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ON NANO (τ_1,τ_2) GENERALIZED β CLOSED SETS AND NANO (τ_1,τ_2) GENERALIZED β OPEN SETS IN NANO BITOPOLOGICAL SPACES

S. B. SHALINI*, G. SINDHU AND K. INDIRANI

Department of Mathematics,
Nirmala College for Women, Coimbatore, Tamil Nadu, India.

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ABSTRACT

The aim of this paper is to introduce Nano (τ_1,τ_2) generalized β closed sets and Nano (τ_1,τ_2) generalized β open sets in Nano bitopological spaces. Also the characteristics and properties of Nano (τ_1,τ_2) g β closed sets and Nano (τ_1,τ_2) g β open sets are studied respectively.

Keywords: *Nano (τ_1,τ_2) β interior, Nano (τ_1,τ_2) β closure, Nano (τ_1,τ_2) generalized β closed sets, Nano (τ_1,τ_2) generalized β open sets.*

1. INTRODUCTION

The notion of Nano topology was introduced by Lellis Thivagar [6] which was defined in terms of approximations and boundary regions of a subset of an universe using an equivalence relation on it and he also defined Nano closed set, Nano interior and Nano closure. Levine [7] introduced generalized closed sets as a generalization of closed sets in topological spaces. Abd El Monsef *et al.* [1] introduced the notion of β-open set in topology, further investigation of Nano β open sets was given by Gnanambal [4]. Shalini *et al.* [8] have introduced Nano generalized β closed sets in Nano topology. Kelly [5] introduced the concept of bitopological space in and Fukutake [3] introduced the generalized closed sets in bitoplogical space. Bhuvaneswari et.al [2] introduced the Nano bitopological space. In this paper we introduce Nano (τ_1,τ_2) generalized β closed sets and Nano (τ_1,τ_2) generalized β open sets and some of its properties are investigated.

2. PRELIMINARIES

Definition 2.1[6]: Let U be the universe, R be an equivalence relation on U and where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

- U and $\phi \in \tau_R(X)$.
- The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.hence $\tau_R(X)$ is called the Nano topology on U with respect to X, $(U,\tau_R(X))$ is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U. Elements of $[\tau_R(X)]^C$ are called Nano closed sets.

Definition 2.2[6]: If $(U,\tau_R(X))$ is a Nano topological space where $X \subseteq U$ and if $A \subseteq U$, then

- The Nano interior of a set A is defined as the union of all Nano open subsets contained in A and is denoted by $N\operatorname{int}(A)$. $N\operatorname{int}(A)$ isthe largest Nano open subset of A.
- The Nano closure of a set A is defined as the intersection of all Nano closed sets containing A and is denoted by $Ncl(A)$. $Ncl(A)$ is the smallest Nano closed set containing A.

*Corresponding Author: S. B. Shalini**

Definition 2.3 [7]: A subset A of (X, τ) is called generalized closed set (briefly g closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

Definition 2.4 [2]: A subset A of $(U, \tau_R(X))$ is called Nano generalized closed set (briefly Ng closed) if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U, \tau_R(X))$.

Definition 2.5 [8]: A subset A of Nano topological space $(U, \tau_R(X))$ is called Nano generalized β closed set (briefly Ng β closed) if $N\beta cl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U, \tau_R(X))$.

Definition 2.6 [3]: A subset A of (X, τ_1, τ_2) is called (τ_i, τ_j) generalized closed set (briefly (τ_i, τ_j) g closed) if $\tau_2 cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 open in (X, τ_1, τ_2).

3. NANO (τ_1, τ_2) GENERALIZED β CLOSED SETS

Definition 3.1: Let U be the universe, R_1 and R_2 are equivalence relations on U and X_1 and X_2 are subsets of U. Then $\tau_{R1}(X_1)$ and $\tau_{R2}(X_2)$ satisfies the following axioms:

- U and $\phi \in \tau_{R1}(X_1)$ and $\tau_{R2}(X_2)$.
- The union of the elements of any sub collection of $\tau_{R1}(X_1)$ is in $\tau_{R1}(X_1)$ and $\tau_{R2}(X_2)$ is in $\tau_{R2}(X_2)$.
- The intersection of the elements of any finite sub collection of $\tau_{R1}(X_1)$ is in $\tau_{R1}(X_1)$ and $\tau_{R2}(X_2)$ is in $\tau_{R2}(X_2)$.

  Hence $\tau_{R1}(X_1)$ and $\tau_{R2}(X_2)$ is called the Nano bitopology on U with respect to X_1 and X_2, $(U, \tau_{R1}(X_1), \tau_{R2}(X_2))$ is called the Nano bitopological space. Elements of the Nano bitopology are known as Nano (τ_1, τ_2) open sets in U and elements of $[\tau_{R1}(X_1)]^C$ and $[\tau_{R2}(X_2)]^C$ are called Nano (τ_1, τ_2) closed sets.

Definition 3.2: If $(U, \tau_{R1}(X_1), \tau_{R2}(X_2))$ is a Nano bitopological space where X_1 and X_2 are subsets of U and if $A \subseteq U$, then

- The Nano (τ_1, τ_2) interior of a set A is defined as the union of all Nano (τ_1, τ_2) open subsets contained in A and is denoted by $N\tau_1\tau_2 \operatorname{int}(A)$. $N\tau_1\tau_2 \operatorname{int}(A)$ is the largest Nano (τ_1, τ_2) open subset of A.
- The Nano (τ_1, τ_2) closure of a set A is defined as the intersection of all Nano (τ_1, τ_2) closed sets containing A and is denoted by $N\tau_1\tau_2 cl(A)$. $N\tau_1\tau_2 cl(A)$ is the smallest Nano (τ_1, τ_2) closed set containing A.

Definition 3.3: Let $(U, \tau_{R1}(X_1), \tau_{R2}(X_2))$ be a Nano bitopological space and $A \subseteq U$. Then A is said to be

(i) Nano (τ_1, τ_2) semi open if $A \subseteq N\tau_2 cl(N\tau_1 \operatorname{int}(A))$
(ii) Nano (τ_1, τ_2) pre open if $A \subseteq N\tau_2 \operatorname{int}(N\tau_1 cl(A))$
(iii) Nano (τ_1, τ_2) α open if $A \subseteq N\tau_1 \operatorname{int}[N\tau_2 cl(N\tau_1 \operatorname{int}(A))]$
(iv) Nano (τ_1, τ_2) regular open if $A = N\tau_2 \operatorname{int}(N\tau_1 cl(A))$
(v) Nano (τ_1, τ_2) β open(Nano (1,2) semi-pre open) if $A \subseteq N\tau_1 cl[N\tau_2 \operatorname{int}(N\tau_1 cl(A))]$

The family of Nano (τ_1, τ_2) semi open (resp. Nano (τ_1, τ_2) pre open, Nano (τ_1, τ_2) α open, Nano (τ_1, τ_2) regular open, Nano (τ_1, τ_2) β open) sets in U is denoted by N(τ_1, τ_2) $SO(U, X)$ (resp. N(τ_1, τ_2) $PO(U, X)$, N(τ_1, τ_2) $\alpha O(U, X)$, N(τ_1, τ_2) $RO(U, X)$ and N(τ_1, τ_2) $\beta O(U, X)$).

The complement of Nano (τ_1,τ_2) semi open (resp. Nano (τ_1,τ_2) pre open, Nano (τ_1,τ_2) α open, Nano (τ_1,τ_2) regular open, Nano (τ_1,τ_2) β open) sets in U is Nano (τ_1,τ_2) semi closed (resp. Nano (τ_1,τ_2) pre closed, Nano (τ_1,τ_2) α closed, Nano (τ_1,τ_2) regular closed, Nano (τ_1,τ_2) β closed).

Definition 3.4: A subset A of $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$ is called Nano (τ_1,τ_2) generalized closed set (briefly N (τ_1,τ_2) g closed) if $N\tau_2 cl(A)\subseteq V$ whenever $A\subseteq V$ and V is Nano τ_1 open in $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$.

Definition 3.5: If $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$ is a Nano bitopological space where X_1 and X_2 are subsets of U and if $A\subseteq U$ then

- The Nano (τ_1,τ_2) β interior of a set A is defined as the union of all Nano (τ_1,τ_2) β open subsets contained in A and is denoted by $N\tau_1\tau_2\beta\operatorname{int}(A)$. $N\tau_1\tau_2\beta\operatorname{int}(A)$ is the largest Nano (τ_1,τ_2) β open subset of A.
- The Nano (τ_1,τ_2) β closure of a set A is defined as the intersection of all Nano (τ_1,τ_2) β closedsets containing A and is denoted by $N\tau_1\tau_2\beta cl(A)$. $N\tau_1\tau_2\beta cl(A)$ is the smallest Nano (τ_1,τ_2) β closed set containing A.

Definition 3.6: A subset A of Nano bitopological space $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$ is called Nano (τ_1,τ_2) generalized β closed set (briefly N (τ_1,τ_2) g β closed) if $N\tau_2\beta cl(A)\subseteq V$ whenever $A\subseteq V$ and V is Nano τ_1 open in $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$.

Theorem 3.8: If A is Nano τ_2 closed set in $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$ then it is Nano (τ_1,τ_2) g β closed set in $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$ but not conversely.

Proof: Since every Nano closed set is Nano g β closed set, the proof follows.

Example 3.9: Let $U=\{a,b,c,d\}$ with $U/R_1=\{\{a\},\{c\},\{b,d\}\}$, $X_1=\{a,b\}$, $\tau_{R1}(X_1)=\{U,\phi,\{a\},\{b,d\},\{a,b,d\}\}$, $U/R_2=\{\{a\},\{b\},\{c\},\{d\}\}$, $X_2=\{a,d\}$, $\tau_{R2}(X_2)=\{U,\phi,\{a,d\}\}$. Here the set $\{a,c,d\}$ is Nano (τ_1,τ_2) g β closed but notNano τ_2 closed in U.

Theorem 3.10: Every Nano τ_2 pre closed set is Nano (τ_1,τ_2) g β closed set but not conversely.

Proof: Let A be Nano τ_2 pre closed set in $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$ and let G be a Nano τ_1 open set such that $A\subseteq G$. Since every Nano pre closed is Nano g β closed, we have $N\tau_2\beta cl(A)\subseteq G$. Hence A is Nano(1,2) g β closed set in $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$.

Example 3.11: Let $U=\{a,b,c,d\}$ with $U/R_2=\{\{a\},\{b\},\{c\},\{d\}\}$, $X_1=\{a,d\}$, $\tau_{R1}(X_1)=\{U,\phi,\{a,d\}\}$, $U/R_2=\{\{a\},\{c\},\{b,d\}\}$, $X_2=\{a,b\}$, $\tau_{R2}(X_2)=\{U,\phi,\{a\},\{b,d\},\{a,b,d\}\}$. Here the set $\{a,b\}$ is Nano (τ_1,τ_2) g β closed but not Nano τ_2 pre closed in U.

Theorem 3.12: Every Nano τ_2 regular closed set is Nano (τ_1,τ_2) g β closed set but not conversely.

Proof: Since every Nano regular closed set is Nano g β closed set, the proof follows.

Example 3.13: Let $U=\{a,b,c,d\}$ with $U/R_1=\{\{a\},\{b\},\{c\},\{d\}\}$, $X_1=\{a,d\}$, $\tau_{R1}(X_1)=\{U,\phi,\{a,d\}\}$, $U/R_2=\{\{a\},\{c\},\{b,d\}\}$, $X_2=\{a,b\}$, $\tau_{R2}(X_2)=\{U,\phi,\{a\},\{b,d\},\{a,b,d\}\}$. Here the set $\{c,d\}$ is Nano (1,2) g β closed but not Nano τ_2 pre closed in U.

Remark 3.14: The union of two Nano (τ_1,τ_2) g β closed sets need not be Nano (τ_1,τ_2) g β closed set which can be seen from the following example.

Example 3.15: Let $U=\{a,b,c,d\}$ with $U/R_1=\{\{a\},\{c\},\{b,d\}\}$, $X_1=\{a,b\}$, $\tau_{R1}(X_1)=\{U,\phi,\{a\},\{b,d\},\{a,b,d\}\}$, $U/R_2=\{\{a\},\{b\},\{c\},\{d\}\}$, $X_2=\{a,d\}$, $\tau_{R2}(X_2)=\{U,\phi,\{a,d\}\}$. Here the sets $\{a\}$ and $\{b,d\}$ are Nano (1,2) g β closed sets but $\{a\}\cup\{b,d\}=\{a,b,d\}$ is notNano (τ_1,τ_2) g β closed set in U.

Theorem 3.16: If a set A isN (τ_1,τ_2) g β closed set in a Nano bitopological space $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$, then $N\tau_2\beta cl(A)-A$ contains no non-empty Nano τ_1 closed set in $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$.

Proof: Suppose that F is a Nano τ_1 closed setsuch that $F\subseteq N\tau_2\beta cl(A)-A$. Now $F\subseteq N\tau_2\beta cl(A)$ and $F\subseteq A^C$ then $A\subseteq U-F$, $U-F$ is Nano τ_1 open set and A is Nano (τ_1,τ_2) g β closed. Therefore $N\tau_2\beta cl(A)\subseteq U-F$. That is $F\subseteq U-(N\tau_2\beta cl(A))$. Hence $F\subseteq N\tau_2\beta cl(A)\cap(U-(N\beta cl(A)))=\phi, F=\phi$. Therefore $N\tau_2\beta cl(A)-A$ contains no non-empty Nano τ_1 closed set in $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$.

Remark 3.17: If a set A in a Nano bitopological space $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$ is Nano $\tau_2\beta$ closed then $N\tau_2\beta cl(A)-A=\phi$.

Proof: Assume that A is Nano $\tau_2\beta$ closed. Since $N\tau_2\beta cl(A)=A$, $N\tau_2\beta cl(A)-A=\phi$.

Theorem 3.18: For each point x of U, a singleton $\{x\}$ is Nano τ_1 closed or $\{x\}^C$ is Nano (τ_1,τ_2) g β closed.

Proof: Suppose $\{x\}$ is not Nano τ_1 closed. Since $\{x\}^C$ is not Nano τ_1 open, a Nano τ_1 open containing $\{x\}^C$ is only U Then $N\tau_2\beta cl(\{x\}^C)\subseteq U$ and $\{x\}^C$ is Nano (1,2) g β closed.

Theorem 3.19: If A is Nano (τ_1,τ_2) g β closed then $N\tau_2\beta cl(x)\cap A\neq\phi$ for some $x\in N\tau_2\beta cl(A)$ but not conversely.

Proof: If $N\tau_2\beta cl(x)\cap A=\phi$ for $x\in N\tau_2\beta cl(A)$, then $A\subseteq(N\tau_2\beta cl(x))^C$. Since A is Nano (τ_1,τ_2) g β closed set, we have $N\tau_2\beta cl(A)\subseteq(N\tau_2\beta cl(x))^C$. This implies $x\notin N\tau_2\beta cl(A)$ which is a contradiction.

Theorem 3.20: If A is Nano (τ_1,τ_2) g β closed in a Nano bitopological space $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$ and $A\subseteq B\subseteq N\tau_2\beta cl(A)$, then B is also Nano (1,2) g β closed in $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$.

Proof: Let $B\subseteq G$ where G is Nano τ_1 open set in U. Then $A\subseteq B$ implies $A\subseteq G$. As A is Nano (1, 2) g β closed, we have $N\tau_2\beta cl(A)\subseteq G$. Now $B\subseteq N\tau_2\beta cl(A)$, implies

$N\tau_2\beta cl(B)\subseteq N\tau_2\beta cl(N\tau_2\beta cl(A))=N\tau_2\beta cl(A)\subseteq G$. Thus $N\tau_2\beta cl(B)\subseteq G$. Therefore B is Nano (1,2) g β closed set in U.

4. NANO (τ_1,τ_2) GENERALIZED β OPEN SETS

Definition 4.1: A subset A of a Nano bitopological space $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$ is called Nano (τ_1,τ_2) generalized β open (brieflyN (τ_1,τ_2) g β open), if its compliment A^C is Nano (τ_1,τ_2) g β closed.

The collection of allNano (τ_1,τ_2) g β open subsets of U is denoted by N (τ_1,τ_2) G $\beta O(U,X)$.

Theorem 4.2: Every Nano τ_2 open set in $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$ is Nano (τ_1,τ_2)gβ open set in $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$ but not conversely.

Proof: Since every Nano open set is Nano g β open, the proof follows.

Example 4.3: Let $U=\{a,b,c,d,e\}$with $U/R1=\{\{a\},\{b,c,d\},\{e\}\}$, $X_1=\{a,b\}$, $\tau_{R1}(X_1)=\{U,\phi,\{a\},\{b,c,d\},\{a,b,c,d\}\}$, $U/R2=\{\{a,b\},\{c,e\},\{d\}\}$, $X_2=\{a,d\}$, $\tau_{R2}(X_2)=\{U,\phi,\{d\},\{a,b\},\{a,b,d\}\}$. Here the set $\{a,c,e\}$is Nano (τ_1,τ_2) g β open but not Nano τ_2 open in$U$.

Remark 4.4: The intersection of two Nano (τ_1,τ_2) g β open sets need not be Nano (τ_1,τ_2) g β open set which can be seen from the following example.

Example 4.5: Let $U=\{a,b,c,d,e\}$with $U/R1=\{\{a\},\{b,c,d\},\{e\}\}$, $X_1=\{a,b\}$, $\tau_{R1}(X_1)=\{U,\phi,\{a\},\{b,c,d\},\{a,b,c,d\}\}$,$U/R_2=\{\{a,b\},\{c,e\},\{d\}\}$, $X_2=\{a,d\}$, $\tau_{R2}(X_2)=\{U,\phi,\{d\},\{a,b\},\{a,b,d\}\}$. Here the sets $\{a,e\}$ and $\{d,e\}$are Nano (τ_1,τ_2)gβopen sets but $\{a,e\}\cap\{d,e\}=\{e\}$ is not Nano (τ_1,τ_2)gβopen set in$U$.

Theorem 4.6: A subset A in a Nano bitopological space$(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$ isN (τ_1,τ_2) g β open if and only if $F\subseteq N\tau_2\beta\operatorname{int}(A)$ whenever F is Nano $\tau_1$closed and$F\subseteq A$.

Proof: Assume that A is N (τ_1,τ_2) g β open setin $(U,\tau_R(X))$. Let$F$ be Nano $\tau_1$closedand $F\subseteq A$, then $A^C\subseteq F^C$ impliesF^C is Nano$\tau_1$open. Since A^C isNano(τ_1,τ_2) gβ closed set$N\tau_2\beta cl(A^C)\subseteq F^C$.Since $(N\tau_2\beta\operatorname{int}(A))^C=N\tau_2\beta cl(A^C)$,$(N\tau_2\beta\operatorname{int}(A))^C\subseteq F^C$. Therefore $F\subseteq N\tau_2\beta\operatorname{int}(A)$.

Conversely assume that $F\subseteq N\tau_2\beta\operatorname{int}(A)$whenever$F$is Nano $\tau_1$closed set and$F\subseteq A$. Then $(N\tau_2\beta\operatorname{int}(A))^C\subseteq F^C$.Thus$N\tau_2\beta cl(A^C)\subseteq F^C$. Hence A^C is Nano (τ_1,τ_2)gβ closed set and A isNano (τ_1,τ_2)gβ open set in$U$.

Theorem 4.7: IfAisa Nano(τ_1,τ_2) gβ open set in a Nano bitopological space $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$and $N\tau_2\beta\operatorname{int}(A)\subseteq B\subseteq A$, then$B$ is also Nano (τ_1,τ_2)gβ open.

Proof: Let$A$ is Nano(τ_1,τ_2) gβ open set and$N\tau_2\beta\operatorname{int}(A)\subseteq B\subseteq A$. Then$A^C\subseteq B^C\subseteq(N\tau_2\beta\operatorname{int}(A))^C$ implies $A^C\subseteq B^C\subseteq N\beta cl(A^C)$. Since$A^C$ isNano (τ_1,τ_2)gβ closed,B^C is alsoNano(τ_1,τ_2) gβ closed. Therefore$B$ isNano (τ_1,τ_2)gβopen.

Theorem: 4.8: If a set$A$ isNano(τ_1,τ_2)gβ open in a Nanobitopologicalspace $(U,\tau_{R1}(X_1),\tau_{R2}(X_2))$, then $G=U$ whenever$G$is Nanoτ_1 openand$N\tau_2\beta\operatorname{int}(A)\cup A^C\subseteq G$.

Proof: Let Abe Nano (τ_1,τ_2)gβ open,Gbe Nano $\tau_1$open set and$N\tau_2\beta\operatorname{int}(A)\cup A^C\subseteq G$. Then $G^C\subseteq(N\tau_2\beta\operatorname{int}(A)\cup A^C)^C=N\tau_2\beta cl(A^C)-A^C$, since G^C is Nano $\tau_1$closed andA^C is Nano(τ_1,τ_2)gβ closed. By theorem 3.13[8] we have $G^C=\phi$. Therefore $G=U$.

REFERENCES

1. Abd EL-Monsef M. E., EL-Deep S. N. and Mahmoud R. A., β-Open Sets and β-continuous mappings, Bull. Fac. Sci., Assiut Univ., 12 (1983), 77-90.
2. Bhuvaneshwari. K and Karpagam. K., Nano generalized closed sets in Nano bitopological Spaces, Inter. J. of Math.and its Applications, 4(1-B) (2016), 149-153.
3. Fukutake. T., On generalized closed sets in bitopological spaces, Bull. Fukuoka Univ. Ed. Part III, 35 (1985), 19-28.
4. Gnanambal. Y, On Nano β open sets, Int. Jr. of Engineering, 1(2) (2015), 1-6.
5. Kelly. J. C., Bitopological spaces, Proc.London Math. Soc., 13 (1963), 71-89.
6. LellisThivagar .M, and Carmel Richard, On Nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, 1(1) (2013), 31 -37.
7. Levine. N, Generalized closed sets in topology, Rend. Circ. Mat. Palermo 19(2) (1970), 89-96.
8. Shalini. S.B, and Indirani. K, Characterisation of nano generalized β closed sets in nano topological spaces, International Journal of Science and Applied Research, 4(1) (2017), 7-11.

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