalised semi-pre regular closed sets in topological spaces.

Theorem 3.3:- Every semi-closed [19] set in X is ws-closed set in X.

Proof: Let A be a semi-closed set in X. Let U be any w-open set in X such that $A \subseteq U$. Since A is semi-closed, we have $scl(A) = A \subseteq U$, we have $scl(A) \subseteq U$. Hence A is ws-closed set in X.

Remark 3.4: The converse of the above Theorem 3.3 need not be true as seen from the following Example 3.5.

Example 3.5: Let X= {a, b, c, d} and τ ={X, ϕ, {a}, {b}, {a, b}, {a, b, c}} then the set A= {a, b, d} is ws-closed set but not semi-closed in X.

Corollary 3.6: In a topological space (X,τ),

- i) Every regular closed [32] set in X is ws-closed set in X.
- ii) Every closed set in X is ws-closed set in X.
- iii) Every α-closed [20] set in X is ws-closed set in X.
- iv) Every $g^{\#}$-closed [37] set in X is ws-closed set in X.
- v) Every $*g\alpha$ -closed [41] set in X is ws-closed set in X.
- vi) Every $g^{\#}s$ –closed [40] set in X is ws-closed set in X.
- vii) Every rb -closed [24] set in X is ws-closed set in X.
- viii) Every $\ddot{g}$-closed set in X is ws-closed set in X.
- ix) Every $g\xi^*$ -closed [17]] set in X is ws-closed set in X.
- x) Every αgp -closed [11] set in X is ws-closed set in X.

Proof:

- i) In view of the fact that every regular closed is semi-closed, therefore by 3.3 every regular closed is ws-closed set.
- ii) In view of the fact that every closed set is semi -closed, therefore by 3.3 every closed set is ws-closed set.
- iii) in view of the fact that every α - closed is semi -closed, therefore by 3.3 every α - closed is ws-closed set.
- iv) Let A be $g^{\#}$-closed set in X. Let U be any w-open set in X s.t $A \subseteq U$. Since A is $g^{\#}$-closed, we have $cl(A) = A \subseteq U$, we have $scl(A) \subseteq U$. Hence A is ws-closed set in X.
- v) Let A be $*g\alpha$ -closed set in X. Let U be any w-open set in X s.t $A \subseteq U$. Since A is $*g\alpha$ -closed, we have $scl(A) = A \subseteq U$, we have $scl(A) \subseteq U$. Hence A is ws-closed set in X.
- vi) Let A be $g^{\#}s$ -closed set in X. Let U be any w-open set in X s.t $A \subseteq U$. Since A is $g^{\#}s$ -closed, we have $scl(A) = A \subseteq U$, we have $scl(A) \subseteq U$. Hence A is ws-closed set in X.
- vii) Let A be rb -closed set in X. Let U be any w-open set in X s.t $A \subseteq U$. Since A is rb -closed, we have $scl(A) = A \subseteq U$, we have $scl(A) \subseteq U$. Hence A is ws-closed set in X.
- viii) Let A be $\ddot{g}$ -closed set in X. Let U be any w-open set in X s.t $A \subseteq U$. Since A is $\ddot{g}$ -closed, we have $scl(A) = A \subseteq U$, we have $scl(A) \subseteq U$. Hence A is ws-closed set in X.
- ix) Let A be $g\xi^*$ -closed set in X. Let U be any w-open set in X s.t $A \subseteq U$. Since A is $g\xi^*$ -closed, we have $scl(A) = A \subseteq U$, we have $scl(A) \subseteq U$. Hence A is ws-closed set in X.
- x) Let A be αgp -closed set in X. Let U be any w-open set in X s.t $A \subseteq U$. Since A is αgp -closed, we have $scl(A) = A \subseteq U$, we have $scl(A) \subseteq U$. Hence A is ws-closed set in X.

Remark 3.7: The converse of the above Corollary 3.6 need not be true as seen from the following Example 3.8.

Example 3.8: Let X= {a, b, c, d} and τ ={X, ϕ, {a}, {b},{a, b}, {a, b, c}} then the sets

- i. regular-closed sets in (X, τ) are X,ϕ,{a, c, d}, {b, c, d}.
- ii. closed sets in (X, τ) are X,ϕ,{d},{c, d},{a, c, d}, {b, c, d}.
- iii. α -closed, sets in (X, τ) are X,ϕ,{c},{d},{c, d},{a, c, d}, {b, c, d}.
- iv. $g^{\#}$ -closed sets in (X, τ) are X,ϕ,{d},{c, d},{a, c, d}, {b, c, d}.
- v. *gα -closed sets in (X, τ) are X,ϕ,{d},{c, d},{a, c, d}, {b, c, d}.
- vi. $g^{\#}s$ -closed sets in (X, τ) are X,ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, c, d}, {b, c, d}.
- vii. rb -closed sets in (X, τ) are X,ϕ,{c, d},{a, c, d}, {b, c, d}.
- viii. $\ddot{g}$ -closed sets in (X, τ) are X,ϕ,{d},{c, d},{a, c, d}, {b, c, d}.
- ix. gξ* -closed sets in (X, τ) are X,ϕ,{d},{c, d},{a, c, d}, {b, c, d}.
- x. αgp -closed sets in (X, τ) are X,ϕ,{c},{d},{c, d},{a, c, d}, {b, c, d}.

and

ws-closed sets in (X, τ) are X,ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, d},{a, c, d}, {b, c, d}.

It is observed that set A= {a, b, d} is ws-closed set but not regular closed (closed, α - closed, $g^{\#}$-closed, *gα –closed, $g^{\#}s$ –closed, rb -closed $\ddot{g}$-closed , gξ* -closed, αgp –closed sets) in X .

Theorem 3.9: Every ws-closed set in X is gspr-closed [10] set in X.

Proof: Let A be a ws-closed set in X. Let U be any regular open set in X such that A⊆U. Since every regular open set is w- open set and A is ws-closed set, we have scl(A) ⊆U. Therefore scl(A) ⊆U. Therefore U is regular open in X. Hence A is gspr -closed in X.

Remark 3.10: The converse of the above Theorem 3.9 need not be true as seen from the following Example 3.11.

Example 3.11: Let X = {a, b, c d}, τ = **{**X,ϕ, {a, b}, {c, d}**}.** Then the set A= {b} is gspr -closed set but not ws-closed set in X.

Corollary 3.12:

- i) Every ws-closed set is gsp-closed [8] set in X.
- ii) Every ws-closed set is rgb-closed [22] set in X.

Proof:

- i) Follow from Govindappa Navalagi et all[8], every gspr-closed set is gsp-closed set and then follows from Theorem 3.9
- ii) Let A be a ws-closed set in X. Let U be any regular open set in X such that A⊆U. Since every regular open set is w- open set and A is ws-closed set, we have scl(A) ⊆U. Therefore scl(A)⊆U. Therefore U is regular open in X. Hence A is rgb -closed in X

The converse of the Corollary 3.12 is need not be true in general as seen from the following Example 3.13.

Example 3.13: Let X = {a, b, c d}, τ = **{**X, ϕ, {a, b},{c, d}**}.** Then the set A = {b} is gsp (rgb) -closed set but not ws-closed set in X.

Remark 3.14: The following Example 3.15, shows that ws-closed sets are independent of gpr-closed [9] sets, wgrα-closed [16] sets, pgrα-closed [5] sets, $\widehat{rg}$ -closed sets [31], gp closed [30] sets, rgw-closed [29] sets, rw-closed [2] sets, rgα-closed [36] sets, βwg**-closed [35] sets.

Example 3.15: Let X = {a, b, c, d}, τ = **{**X, ϕ, {a}, {b}, {a, b}, {a, b, c}**}.** Then

- i) closed sets in (X, τ) are X, ϕ,{d},{c, d},{a, c, d}, {b, c, d}.
- ii) ws-closed sets in (X, τ) are X, ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
- iii) gpr -closed sets in (X, τ) are X, ϕ,{c},{d},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, c}, {a, b, d}, {a, c, d},{b, c, d}.
- iv) wgrα -closed sets in (X, τ) are X, ϕ,{c},{d},{a, b},{a, c},{a, d},{b, c},{b, d}, {c, d},{a, b, c}{a, b, d}, {a, c, d},{b, c, d}.
- v) pgrα -closed sets in (X, τ) are X, ϕ,{c},{d},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d}, {a, b, c},{a, b, d}, {a, c, d},{b, c, d}.

vi) $\widehat{rg}$ -closed sets in (X, τ) are X, ϕ,{c},{d},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d}, {a, b, c},{a, b, d}, {a, c, d},{b, c, d}.
vii) gprw–closed sets in (X, τ) are X, ϕ,{c},{d},{a, b},{c, d},{a, b, c},{a, b, d}, {a, c, d},{b, c, d}.
viii) rgw–closed sets in (X, τ) are X, ϕ,{c},{d},{a, b},{c, d},{a, b, c},{a, b, d}, {a, c, d}, {b, c, d}.
ix) rw-closed sets in (X, τ) are X, ϕ,{d},{a, b},{c, d},{a, b, c},{a, b, d},{a, c, d}, {b, c, d}.
x) rgα-closed sets in (X, τ) are X, ϕ,{c}{d},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d}, {a, b, c},{a, b, d}, {a, c, d},{b, c, d}.
xi) βwg**-closed sets in (X, τ) are X, ϕ,{c}{d},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, c},{a, b, d}, {a, c, d},{b, c, d}.

Therefore {a} is ws-closed in X but not gpr-closed (resp. wgrα-closed, pgrα-closed, $\widehat{rg}$ -closed, gprw-closed, rgw-closed, rw-closed, rgα-closed, βwg**-closed) set in X.

Remark 3.16: The following Example 3.17, shows that ws-closed sets are independent of sets, wg-closed[23], gwα-closed [3] sets, g*p-closed[39] sets, βwg*-closed[7] sets,**gα-closed[41] sets, $\hat{g}$-closed[38] sets, $\tilde{g}$-closed[14]sets, #gα-closed [6] sets, g*-preclosed [39] sets and g#p#-closed sets [28].

Example 3.17: Let X = {a, b, c }, τ_1 = {X, ϕ, {a}, {b},{a, b}} and τ_2 = {X, ϕ, {a}, {b, c}}. Then
i) closed sets in (X, τ_1) are X, ϕ,{c},{a, c},{b, c}.
ii) ws-closed sets in (X, τ_1) are X, ϕ,{a}, {b},{c},{a, c},{b, c}.
iii) wg-closed sets in (X, τ_1) are X, ϕ, {c},{a, c},{b, c}.
iv) gwα-closed sets in (X, τ_1) are X, ϕ, {c},{a, c},{b, c}.
v) g*p-closed sets in (X, τ_1) are X, ϕ, {c},{a, c},{b, c}.
vi) βwg*-closed sets in (X, τ_1) are X, ϕ, {c},{a, c},{b, c}.
vii) **gα-closed sets in (X, τ_1) are X, ϕ, {c},{a, c},{b, c}.
viii) $\hat{g}$-closed sets, in (X, τ_1) are X, ϕ, {c},{a, c},{b, c}.
ix) $\hat{g}$-closed sets, in (X, τ_1) are X, ϕ, {c},{a, c},{b, c}.
x) #gα-closed sets in (X, τ_1) are X, ϕ, {c},{a, c},{b, c}.
xi) g*-preclosed sets in (X, τ_1) are X, ϕ, {c},{a, c},{b, c}.
xii) g#p#-closed sets in (X, τ_1) are X, ϕ, {c},{a, c},{b, c}. and also

i) closed sets in (X, τ_2) are X, ϕ,{a},{b, c}.
ii) ws-closed sets in (X, τ_2) are X, ϕ,{a} ,{b, c}.
iii) wg-closed set in (X, τ_2) are X, ϕ, {a}, {b},{c},{a, b},{a, c},{b, c}.
iv) gwα-closed sets in (X, τ_2) are X, ϕ, {a}, {b},{c},{a, b},{a, c},{b, c}.
v) g*p-closed sets in (X, τ_2) are X, ϕ, {a}, {b},{c},{a, b},{a, c},{b, c}.
vi) βwg*-closed sets in (X, τ_2) are X, ϕ, {a}, {b},{c},{a, b},{a, c},{b, c}.
vii) **gα-closed sets in (X, τ_2) are X, ϕ, {a}, {b},{c},{a, b},{a, c},{b, c}.
viii) $\hat{g}$-closed sets in (X, τ_2) are X, ϕ, {a}, {b},{c},{a, b},{a, c},{b, c}.
ix) $\tilde{g}$-closed sets in (X, τ_2) are X, ϕ, {a}, {b},{c},{a, b},{a, c},{b, c}.
x) #gα-closed sets in (X, τ_2) are X, ϕ,{a}, {b},{c},{a, b},{a, c},{b, c}.
xi) g*-preclosed sets in (X, τ_2) are X, ϕ, {a}, {b},{c},{a, b},{a, c},{b, c}.
xii) g#p#-closed sets in (X, τ_2) are X, ϕ, {a}, {b},{c},{a, b},{a, c},{b, c}.

Therefore {b} is ws-closed set in (X, τ_1) but not in wg-closed (resp., gwα-closed, g*p-closed, βwg*-closed, **gα-closed, $\widehat{g}$ -closed, $\tilde{g}$ -closed, #gα-closed, g*-preclosed, g#p#-closed) set in (X, τ_1).

Meanwhile {b} in in wg-closed (resp., gwα-closed, g*p-closed, βwg*-closed, **gα-closed, $\widehat{g}$ -closed, $\tilde{g}$ -closed , #gα-closed , g*-preclosed, g#p#-closed)set in (X, τ_2) but not ws-closed set in (X, τ_2).

Remark 3.18: The following Example 3.19 shows that ws-closed sets are independent of sets g-closed[18] sets, sg-closed[14] sets, gα-closed[21] sets, sgb-closed[13] sets, rg*b-closed[12] sets, pgpr-closed[1] sets, gαb-closed[42] sets and rps-closed[34] sets

Example 3.19: Let X = {a, b, c, d}, τ_1 ={X, ϕ, {a}, {a, b},{a, b, c}} and τ_2 ={ X, ϕ, {a, b}, {c, d}}. Then
i) closed sets in (X, τ_1) are X, ϕ,{d},{c, d},{b, c, d}.
ii) ws-closed sets in (X, τ_1) are X, ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d}, {a, b, d},{a, c, d}, {b, c, d}.

iii) g-closed sets in (X, τ_1) are X, ϕ,{d},{a, d},{b, d},{c, d}{a, b, d},{a, c, d},{b, c, d}.
iv) sg-closed sets in (X, τ_1) are X, ϕ,{b},{c},{d},{b, c},{b, d},{c, d},{b, c, d}.
v) gα- closed sets in (X, τ_1) are X, ϕ,{b},{c},{d},{b, c},{b, d},{c, d},{b, c, d}.
vi) sgb -closed sets in (X, τ_1) are X, ϕ,{b},{c},{d},{b, c},{b, d},{c, d},{b, c, d}.
vii) rg*b- closed sets in (X, τ_1) are X, ϕ,{b},{c},{d},{b, c},{b, d},{c, d},{b, c, d}
viii) pgpr- closed sets in (X, τ_1) are X, ϕ,{b},{c},{d},{b, c},{b, d},{c, d},{b, c, d}
ix) gαb- closed sets in (X, τ_1) are X, ϕ,{b},{c},{d},{b, c},{b, d},{c, d},{b, c, d}
x) rps- closed sets in (X, τ_1) are X, ϕ,{b},{c},{d},{b, c},{b, d},{c, d},{b, c, d} and also
xi) closed sets in (X, τ_2) are X, ϕ,{c, d},{a, b}.
xii) ws-closed sets in (X, τ_2) are X, ϕ, {a, b},{c, d}.
xiii) g-closed sets in (X, τ_2) are X, ϕ,{a},{b},{c},{d},{a, b},{a, c},{a, d},{b, c},{b, d}, {c, d},{a, b, c}, {a, b, d},{a, c, d},{b, c, d}.
xiv) sg- closed sets in (X, τ_2) are X, ϕ,{a},{b},{c},{d},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d}, {a, b, c}, {a, b, d},{a, c, d},{b, c, d}.
xv) gα- closed sets in (X, τ_2) are X, ϕ,{a},{b},{c},{d},{a, b},{a, c},{a, d},{b, c}, {b, d},{c, d}, {a, b, c}, {a, b, d},{a, c, d},{b, c, d}.
xvi) sgb -closed sets in (X, τ_2) are X, ϕ,{a},{b},{c}{d},{a, b},{a, c},{a, d},{b, c}, {b, d},{c, d}, {a, b, c} {a, b, d},{a, c, d},{b, c, d}.
xvii) rg*b- closed sets in (X, τ_2) are X, ϕ,{a},{b},{c}{d},{a, b},{a, c},{a, d},{b, c}, {b, d},{c, d},{a, b, c}, {a, b, d},{a, c, d},{b, c, d}.
xviii) pgpr- closed sets in (X, τ_2) are X, ϕ,{a},{b},{c}{d},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, c}, {a, b, d},{a, c, d},{b, c, d}.
xix) gαb- closed sets in (X, τ_2) are X, ϕ,{a},{b},{c}{d},{a, b},{a, c},{a, d},{b, c},{b, d}, {c, d}, {a, b, c}, {a, b, d},{a, c, d},{b, c, d}.
xx) rps- closed sets in (X, τ_2) are X, ϕ,{a},{b},{c}{d},{a, b},{a, c},{a, d},{b, c},{b, d}, {c, d}, {a, b, c}, {a, b, d},{a, c, d},{b, c, d}.

Therefore {a} is ws-closed set in (X, τ_1) but not g-closed (resp. sg-closed, gα-closed, sgb-closed sets, rg*b-closed, pgpr-closed, gαb-closed, rps-closed) set in (X, τ_1).

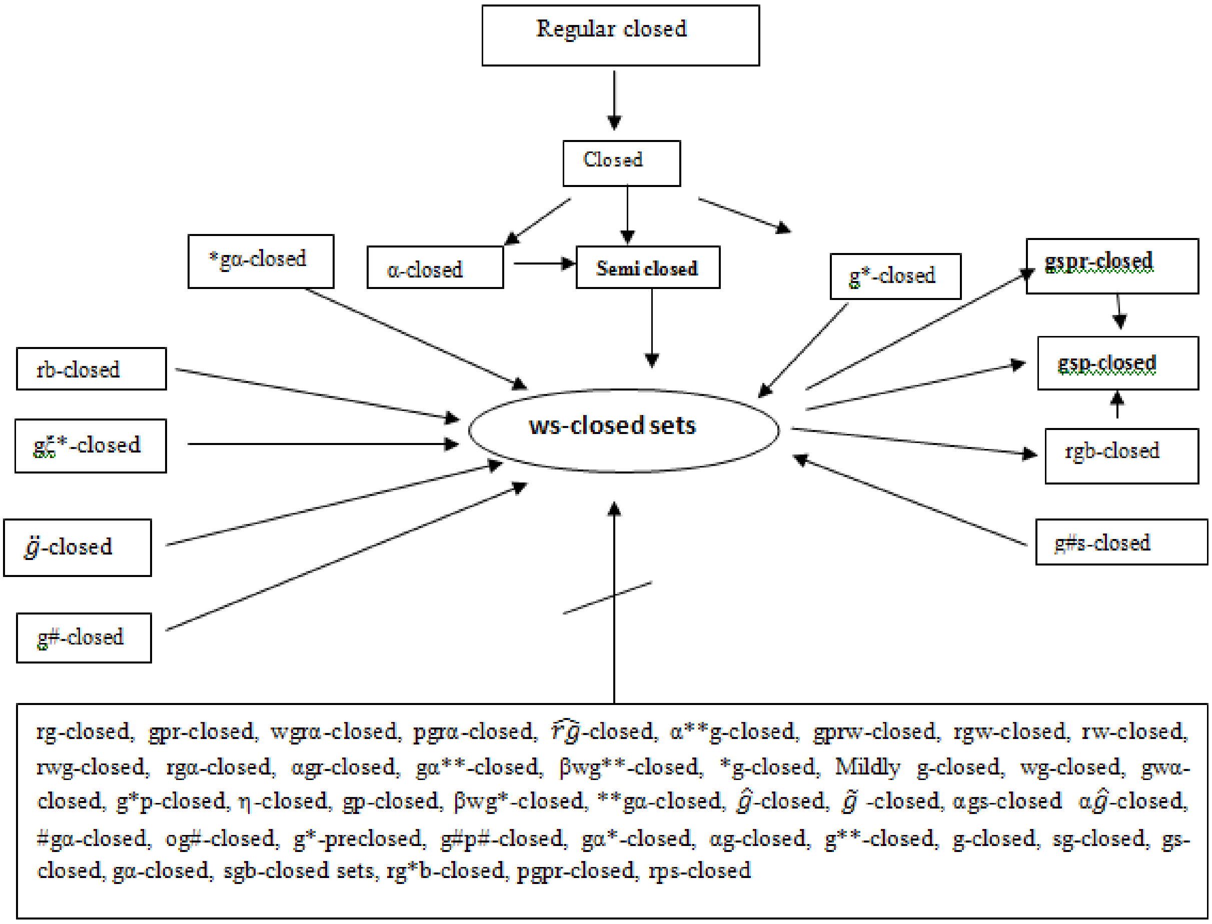
Meanwhile {a} is g-closed (resp. sg-closed, gα-closed, sgb-closed, rg*b-closed, pgpr-closed, gαb-closed, rps-closed) set in (X, τ_2) but not ws-closed set in (X, τ_2).

Remark 3.20: The following Example 3.21, shows that ws-closed sets are independent of R*-closed[15] sets, rgβ- closed[26] sets, pgrα-closed[5] sets, rgw-closed[29] sets and gprw-closed[30] sets.

Example 3.21: Let X = {a, b, c}, τ = {X, ϕ,{a}, {b}, {a, b}}. Then
i) closed sets in (X, τ) are X, ϕ,{c},{a, c},{b, c}.
ii) ws-closed sets in (X, τ) are X, ϕ, {a},{b},{c},{b, c}, {a, c}.
iii) R* -closed sets in (X, τ) are X, ϕ,{c},{a, b},{b, c},{a, c}.
iv) rgβ -closed sets in (X, τ) are X, ϕ,{c},{a, b},{b, c},{a, c}.
v) pgrα- closed sets in (X, τ) are X, ϕ,{c},{a, b},{b, c},{a, c}
vi) rgw- closed sets in (X, τ) are X, ϕ,{c},{a, b},{b, c},{a, c}
vii) gprw -closed sets in (X, τ) are X, ϕ,{c},{a, b},{b, c},{a, c}

Therefore {a} is ws-closed set in X but not R*-closed (resp. rgβ- closed, pgrα-closed, rgw-closed, gprw-closed) set in X.

Remark 3.22: From the above discussion and results we have the following implications.



A $\longrightarrow$ B means A implies B, but converse is not true.

A $\not\longleftrightarrow$ B means A and B are independent of each other

Theorem 3.23: The intersection of two ws-closed subsets of X is ws-closed set in X.

Proof: Let A and B be are ws-closed sets in X. Let U be any semiopen set in X such that $(A \cap B) \subseteq U$ that is $A \subseteq U$ and $B \subseteq U$. Since A and B are ws-closed sets then $scl(A) \subseteq U$ and $scl(B) \subseteq U$ and we know that $(scl(A) \cap scl(B)) = scl(A \cap B) \subseteq U$. Therefore $A \cap B$ is ws-closed set in X.

Remark 3.24: The union of two ws-closed sets in X is generally not a ws-closed set in X.

Example 3.25: Let X = {a, b, c} and τ = {ϕ, X, {a}, {b},{a, b}} then the sets A={a} and B={b}are ws-closed sets in X but $A \cup B$ ={a, b} is not a ws-closed set in X.

Theorem 3.26: If a subset A of a topological space X is ws-closed set in X then scl(A)-A does not contain any non-empty open set in X but converse is not true.

Proof: Let A is an ws-closed set in X and suppose F be an non empty w-closed subset of scl(A)-A.
$F \subseteq scl(A)-A \Longrightarrow F \subseteq scl(A) \cap (X-A) \Longrightarrow F \subseteq scl(A) \longrightarrow (1)$ & $F \subseteq X-A$
$\Longrightarrow A \subseteq X-F$ and X-F is w-open set and A is a ws-closed set, $scl(A) \subseteq X-F$
$\Longrightarrow F \subseteq X-scl(A) \longrightarrow (2)$ from equations (1) and (2) we get $F \subseteq scl(A) \cap (X-scl(A)) = \phi$
$\Longrightarrow F = \Phi$ thus scl(A)-A does not contain any non-empty w-closed set in X.

Remark 3.27: The converse of the above Theorem need not be true as seen from the following Example 3.28.

Example 3.28: Let X={a, b, c, d} τ= **{**X,ϕ,{a},{a, b},{a, b, c}**}** then the set A={b} scl{b}={b}, scl{A}-A={b} does not contain any non-empty w-closed set in X but A is not ws-closed set.

Theorem 3.29: If A is a ws-closed set in X and A⊆B⊆scl(A) then B is also ws-closed set in X**.**

Proof: Let A be a ws-closed set in X such that B⊆scl(A). Let U be a w-open set of X such that B⊆U then A⊆U. Since A is ws-closed set, we have scl(A)⊆U and A⊆U. Now B⊆scl(A) ⟹ scl(B)⊆scl(scl(A))=scl(A)⊆U. That is scl(B)⊆U. Therefore B is a ws-closed set in X.

Remark 3.30: The converse of the above Theorem 3.29 is need not be true as seen from the following Example 3.31.

Example 3.31: Let X = {a, b, c }, τ = {ϕ, X, {a}, {b}, {a, b}}, then the set A={a}, B={a,c} such that A and B are ws-closed sets in X but A⊆B⊈scl(A) because scl(A)={a}.

Theorem 3.32: Let (X,τ) be a topological space then for each x∈ X the set X-{x} is ws-closed or semi open.

Proof: Let x∈X. Suppose X-{x} is not a semiopen set. Then X is the only semiopen set containing X-{x}, that is X-{x} ⊆X ⟹ cl(X-{x}) ⊆cl(X) ⟹ cl(X-{x})⊆X. Therefore X-{x} is ws-closed set in X.

Theorem 3.33: Let X and Y are topological spces and A⊆Y⊆X. Suppose that A is ws-closed set in X then A is ws-closed relative to Y.

Proof: Let A⊆Y∩G, where G is a w-open. Since A is a ws-closed set in X, then A⊆G and scl(A)⊆G. This implies that Y∩scl(A) ⊆Y∩G where Y ∩scl(A) is closed set of A in Y. Thus A is a ws-closed relative to Y.

Theorem 3.34: In a topological space X if SO(X) ={X,ϕ} then every subset of X is a ws-closed set**.**

Proof: Let X be a topological space and SO(X) = {X,ϕ}. Let A be any subset of X. Suppose A=ϕ. Then ϕ is ws-closed set. Suppose A≠ϕ. Then X is the only semiopen set containing A and so scl(A)⊆X. Hence A is a ws-closed set in X.

Remark 3.35: The converse of the above Theorem need not be true in general as seen from the following Example 3.36..

Example 3.36: Let X = {a, b, c}, τ = {ϕ, X, {a}, {b, c}}. Then every subset of (X,τ) is a ws-closed set in X but SO={ϕ, X, {a}, {b, c}}.

Theorem 3.37: If A is regular open and gspr-closed set in X then A is ws-closed set in X.

Proof: Let A be a regular open and gspr-closed set in X. Let U be any w-open set in X such that A⊆U. Since A is regular open and gspr-closed set in X, by definition, scl(A)⊆A then scl(A)⊆A⊆U. Hence A is ws-closed set in X.

Theorem 3.38: If A is regular open and rgb-closed set then A is ws-closed set in X.

Proof: Let A be a regular open and rgb-closed in X. Let U be any w-open set in X such that A⊆U. Since A is regular open and rgb-closed in X, by definition, scl(A)⊆A then scl(A)⊆A⊆U. Hence A is ws-closed set in X.

Theorem 3.39: If A is semiopen and swg*-closed then A is ws-closed set in X.

Proof: Let A be a semiopen and swg*-closed in X. Let U be any w- open set in X such that A⊆U. Since A is semiopen and swg*-closed in X, by definition, scl(A)⊆A then scl(A)⊆A⊆U. Hence A is ws-closed set in X.

Theorem 3.40: If A is semiopen and swg-closed then A is ws-closed set in X.

Proof: Let A be a semiopen and swg-closed in X. Let U be any w- open set in X such that A⊆U. Since A is semiopen and swg-closed in X, by definition, scl(A)⊆A then scl(A)⊆A⊆U. Hence A is ws-closed set in X.

Theorem 3.41: If A is semiopen and sg-closed then A is ws-closed set in X.

Proof: Let A be a semiopen and sg-closed in X. Let U be any w- open set in X such that A⊆U. Since A is semiopen and sg-closed in X, by definition, scl(A)⊆A then scl(A)⊆A⊆U. Hence A is ws-closed set in X.

Theorem 3.42: If A is semiopen and sgb-closed then A is ws-closed set in X.

Proof: Let A be a semiopen and sgb-closed in X. Let U be any w- open set in X such that A⊆U. Since A is semiopen and sgb-closed in X, by definition, scl(A)⊆A then scl(A)⊆A⊆U. Hence A is ws-closed set in X.

Theorem 3.43: If A is semiopen and αgs-closed then A is ws-closed set in X.

Proof: Let A be a semiopen and αgs -closed in X. Let U be any w- open set in X such that A⊆U. Since A is semiopen and αgs -closed in X, by definition, scl(A)⊆A then scl(A)⊆A⊆U. Hence A is ws-closed set in X.

Theorem 3.44: If A is β-open and βwg*-closed then A is ws-closed set in X**.**

Proof: Let A be a β-open and βwg*-closed in X. Let U be any regular semiopen set in X such that A⊆U. Since A is β-open and βwg*-closed in X, by definition, gcl(A)⊆A then gcl(A)⊆A⊆U. Hence A is ws-closed set in X.

Theorem 3.45: If A is both open and g-closed then A is ws-closed set in X**.**

Proof: Let A be open and g-closed set in X. Let U be any regular open set in X such that A⊆U. By definition, cl(A)⊆A⊆U and gcl(A)=A. This implies that cl(A)⊆gcl(A) ⊆A⊆U ⟹gcl(A)⊆U. Hence A is ws-closed set.

Theorem 3.46: If A is regular semiopen and rw-closed then A is ws-closed set in X**.**

Proof: Let A be a regular semiopen and rw-closed set in X. Let U be any w-open set in X such that A⊆U. Now A⊆A by hypothesis cl(A)⊆A then we know that cl(A) ⊆scl(A)⊆A. Hence scl(A)⊆U therefore A is ws-closed set in X.

Theorem 3.47: If A is regular semiopen and R*-closed then A is ws-closed set in X.

Proof: Let A be a regular semiopen and R*-closed set in X. Let U be any w-open set in X such that A⊆U. Now A⊆A by hypothesis cl(A)⊆A then we know that cl(A) ⊆scl(A) ⊆A. Hence scl(A)⊆U therefore A is ws-closed set in X.

Theorem 3.48: If A is regular semiopen and gprw-closed then A is ws-closed set in X.

Proof: Let A be a regular semiopen and gprw -closed set in X. Let U be any w-open set in X such that A⊆U. Now A⊆A by hypothesis cl(A)⊆A then we know that cl(A) ⊆scl(A) ⊆A. Hence scl(A)⊆U therefore A is ws-closed set in X.

Theorem 3.49: If A is regular semiopen and rgw-closed then A is ws-closed set in X.

Proof: Let A be a regular semiopen and rgw -closed set in X. Let U be any w-open set in X such that A⊆U. Now A⊆A by hypothesis cl(A)⊆A then we know that cl(A) ⊆scl(A) ⊆A. Hence scl(A)⊆U therefore A is ws-closed set in X.

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