International Journal of Mathematical Archive-8(9), 2017, 52-58
JMA Available online through www.ijma.info ISSN 2229 – 5046

# PATH RELATED NEAR MEAN CORDIAL GRAPHS

L. PANDISELVI*, S. NAVANEETHAKRISHNAN AND A. NAGARAJAN

PG and Research Department of Mathematics,
V. O. Chidambaram College, Tuticorin-628008, Tamil Nadu, India.

*(Received On: 04-08-17; Revised & Accepted On: 31-08-17)*

*ABSTRACT*

*Let G = (V, E) be a simple graph. A* ***Near Mean Cordial Labeling*** *of G is a function in f : V(G) → {1, 2, 3, . . . , p−1, p+1} such that the induced map f* defined by*

$$f^*(uv) = \begin{cases} 1 & if\big(f(u)+f(v)\big) \equiv 0\ (mod 2) \\ 0 & else \end{cases}$$

*and it satisfies the condition* $|e_f(0) - e_f(1)| \leq 1$*, where* $e_f(0)$ *and* $e_f(1)$ *represent the number of edges labeled with 0 and 1 respectively. A graph is called a* ***Near Mean Cordial Graph*** *if it admits a near mean cordial labeling.*

*In this paper, It is to be proved that* $P_n \times K_2$ , $P_n@2K_{1,n}$ *and* $H_n^+$ *are* ***Near Mean Cordial*** *graphs.*

***AMS Mathematics subject classification 2010:*** *05C78.*

***Keywords and Phrases:*** *Cordial labeling, Near Mean Cordial Labeling and Near Mean Cordial Graph.*

## 1. INTRODUCTION

By a graph, it means a finite undirected graph without loops or multiple edges. For graph theoretic terminology, Harary [4] and G.J. Gallian[1] are referred.

A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge *uv* a label depending on the vertex labels of *u* and *v*.

A graph *G* is said to be labeled if the *n* vertices are distinguished from a given set, which induces distinguish edge values satisfying certain conditions. The concept of graceful labeling was introduced by Rosa [3] in 1967 and subsequently by Golomb [2].

In this paper, It is to be proved that $P_n \times K_2$ , $P_n@2K_{1,n}$ and $H_n^+$ are **Near Mean Cordial** graphs.

## 2. PRELIMINARIES

**Definition 2.1:** Let G = (V, E) be a simple graph. Let f:V(G)→{0,1} and for each edge uv, assign the label |f(u) − f(v)|. f is called a **cordial labeling** if the number of vertices labeled 0 and the the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the the number of edges labeled 1 differ by atmost 1. A graph is called **Cordial** if it has a cordial labeling.

**Definition 2.2:** Let G = (V, E) be a simple graph. A **Near Mean Cordial Labeling** of G is a function in f : V(G) → {1, 2, 3, . . . , p−1, p+1}such that the induced map f* defined by

$$f^*(uv) = \begin{cases} 1 & if\big(f(u)+f(v)\big) \equiv 0\ (mod 2) \\ 0 & else \end{cases}$$

and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a **Near Mean Cordial Graph** if it admits a near mean cordial labeling.

*Corresponding Author: L. Pandiselvi**,
*PG and Research Department of Mathematics,*
*V. O. Chidambaram College, Tuticorin-628008, Tamil Nadu, India.*

**Definition 2.3:** $P_n@2K_{1,n}$ is a graph which is obtained by joining the root of the star $K_{1,n}$ to the end vertex of the path $P_n$.

**Definition 2.4:** Define the product $G_1 \times G_2$ , by consider any two vertices u = ($u_1$ ,$u_2$), and v = ($v_1$ , $v_2$) in $V_1 \times V_2$ Then u and v are adjacent in $G_1 \times G_2$.whenever ($u_1 = v_1$ and $u_2$ adj to $v_2$ ) or ($u_2 = v_2$ and $u_1$ adj to $v_1$ ).

The product $P_m \times P_n$ is called planar grids and $\mathbf{K_2 \times P_n}$ is called Ladder. The product $C_m \times P_n$ is called Grids on cylinder of order mn. In particular, $D_n = C_n \times K_2$ is called a prism and $B_m = K_{1,m} \times K_2$ is called a book.

**Definition 2.5:** $G^+$ is a graph obtained from G by attaching a pendant vertex from each vertex of the graph G.

## 3. MAIN RESULTS

**Theorem 3.1:** $P_n \times K_2$ is a Near Mean Cordial Graph $\forall\, n \geq 2$ .

**Proof:** Let V($P_n \times K_2$) = $\{u_i : 1 \leq i \leq n,\ v_i : 1 \leq i \leq n\}$.
Let E ( $P_n \times K_2$ ) = $\{(u_i v_i) : 1 \leq i \leq n\} \cup \{(u_i u_{i+1}) :\ 1 \leq i \leq n-1\} \cup \{(v_i v_{i+1}) :\ 1 \leq i \leq n-1\}$

**Case (i): when n = 2 and n = 3**

Define f : V( $P_n \times K_2$) → {1, 2, 3, . . . , 2n−1 , 2n+1} by

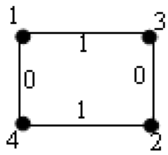


**Figure: 3.1.1**

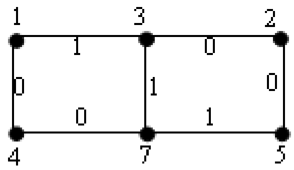


**Figure: 3.1.2**

**Case (ii): when n > 3**

Define f : V( $P_n \times K_2$) → {1, 2, 3, . . . , 2n−1 , 2n+1} by

**when n is even:**

Let f($u_1$) = 1
f($u_{2i+1}$) = $\frac{n}{2}$+i+1, $1 \leq i \leq \frac{n-2}{2}$
f($u_{2i}$) = 1+i, $1 \leq i \leq \frac{n}{2}$
f($v_{2i-1}$) = $n + i$, $1 \leq i \leq \frac{n}{2}$
f($v_{2i}$) = $\frac{3n}{2} + i$, $1 \leq i \leq \frac{n-2}{2}$
f($v_n$) = 2n+1

**when n is odd :**

Let f($u_1$) = 1, f($u_2$) = 2
f($u_{2i+1}$) = 2+$i$, \ $1 \leq i \leq \frac{n-1}{2}$
f($u_{2i}$) = $\frac{n+3}{2} + i - 1$, $2 \leq i \leq \frac{n-3}{2}$
f($v_{2i-1}$) = $n + i$, $1 \leq i \leq \frac{n+1}{2}$
f($v_{2i}$) = $\frac{3(n+1)}{2} + (i-1)$, $1 \leq i \leq \frac{n-3}{2}$
f($v_{n-1}$) = 2n+1

The induced edge labeling are

$$f^*(u_i\, v_i) = \begin{cases} 1 & \text{if } f(u_i) + f(v_i) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases} \quad , 1 \leq i \leq n$$

$$f^*(u_i\, u_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases} \quad , 1 \leq i \leq n-1$$

$$f^*(v_i\, v_{i+1}) = \begin{cases} 1 & \text{if } f(v_i) + f(v_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases} \quad , 1 \leq i \leq n-1$$

(i) Let $n = 2k,\ (k \epsilon N)$
Here, $e_f(1) = e_f(0) = n + k - 1$.

(ii) Let $n = 2k + 1$, $(2, 4, 6, \ldots \epsilon N)$
Here, $e_f(0) = n + k$ and $e_f(1) = n + k - 1$
(iii) Let $n = 2k + 1$, $(3, 5, \ldots \epsilon N)$
Here, $e_f(0) = n + k - 1$ and $e_f(1) = n + k$

Hence, it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, $P_n \times K_2$ is a Near Mean Cordial Graph.

For example, the Near Mean Cordial Labeling of $P_6 \times K_2$, $P_7 \times K_2$ and $P_9 \times K_2$ are shown in Figures 3.1.3 - 3.1.5.

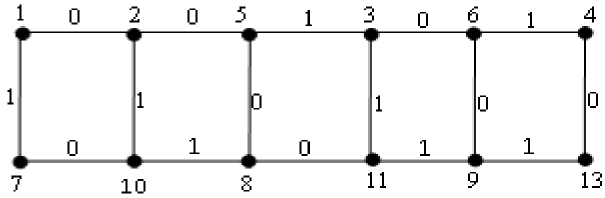

**Figure: 3.1.3**

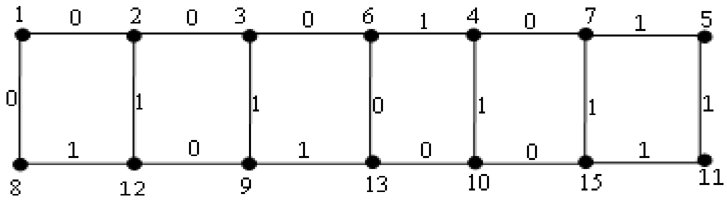

**Figure: 3.1.4**

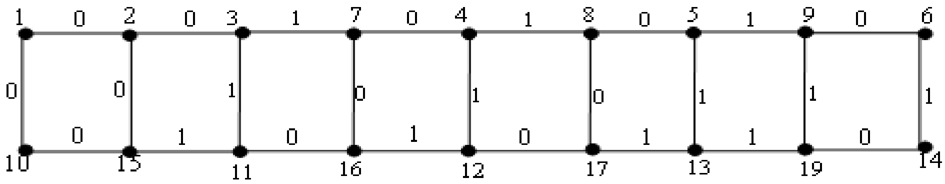

**Figure: 3.1.5**

**Theorem 3.2:** $P_n @2\ K_{1,n}$ is a Near Mean Cordial Graph.

**Proof:** Let $V(P_n @2\ K_{1,n}) = \{u_i : 1 \leq i \leq n,\ v_i : 1 \leq i \leq n, w_i : 1 \leq i \leq n\}$.
Let $E(P_n @2\ K_{1,n}) = \{(u_i w_i) : 1 \leq i \leq n\} \cup \{(w_i w_{i+1}) : 1 \leq i \leq n-1\} \cup \{w_i v_i : 1 \leq i \leq n\}$.

**When n ≡ $\mathbf{0}\ (\boldsymbol{mod}\ \mathbf{4})$:**

Define f: $V(P_n @2\ K_{1,n}) \rightarrow \{1, 2, 3, \ldots, 3n-1, 3n+1\}$ by
$f(u_i) = 2i - 1, \quad 1 \leq i \leq n$
$f(v_i) = 2i, \quad 1 \leq i \leq n$
$f(w_{2i-1}) = 2n + i, \quad 1 \leq i \leq \frac{n}{2}$
$f(w_2) = 3n+1$
$f(w_{2(i+1)}) = 3n - i, \quad 1 \leq i \leq \frac{n-2}{2}$

**When n ≡ $\mathbf{1}\ (\boldsymbol{mod}\ \mathbf{4})$:**

Define f : $V(P_n @2\ K_{1,n}) \rightarrow \{1, 2, 3, \ldots, 3n-1, 3n+1\}$ by
$f(u_i) = 2i - 1, \quad 1 \leq i \leq n$
$f(v_i) = 2i, \quad 1 \leq i \leq n$
$f(w_{2i-1}) = 2n + i, \quad 1 \leq i \leq \frac{n+1}{2}$
$f(w_2) = 3n+1$
$f(w_{2(i+1)}) = 3n - i, \quad 1 \leq i \leq \frac{n-3}{2}$

 

**When $n \equiv 2\ (mod\ 4)$:**

Define f : V($P_n$@2 $K_{1,n}$) → {1, 2, 3, . . . , 3n−1 , 3n+1} by

f($u_i$) = $2n + i$, $1 \le i \le n-1$

f($u_n$) = 3n+1

f($v_i$) = $n + i$, $1 \le i \le n$

f($w_{2i-1}$) = $i$, $1 \le i \le \frac{n}{2}$

f($w_{2i}$) = $\frac{n}{2} + i$, $1 \le i \le \frac{n}{2}$

**When $n \equiv 3\ (mod\ 4)$:**

Define f : V($P_n$@2 $K_{1,n}$) → {1, 2, 3, . . . , 3n−1 , 3n+1} by

f($u_i$) = $2n + i$, $1 \le i \le n-1$

f($u_n$) = 3n+1

f($v_i$) = $n + i$, $1 \le i \le n$

f($w_{2i-1}$) = $i$, $1 \le i \le \frac{n+1}{2}$

f($w_{2i}$) = $\frac{n+1}{2} + i$, $1 \le i \le \frac{n-1}{2}$

From all the cases, The induced edge labelings are

$f^*(u_i w_i) = \begin{cases} 1 & \text{if } f(u_i) + f(w_i) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$, $1 \le i \le n$

$f^*(w_i w_{i+1}) = \begin{cases} 1 & \text{if } f(w_i) + f(w_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$, $1 \le i \le n-1$

$f^*(w_i v_i) = \begin{cases} 1 & \text{if } f(w_i) + f(v_i) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$, $1 \le i \le n$

Let $n = 2k + 1$, $(k \epsilon N)$

Here, $e_f(1) = e_f(0) = n + k$.

Let $n = 2k$, $(k \epsilon N)$

Here, $e_f(1) = n + k - 1$ and $e_f(0) = n + k$, (when $k \equiv 0\ (mod\ 2)$) .

Here, $e_f(0) = n + k - 1$ and $e_f(1) = n + k$, (when $k \equiv 1\ (mod\ 2)$).

So, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, $P_n$@2 $K_{1,n}$ is a Near Mean Cordial Graph .

For example, the Near Mean Cordial Labeling of $P_8$@2 $K_{1,8}$ , $P_9$@2 $K_{1,9}$ , $P_6$@2 $K_{1,6}$ and $P_7$@2 $K_{1,7}$ are shown in Figures 3.2.1 - 3.2.4.

**When n ≡ 0 (mod 4):**

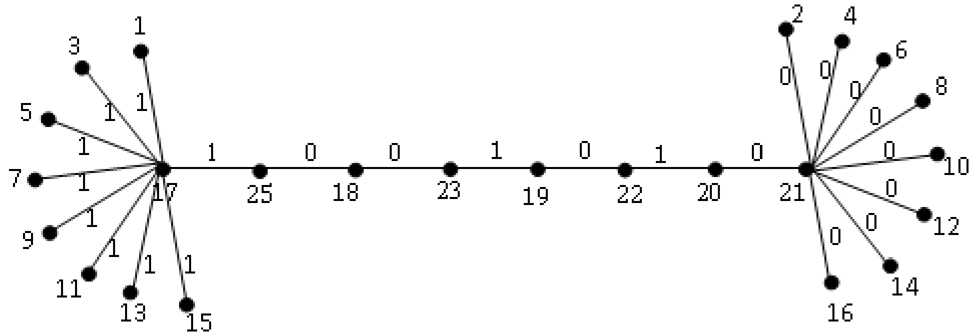


**Figure: 3.2.1**

**When n ≡ 1 (mod 4):**

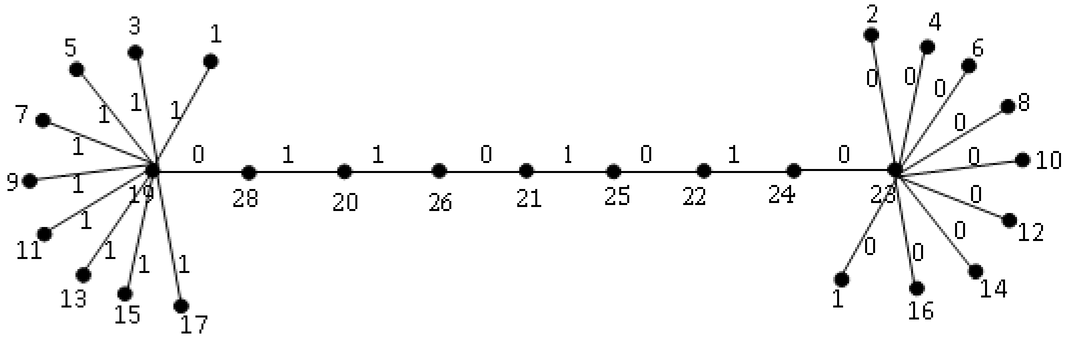


**Figure: 3.2.2**

**When n ≡ 2 (mod 4):**

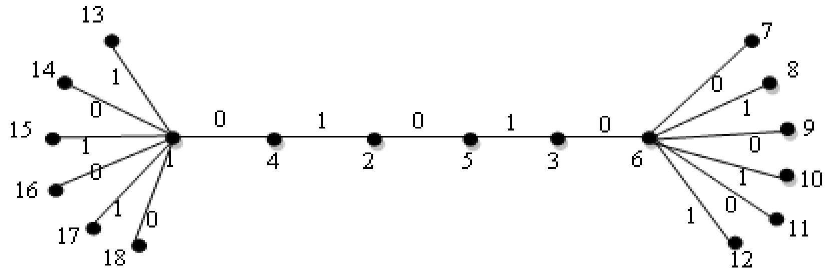


**Figure: 3.2.3**

**When n ≡ 3 (mod 4):**

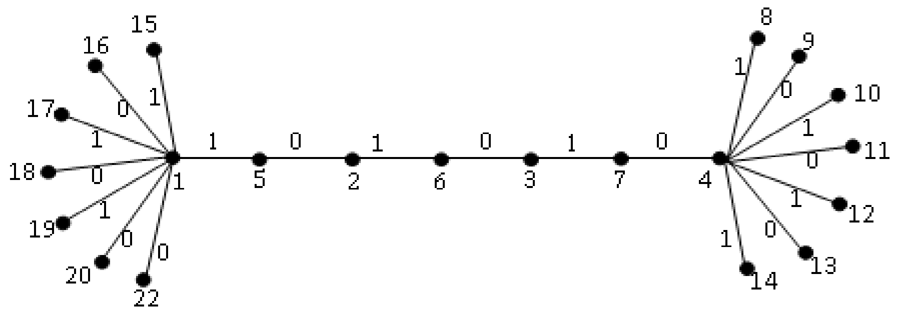


**Figure: 3.2.4**

**Theorem 3.3:** $H_n^+$ (n : odd) is a Near Mean Cordial Graph.

**Proof:** Let V($H_n^+$) = $\{u_i, v_i : 1 \le i \le n, u_i', v_i' : 1 \le i \le n\}$.

Let E($H_n^+$) = $\{(u_i u_{i+1}), (v_i v_{i+1}) : 1 \le i \le n-1\} \cup \{u_i u_i', v_i v_i' : 1 \le i \le n\} \cup \{u_{\left(\frac{n+1}{2}\right)} v_{\left(\frac{n+1}{2}\right)}\}$.

Define f : V($H_n^+$) → {1, 2, 3, . . . , 4n−1 , 4n+1} by

$f(u_{2i-1}) = 2n - 2(i-1), \quad 1 \le i \le \frac{n+1}{2}$

$f(u_{2i}) = 2i, \quad 1 \le i \le \frac{n-1}{2}$

$f(u'_{2i-1}) = 2i - 1, \quad 1 \le i \le \frac{n+1}{2}$

$f(u'_{2i}) = 2n - 1 - 2(i-1), \quad 1 \le i \le \frac{n-1}{2}$

$f(v_1) = 4n + 1$

$f(v_{2i+1}) = 4n - 2 - 2(i-1), \quad 1 \le i \le \frac{n-1}{2}$

$f(v_{2i}) = 2n + 2 + 2(i-1), \quad 1 \le i \le \frac{n-1}{2}$

$f(v'_{2i-1}) = 2n + 1 + 2(i-1), \quad 1 \le i \le \frac{n+1}{2}$

$f(v'_{2i}) = 4n - 1 - 2(i-1), \quad 1 \le i \le \frac{n-1}{2}$

The induced edge labelings are

$$f^*(u_i u_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n-1$$

$$f^*(u_i u_i') = \begin{cases} 1 & \text{if } f(u_i) + f(u_i') \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n$$

$$f^*(v_i v_{i+1}) = \begin{cases} 1 & \text{if } f(v_i) + f(v_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n-1$$

$$f^*(v_i v_i') = \begin{cases} 1 & \text{if } f(v_i) + f(v_i') \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n$$

$$f^*(u_{\left(\frac{n+1}{2}\right)} v_{\left(\frac{n+1}{2}\right)}) = 1$$

Here, $e_f(0) = 2n$ and $e_f(1) = 2n-1$

So, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, $H_n^+$ is a Near Mean Cordial Graph .

For example, the Near Mean Cordial Labeling of $H_7^+$ is shown in the Figure 3.3.1.

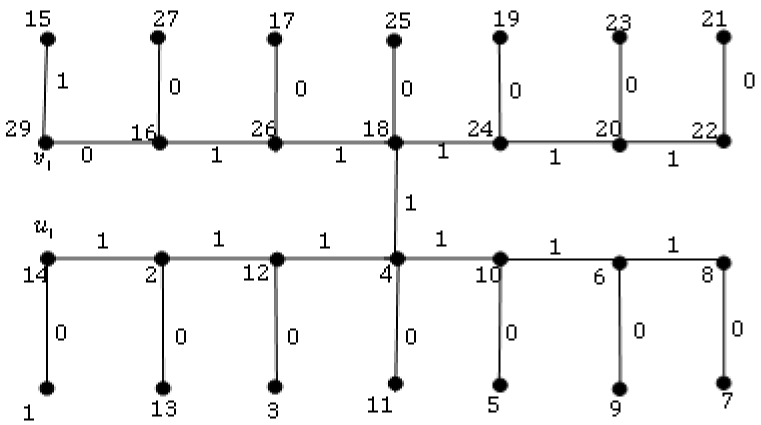


**Figure: 3.3.1**

**Theorem 3.4:** $H_n^+$ (n : even) is a Near Mean Cordial Graph .

**Proof:** Let V $(H_n^+) = \{u_i, v_i : 1 \le i \le n, u_i', v_i' : 1 \le i \le n\}$.

Let E $(H_n^+) = \{(u_i u_{i+1}), (v_i v_{i+1}) : 1 \le i \le n-1\} \cup \{u_i u_i', v_i v_i' : 1 \le i \le n\} \cup \{u_{\left(\frac{n}{2}+1\right)} v_{\left(\frac{n}{2}\right)}\}$ .

Define f : V$(H_n^+) \to \{1, 2, 3, \ldots, 4n-1, 4n+1\}$ by

$$f(u_{2i-1}) = 2n - 2(i-1), \quad 1 \le i \le \frac{n}{2}$$
$$f(u_{2i}) = 2i, \quad 1 \le i \le \frac{n}{2}$$
$$f(u_{2i-1}') = 2i - 1, \quad 1 \le i \le \frac{n}{2}$$
$$f(u_{2i}') = 2n - 1 - 2(i-1), \quad 1 \le i \le \frac{n}{2}$$
$$f(v_1) = 4n + 1$$
$$f(v_{2i+1}) = 4n - 2 - 2(i-1), \quad 1 \le i \le \frac{n-2}{2}$$
$$f(v_{2i}) = 2n + 2 + 2(i-1), \quad 1 \le i \le \frac{n}{2}$$
$$f(v_{2i-1}') = 2n + 1 + 2(i-1), \quad 1 \le i \le \frac{n}{2}$$
$$f(v_{2i}') = 4n - 1 - 2(i-1), \quad 1 \le i \le \frac{n}{2}$$

The induced edge labelings are

$$f^*(u_i u_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n-1$$

$$f^*(u_i u_i') = \begin{cases} 1 & \text{if } f(u_i) + f(u_i') \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n$$

$$f^*(v_i v_{i+1}) = \begin{cases} 1 & \text{if } f(v_i) + f(v_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n-1$$

 

$$f^*(v_i v_i') = \begin{cases} 1 & \text{if } f(v_i) + f(v_i') \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n$$

$$f^*(u_{(\frac{n}{2}+1)} v_{(\frac{n}{2})}) = 1$$

Here, $e_f(0) = 2n$ and $e_f(1) = 2n - 1$

So, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, $H_n^+$ is a Near Mean Cordial Graph.

For example, the Near Mean Cordial Labeling of $H_8^+$ is shown in the Figure 3.4.1.

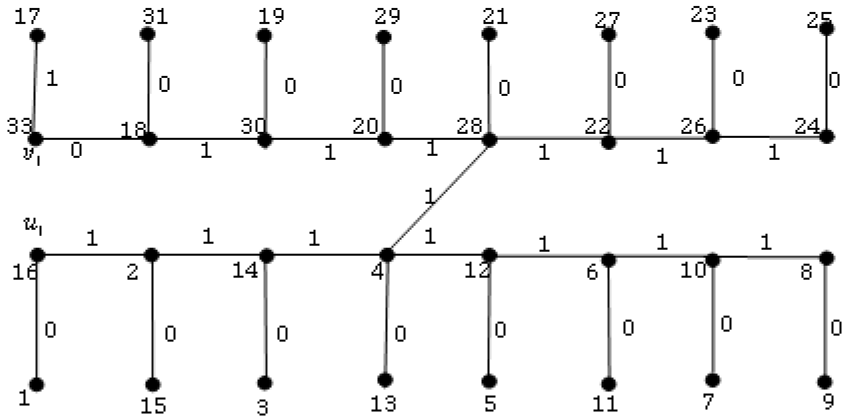


**Figure: 3.4.1**

## 4. REFERENCES

1. G.J. Gallian, *A Dynamic survey of graph labeling*, The electronic journal of combinotorics, 16 (2009), #DS6.
2. S.W. Golombo, *How to number a graph in graph Theory and Computing*, R.C.Read, ed., Academic Press, New York (1972) 23-37.
3. A. Rosa, *On certain valuations of the vertices of a graph*, Theory of graphs (International Symposium, Rome), July (1966).
4. Frank Harary, *Graph Theory*, Narosa publishing house pvt. Ltd., 10th reprint 2001.
5. J. Gross and J. Yellen, *Hand book of graph theory*, CRC Press, 2004.
6. F. Harary, *Graph Theory*, Addition- Wesley, Reading, Mass, 1972.
7. Proceedings, UGC National Seminor on "Recent Developments in Functional Analysis, Topology and Graph Theory", M. D. T. Hindu College, Tirunelveli, 26th and 27th March 2015 *Near Mean Cordial Labeling of Path Related Graphs.*
8. L. Pandiselvi, A. Nellai Murugan, and S. Navaneethakrishnan, *Some Results On Near Mean Cordial Graphs*, Global Journal of Mathematics, Vol.4. No.2 October 6, 2015 ISSN 2395-4760.
9. L. Pandiselvi, S. Navaneethakrishnan and A. NellaiMurugan, *Near Mean Cordial-Path Related Graphs*, International Journal Of Scientific Research and Development Vol. 4, Issue 8, Oct 2016. Pg. 62 – 64.
10. L. Pandiselvi, S. Navaneethakrishnan and A. Nagarajan, Cycle and *Path Related Near Mean Cordial* Graphs, Global Journal of Pure and Applied Mathematics (Accepted).
11. L. Pandiselvi, S. Navaneethakrishnan and A. Nagarajan, Twig and Cycle *Related Near Mean Cordial* Graphs, International Journal of Mathematics And its Applications (Accepted).

**Source of support: Nil, Conflict of interest: None Declared.**