

**COLORFUL TOTAL DOMINATION  
 WITH RESPECT TO COLOR CLASS DOMINATION PARTITION**

**V. PRABA\*<sup>1</sup>, P. ARISTOTLE<sup>2</sup> AND V. SWAMINATHAN<sup>3</sup>**

<sup>1</sup>Research scholar, Sri Chandrasekharendra Saraswathi Viswa Mahavidyalaya,  
 Kanchipuram – 631 561, Tamil Nadu, India.

<sup>1</sup>Department of Mathematics,  
 Rajalakshmi Engineering College, Chennai – 602 105, Tamil Nadu, India.

<sup>2</sup>PG & Research Department of Mathematics,  
 Raja Doraisingam Government Arts College, Sivagangai – 630 561, Tamil Nadu, India.

<sup>3</sup>Ramanujan Research Center in Mathematics,  
 Saraswathi Narayanan College, Madurai – 625 022, Tamil Nadu, India.

(Received On: 22-07-17; Revised & Accepted On: 29-08-17)

**ABSTRACT**

Let  $G$  be a finite, simple and undirected graph. A partition of  $V(G)$  into independent sets such that each element of the partition is dominated by a vertex of  $G$  is called color class domination partition of  $G$  [1],[2],[6],[7]. The minimum cardinality of such a partition is called the color class domination partition number of  $G$  and is denoted by  $\chi_{cd}(G)$ . If  $S$  is a  $\chi_{cd}$ -partition of  $G$ , then the set consisting of one vertex from each element of the partition need not be a dominating set. Since  $\Pi = \{\{u_1\}, \{u_2\}, \dots, \{u_n\}\}$  where  $V(G) = \{u_1, u_2, \dots, u_n\}$  is a  $cd$ -partition of  $G$  such that the set consisting of one element from each partition is a dominating set of  $G$ . The minimum cardinality of a  $cd$ -partition in which the set consisting one element from each set of the partition is a dominating set is called the colorful domination number of  $G$  with respect to  $cd$ -partition of  $G$  and is denoted by  $\chi_y^{cd}(G)$ . If  $G$  has no isolates, then the trivial partition is a  $cd$ -partition which gives rise to a total dominating set of  $G$ . The minimum cardinality of a  $cd$ -partition which gives rise to a total dominating set of  $G$  is called the colorful total domination number of  $G$  and is denoted by  $\chi_t^{cd}(G)$ . In this paper, a study of this new parameter is initiated.  $\chi_t^{cd}(G)$  for well known graphs are found, bounds are obtained and for bipartite graph in certain conditions, the value of  $\chi_{cd}$  is obtained and bound for  $\chi_t^{cd}$  is also derived.

**Keywords:** Color class domination, total domination.

**1. DEFINITION AND  $\chi_t^{cd}(G) / \chi_t^{cd}(G)$  FOR WELL KNOWN GRAPHS**

**Definition 1.1:** Let  $G$  be a simple, finite, undirected graph without isolates. Let  $\Pi = \{V_1, V_2, \dots, V_k\}$  be a  $cd$ -partition of  $G$ . Let  $x_i \in V_i$ ,  $(1 \leq i \leq k)$ , be such that  $\{x_1, x_2, \dots, x_k\}$  is a total dominating set of  $G$ . Then  $\Pi$  is called a  $\Pi_t^{cd}$ -partition of  $G$ . The minimum cardinality of a  $\Pi_t^{cd}$ -partition of  $G$  is denoted by  $\chi_t^{cd}(G)$ .

**Note 1.2:** The existence of  $\Pi_t^{cd}$ -partition of  $G$  is guaranteed, Since  $\Pi = \{\{u_1\}, \{u_2\}, \dots, \{u_n\}\}$  where  $V(G) = \{u_1, u_2, \dots, u_n\}$  is a  $\Pi_t^{cd}$ -partition of  $G$ .

**Corresponding Author: V. Praba\*<sup>1</sup>,**

<sup>1</sup>Research scholar, Sri Chandrasekharendra Saraswathi Viswa Mahavidyalaya,  
 Kanchipuram – 631 561, Tamilnadu, India.

**Proposition 1.3:**

- (1)  $\chi_t^{cd}(K_n) = n$ .
- (2)  $\chi_t^{cd}(D_{r,s}) = 2$ .
- (3)  $\chi_t^{cd}(K_{1,n}) = 2$ .
- (4)  $\chi_t^{cd}(K_{m,n}) = 2$ .
- (5)  $\chi_t^{cd}(W_n) = \begin{cases} 4 & \text{if } n \text{ is even} \\ 3 & \text{if } n \text{ is odd} \end{cases}$
- (6)  $\chi_t^{cd}(P) = 6$ , where P is the Petersen Graph.

**Remark 1.4:** If G has a full degree vertex, then  $\chi_t^{cd}(G) = \chi(G)$ .

## 2. BOUNDS

**Observation 2.1:**

- (i)  $\chi_{cd}(G) = \chi_t^{cd}(G)$
- (ii)  $\max\{\chi_{cd}(G), \gamma_t(G)\} \leq \chi_t^{cd}(G)$   
When  $G = K_n$ , with  $n \geq 3$ ,  $\chi_{cd}(G) = n = \gamma_t(G) = n$ ,  $\chi_t^{cd}(G) = n$ . Therefore Equality holds.
- (iii)  $\chi_Y^{cd}(G) \leq \chi_t^{cd}(G)$ .
- (iv) Let  $G \neq K_1$ ,  $2 \leq \chi_t^{cd}(G) \leq n$  and the bounds are sharp.

**Proof:** When  $G = K_n$ ,  $n \geq 2$ ,  $\chi_t^{cd}(G) = n$ . When  $G = K_2$ ,  $\chi_t^{cd}(G) = 2$ .

**Theorem 2.2:**  $\chi_t^{cd}(G) = 2$  if and only if G is a bipartite graph without isolates with bipartition  $V_1, V_2$  such that there exists  $x \in V_1$  which is adjacent with every vertex of  $V_2$  and there exists  $y \in V_2$  which is adjacent with every vertex of  $V_1$ .

**Proof:** If G is a bipartite graph as in the hypothesis of the theorem, then  $\Pi = \{V_1, V_2\}$  is a  $\pi_t^{cd}$ -partition of G. Clearly  $\Pi$  is a  $\chi_t^{cd}$ -partition of G.

Therefore  $\chi_t^{cd}(G) = 2$ . Conversely, suppose  $\chi_t^{cd}(G) = 2$ . Then there exist a partition  $\Pi = \{V_1, V_2\}$  of V such that an element x from  $V_1$  and an element y from  $V_2$  constitute a total dominating set of G. Therefore x is adjacent with every vertex of  $V_2$  and y is adjacent with every vertex of  $V_1$ . Therefore G is a bipartite graph as in the hypothesis of the theorem.

**Theorem 2.3:** Let G be a simple, finite graph without isolates.

Then  $\chi_t^{cd}(G) \leq (\chi_{cd}(G) + \gamma_t(G) - 2)$ .

**Proof:** Let  $\Pi = \{V_1, V_2, \dots, V_k\}$  be a  $\chi_{cd}$ -partition of G, where  $r = \chi_{cd}(G)$ .

Let  $D = \{x_1, x_2, \dots, x_{\gamma_t}\}$  be a minimum total dominating set of G.

Let  $x_{\gamma_t}(G)$  be adjacent with  $x_i$  for some  $i$ ,  $(1 \leq i \leq \gamma_t - 1)$ . Assign colors  $\chi_{cd}(G) + 1, \dots, \chi_{cd}(G) + \gamma_t(G) - 2$  to the vertices  $x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_{\gamma_t-1}$  leaving the other vertices colored as before. Let  $D_1 = D - \{x_i, x_{\gamma_t}\}$ . Let  $\Pi_1 = \{V_1 - D_1, V_2 - D_1, \dots, V_r - D_1, \{x_1\}, \{x_2\}, \dots, \{x_{i-1}\}, \{x_{i+1}\}, \dots, \{x_{\gamma_t-1}\}\}$ . Since  $\Pi$  is a cd-partition of G, each  $V_i$ ,  $(1 \leq i \leq r)$ , is dominated by a vertex  $z_i$  of G and hence each  $V_i - D_1$  is dominated by  $z_i$   $(1 \leq i \leq r)$  of G. Therefore  $\Pi_1$  is a cd-partition of G.  $|\Pi_1| = r + \gamma_t(G) - 2 = \chi_{cd}(G) + \gamma_t(G) - 2$ . Since  $(V_1 - D_1) \cup (V_2 - D_1) \cup (V_r - D_1) = V - D_1$ ,  $x_{\gamma_t} \in V - D_1$ . Therefore  $x_{\gamma_t} \in V_j - D_1$  for some  $j$ ,  $(1 \leq i \leq r)$ . Since  $x_i$  is adjacent with  $x_{\gamma_t}$ ,  $x_i \notin V_j - D_1$ . Therefore  $x_i \in V_k - D_1$  where  $k \neq j$ ,  $1 \leq k \leq r$ . Choose  $z_k = x_i$  and  $z_j = x_{\gamma_t}$ . Then  $\{z_1, z_2, \dots, z_{i-1}, x_i, z_{i+1}, \dots, z_{j-1}, x_{\gamma_t}, z_{j+1}, \dots, z_r, x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_{\gamma_t-1}\}$  is a total dominating set of G. Therefore  $\Pi_1$  is a  $\Pi_t^{cd}$ -partition of G. Therefore  $\chi_t^{cd}(G) \leq |\Pi_1| = (\chi_{cd}(G) + \gamma_t(G) - 2)$ .

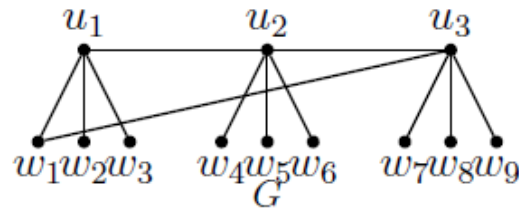
**Remark 2.4:** The above bound is sharp.

For: let  $G = K_n$ .  $\chi_{cd}(G) = n$ ,  $\gamma_t(G) = 2$  and  $\chi_t^{cd}(G) = n$ .

Therefore  $(\chi_{cd}(G) + \gamma_t(G) - 2) = n + 2 - 2 = n = \chi_t^{cd}(G)$ .

**Theorem 2.5:** Given a positive integer k, there exist a connected graph G such that  $\chi_t^{cd}(G) - \chi_{cd}(G) = k$ .

**Proof:** Let G be the graph given below:



Let  $\Pi = \{\{u_1, u_3, w_4, w_5, w_6\}, \{u_2, w_1, w_7, w_8, w_9\}, \{w_2, w_3\}\}$  is a cd-partition of  $G$ . Also  $\chi_{cd}(G)=3$ . Let  $\Pi_1 = \{\{u_1, w_4, w_5, w_6\}, \{u_2, w_1, w_7, w_8, w_9\}, \{w_2, w_3\}, \{u_3\}\}$ .  $\Pi_1$  is a cd-partition of  $G$  and  $D = \{u_1, u_2, w_2, u_3\}$  is a total dominating set of  $G$ . Therefore  $\chi_t^{cd}(G) \leq |\Pi_1| = 4$ . Therefore any  $\chi_{cd}$ -partition of  $G$  will not give a total dominating set of  $G$ . Therefore  $\chi_t^{cd}(G) \leq \chi_{cd}(G) = 3$ . Therefore  $\chi_t^{cd}(G) - \chi_{cd}(G) = 4 - 3 = 1$ .

### 3. BIPARTITE GRAPHS

**Theorem 3.1:** Let  $G$  be a bipartite graph with bipartition  $V_1, V_2$ . Let  $V_1$  contain a vertex  $v_1$  which is adjacent with every vertex of  $V_2$  and let  $V_{1,1}, V_{1,2}, \dots, V_{1,r}, V_{1,r+1}, \dots, V_{1,t}$  be a minimum partition of  $V_1$  such that  $V_{1,i}, (1 \leq i \leq t)$  is dominated by a vertex of  $V_2$  are singletons. Then  $\chi_{cd}(G)$  is  $t+1$  and  $\chi_t^{cd}(G) \leq 2t$ .

**Proof:** Let  $V_{1,1}, V_{1,2}, \dots, V_{1,r}, V_{1,r+1}, \dots, V_{1,t}$  be dominated by  $y_1, y_2, \dots, y_t \in V_2$ . Let  $\Pi = \{V_2 - \{y_2, y_3, \dots, y_t\}, \{y_2\}, \{y_3\}, \dots, \{y_t\}, V_{1,1}, V_{1,2}, \dots, V_{1,r}, V_{1,r+1}, \dots, V_{1,t}\}$ . Then  $\Pi$  is a cd-partition of  $G$ . Choose  $y$  from  $V_2 - \{y_2, y_3, \dots, y_t\}$ , and  $w_1, w_2, \dots, w_t$  from  $V_{1,1}, V_{1,2}, \dots, V_{1,r}, V_{1,r+1}, \dots, V_{1,t}$  respectively such that  $v_1 = w_j$  for some  $j, (1 \leq j \leq t)$ . Since  $v_1$  is adjacent with every vertex of  $V_2$ ,  $\{y_1, y_2, \dots, y_t, w_1, w_2, \dots, w_t\}$  is a  $\Pi_t^{cd}$ -partition of  $G$ . Therefore  $\chi_t^{cd}(G) \leq 2t$ .

**Remark 3.2:**  $\chi_t^{cd}(G) = 2t$  in the above theorem.

For: let  $\{V_{1,1}, V_{1,2}, \dots, V_{1,r}, V_{1,r+1}, \dots, V_{1,t}\}$  be minimum partition of  $V_1$  such that  $V_{1,r+1}, \dots, V_{1,t}$  are all singletons and other contain more than one element. For total domination the dominating vertices for the singleton sets  $V_{1,r+1}, \dots, V_{1,t}$  are to be taken. Hence  $\chi_t^{cd}(G) = 2t$ .

### REFERENCES

1. S. Chitra, Studies in Coloring in Graph with Special Reference to Color Class Domination, Ph.D. Thesis, M.K. University, 2012.
2. S. Chitra, Gokilamani and V. Swaminathan, Color Class Domination in Graphs, Mathematical and Experimental Physics, Narosa Publishing House, pp. 24--28, 2010.
3. T. W. Haynes, S. T. Hedetniemi and P. J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker Inc., 1998.
4. V. Praba, P. Aristotle and V. Swaminathan, Color Class Domination and Colorful Domination, Communicated.
5. M. A. Shalu and T. P. Sandhya, The cd-coloring of graphs, Algorithms and Discrete Applied Mathematics, 2<sup>nd</sup> International Conference Caldam 2016, Springer 337--348.
6. Y. B. Venkatakrishnan and V. Swaminathan, Color class domination number of middle graph and center graph of  $K_{1,n}, C_n, P_n$ , Advanced Modeling and Optimization, 12, 233--237, 2010.
7. Y. B. Venkatakrishnan and V. Swaminathan, Color class domination numbers of some classes of graphs, Algebra and Discrete Mathematics, 18, 301--305, 2014.
8. A. Vijayalakshmi, Total dominator colorings in graphs, International Journal of Advancements in Research and Technology, Vol. 1, Issue 4, 1--6, Sep 2012.

Source of support: Nil, Conflict of interest: None Declared.

[Copy right © 2017. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]