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COLORFUL TOTAL DOMINATION WITH RESPECT TO COLOR CLASS DOMINATION PARTITION

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ABSTRACT

Let G be a finite, simple and undirected graph. A partition of V(G) into independent sets such that each element of the partition is dominated by a vertex of G is called color class domination partition of G [1],[2],[6],[7]. The minimum cardinality of such a partition is called the color class domination partition number of G and is denoted by $\chi_{cd}(G)$. If S is a χ_{cd}-partition of G, then the set consisting of one vertex from each element of the partition need not be a dominating set. Since Π = {{u₁},{u₂},....., {uₙ}} where V(G) ={u₁,u₂,....., uₙ} is a cd-partition of G such that the set consisting of one element from each partition is a dominating set of G. The minimum cardinality of a cd-partition in which the set consisting one element from each set of the partition is a dominating set is called the colorful domination number of G with respect to cd-partition of G and is denoted by $\chi_{\gamma}^{cd}(G)$. If G has no isolates, then the trivial partition is a cd-partition which gives rise to a total dominating set of G. The minimum cardinality of a cd-partition which gives rise to a total dominating set of G is called the colorful total domination number of G and is denoted by $\chi_t^{cd}(G)$. In this paper, a study of this new parameter is initiated. χ_t^{cd}(G for well known graphs are found, bounds are obtained and for bipartite graph in certain conditions, the value of χ_{cd} is obtained and bound for χ_t^{cd} is also derived.

Keywords: *Color class domination, total domination.*

1. DEFINITION AND $\chi_t^{cd}(G$ / $\chi_t^{cd}(G)$ FOR WELL KNOWN GRAPHS

Definition 1.1: Let G be a simple, finite, undirected graph without isolates. Let $\Pi = \{V_1, V_2, \ldots V_k\}$ be a cd-partition of G. Let $x_i \in V_i$, $(1 \le i \le k)$, be such that $\{x_1, x_2, \ldots x_k\}$ is a total dominating set of G. Then Π is called a Π_t^{cd}-partition of G. The minimum cardinality of a Π_t^{cd}- partition of G is denoted by $\chi_t^{cd}(G)$

Note 1.2: The existence of Π_t^{cd}- partition of G is guaranteed, Since $\Pi = \{\{u_1\}, \{u_2\}, \ldots, \{u_n\}\}$ where $V(G) = \{u_1, u_2, \ldots, u_n\}$ is a Π_t^{cd} -partition of G.

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Proposition 1.3:
(1) $\chi_t^{cd}(K_n) = n$.
(2) $\chi_t^{cd}(D_{r,s}) = 2$.
(3) $\chi_t^{cd}(K_{1,n}) = 2$.
(4) $\chi_t^{cd}(K_{m,n}) = 2$.
(5) $\chi_t^{cd}(W_n) = \begin{cases} 4 & if\ n\ is\ even \\ 3 & if\ n\ is\ odd \end{cases}$
(6) $\chi_t^{cd}(P) = 6$, where P is the Petersen Graph.

Remark 1.4: If G has a full degree vertex, then $\chi_t^{cd}(G) = \chi(G)$.

2. BOUNDS

Observation 2.1:
(i) $\chi_{cd}(G) = \chi_t^{cd}(G)$
(ii) **max{** $\chi_{cd}(G), \gamma_t(G)\} \leq \chi_t^{cd}(G)$
When G = K_n, with $n \geq 3$, $\chi_{cd}(G) = n = \gamma_t(G) = n$, $\chi_t^{cd}(G) = n$. Therefore Equality holds.
(iii) $\chi_\gamma^{cd}(G) \leq \chi_t^{cd}(G)$.
(iv) Let $G \neq K_1$, $2 \leq \chi_t^{cd}(G) \leq n$ and the bounds are sharp.

Proof: When G = K_n, $n \geq 2$, $\chi_t^{cd}(G) = n$. When G = K_2, $\chi_t^{cd}(G) = 2$.

Theorem 2.2: $\chi_t^{cd}(G) = 2$ if and only if G is a bipartite graph without isolates with bipartition V_1, V_2 such that there exists $x \in V_1$ which is adjacent with every vertex of V_2 and there exists $y \in V_2$ which is adjacent with every vertex of V_1.

Proof: If G is a bipartite graph as in the hypothesis of the theorem , then $\Pi = \{V_1, V_2\}$ is a π_t^{cd}- partition of G. Clearly Π is a χ_t^{cd}- partition of G.

Therefore $\chi_t^{cd}(G) = 2$. Conversely, suppose $\chi_t^{cd}(G) = 2$. Then there exist a partition $\Pi = \{V_1, V_2\}$ of V such that an element x from V_1 and an element y from V_2 constitute a total dominating set of G. Therefore x is adjacent with every vertex of V_2 and y is adjacent with every vertex of V_1.Therefore G is a bipartite graph as in the hypothesis of the theorem.

Theorem 2.3: Let G be a simple, finite graph without isolates.
Then $\chi_t^{cd}(G) \leq (\chi_{cd}(G) + \gamma_t(G) - 2)$.

Proof: Let $\Pi = \{V_1, V_2, ...V_k\}$ be a χ_{cd}-partition of G, where r = $\chi_{cd}(G)$.

Let $D = \{x_1, x_2, \dots, x_{\gamma_t}\}$ be a minimum total dominating set of G.

Let $x_{\gamma_t}(G)$ be adjacent with x_i for some i, $(1 \leq i \leq \gamma_t - 1)$.. Assign colors $\chi_{cd}(G) + 1$, ..., $\chi_{cd}(G) + \gamma_t(G) - 2$ to the vertices $x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_{\gamma_t - 1}$ leaving the other vertices colored as before. Let $D_1 = D - \{x_i, x_{\gamma_t}\}$. Let $\Pi_1 = \{V_1 - D_1, V_2 - D_1, \dots, V_r - D_1, \{x_1\}, \{x_2\}, \dots, \{x_{i-1}\}, \{x_{i+1}\}, \dots, \{x_{\gamma_t - 1}\}\}$. Since Π is a cd-partition of G, each V_i, $(1 \leq i \leq r)$, is dominated by a vertex z_i of G and hence each $V_i - D_1$ is dominated by z_i $(1 \leq i \leq r)$ of G. Therefore Π_1 is a cd-partition of G. $|\Pi_1| = r + \gamma_t(G) - 2 = \chi_{cd}(G) + \gamma_t(G) - 2$. Since $(V_1 - D_1) \cup (V_2 - D_1) \cup (V_r - D_1) = V - D_1$, $x_{\gamma_t} \in V - D_1$. Therefore $x_{\gamma_t} \in V_j - D_1$ for some j, $(1 \leq i \leq r)$. Since x_i is adjacent with x_{γ_t}, $x_i \notin V_j - D_1$. Therefore $x_i \in V_k - D_1$ where $k \neq j$, $1 \leq k \leq r$. Choose $z_k = x_i$ and $z_j = x_{\gamma_t}$ Then $\{z_1, z_2, \dots, z_{i-1}, x_i, z_{i+1}, \dots, z_{j-1}, x_{\gamma_t}, z_{j+1}, \dots, z_r, x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_{\gamma_t - 1}\}$ is a total dominating set of G. Therefore Π_1 is a Π_t^{cd} - partition of G. Therefore $\chi_t^{cd}(G) \leq |\Pi_1| = (\chi_{cd}(G) + \gamma_t(G) - 2)$.

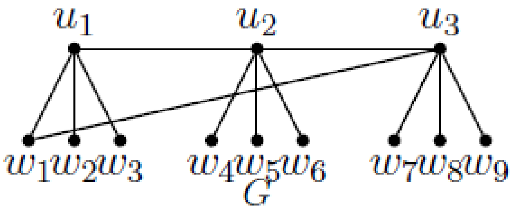
Remark 2.4: The above bound is sharp.

For: let = K_n. $\chi_{cd}(G) = n$, $\gamma_t(G) = 2$ and $\chi_t^{cd}(G) = n$.

Therefore $(\chi_{cd}(G) + \gamma_t(G) - 2) = n+2-2 = n = \chi_t^{cd}(G)$.

Theorem 2.5: Given a positive integer k, there exist a connected graph G such that $\chi_t^{cd}(G) - \chi_{cd}(G) = k$.

Proof: Let G be the graph given below:



Let $\Pi = \{\{u_1, u_3, w_4, w_5, w_6\}, \{u_2, w_1, w_7, w_8, w_9\}, \{w_2, w_3\}\}$ is a cd-partition of G. Also χ_{cd}(G)=3. Let $\Pi_1 = \{\{u_1, w_4, w_5, w_6\}, \{u_2, w_1, w_7, w_8, w_9\}, \{w_2, w_3\}, \{u_3\}\}$. Π_1 is a cd-partition of G and D = $\{u_1, u_2, w_2, u_3\}$ is a total dominating set of G. Therefore χ_t^{cd}(G) ≤ $|\Pi_1|$ = 4. Therefore any χ_{cd}- partition of G will not give a total dominating set of G. Therefore χ_t^{cd}(G) ≤ χ_{cd}(G) = 3. Therefore χ_t^{cd}(G) − χ_{cd}(G) = 4 − 3 = 1.

3. BIPARTITE GRAPHS

Theorem 3.1: Let G be a bipartite graph with bipartition V_1, V_2. Let V_1 contain a vertex v_1 which is adjacent with every vertex of V_2 and let $V_{1,1}, V_{1,2}, \ldots, V_{1,r}, V_{1,r+1}, \ldots, V_{1,t}$ be a minimum partition of V_1 such that $V_{1,i}$, $(1 \le i \le t)$ is dominated by a vertex of V_2 are singletons. Then χ_{cd}(G) is t+1 and χ_t^{cd}(G) ≤ 2t.

Proof: Let $V_{1,1}, V_{1,2}, \ldots, V_{1,r}, V_{1,r+1}, \ldots, V_{1,t}$ be dominated by $y_1, y_2, \ldots, y_t \in V_2$. Let $\Pi = \{V_2 - \{y_2, y_3, \ldots, y_t\}, \{y_2\}, \{y_3\}, \ldots, \{y_t\}, V_{1,1}, V_{1,2}, \ldots, V_{1,r}, V_{1,r+1}, \ldots, V_{1,t}\}$. Then Π is a cd-partition of G. Choose y from $V_2 - \{y_2, y_3, \ldots, y_t\}$, and $w_1, w_2, \ldots, w_t$ from $V_{1,1}, V_{1,2}, \ldots, V_{1,r}, V_{1,r+1}, \ldots, V_{1,t}$ respectively such that $v_1 = w_j$ for some j, $(1 \le j \le t)$. Since v_1 is adjacent with every vertex of V_2, $\{y_1, y_2, \ldots, y_t, w_1, w_2, \ldots, w_t\}$ is a Π_t^{cd} - partition of G. Therefore χ_t^{cd}(G) ≤ 2t.

Remark 3.2: $\chi_t^{cd}(G) = 2t$ in the above theorem.

For: let $\{V_{1,1}, V_{1,2}, \ldots, V_{1,r}, V_{1,r+1}, \ldots, V_{1,t}\}$ be minimum partition of V_1 such that $V_{1,r+1}, \ldots, V_{1,t}$ are all singletons and other contain more than one element. For total domination the dominating vertices for the singleton sets $V_{1,r+1}, \ldots, V_{1,t}$ are to be taken. Hence χ_t^{cd}(G) = 2t.

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