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DIFFERENT TYPES OF CUBE DIFFERENCE LABELING

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ABSTRACT

In this paper different types of cube difference labeling are defined. Cube difference labeling was defined and proved that different types of graphs are cube difference graphs in [5], in continuation to this and [5], [6], [7], [8] the cube difference prime labeling, cube difference prime 2- equitable labeling, cube difference 6- equitable labeling are defined and some graphs which satisfies these labeling are investigated.

Key words: *Cube difference prime labeling, cube difference prime 2- equitable labeling and cube difference 6- equitable labeling.*

INTRODUCTION

Throughout this paper by a graph we mean a finite simple graph G (V, E) with p vertices q edges. A detailed survey of graph labeling can be found in [2]. Cube difference labeling of some graphs are proved in [5]. In this paper we introduce different types of cube difference labeling namely cube difference prime labeling, cube difference prime 2- equitable labeling, cube difference 6- equitable labeling and cube difference cordial labeling. In a similar way square difference 3- equitable labeling was defined and proved some graphs are square difference 3- equitable graphs.

Definition 1.1: Let G = (V(G),E(G)) be a graph .G is said to have cube difference prime labeling if there exist a bijection f: $V(G)\rightarrow\{0,1,2,\ldots.p-1\}$ such that the induced function $f^*:E(G)\rightarrow\{1\}$ given by $f^*(u\,v) = |[f^*(u)]^3 - [f^*(v)]^3|$ (mod 6) are unity.

Definition 1.2: The graphs which satisfies the cube difference prime labeling is called the cube difference prime graphs.

Definition 1.3: Let G = (V(G),E(G)) be a graph .G is said to have cube difference prime 2- equitable labeling if there exist a bijection f: $V(G) \rightarrow \{0,1,2,\ldots.p-1\}$ such that the induced function $f^*: E(G)\rightarrow\{1,3\}$ given by $f^*(u\,v) = |[f^*(u)]^3 - [f^*(v)]^3|$ (mod 4) are the primes 1 and 3 with $|e_j(1) - e_j(3)| \leq 1$

Definition 1.4: The graphs which satisfies the cube difference prime 2- equitable labeling is called the cube difference prime 2- equitable graphs

Definition 1.5: Let G = (V(G), E(G)) be a graph. G is said to have cube difference 6- equitable labeling if there exist a bijection f: $V(G)\rightarrow\{0,1,2,\ldots.p-1\}$ such that the induced function $f^*:E(G) \rightarrow N$ given by $f^*(u\,v) = |[f^*(u)]^3 - [f^*(v)]^3|$ (mod 6) are the numbers 0,1,2,3,4,5

Definition 1.6: The graphs which satisfies the cube difference 6- equitable labeling is called the cube difference 6- equitable graphs

Definition 1.7: Let G = (V(G),E(G)) be a graph .G is said to have cube difference cordial labeling if there exist a bijection f:$V(G)\rightarrow\{0,1,2,\ldots.p-1\}$ such that the induced function $f^*:E(G)\rightarrow\{0, 1\}$ given by $f^*(u\,v) = |[f^*(u)]^3 - [f^*(v)]^3|$ is 1 if the edge weight is odd and 0 if the edge weight is even with $|e_j(1) - e_j(0)| \leq 1$

Definition 1.8: The graphs which satisfies the cube difference cordial labeling is called the cube difference cordial graphs.

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2. THE GRAPHS WITH DIFFERENT CD LABELING

Theorem 2.1: The path P_n admits cube difference prime labeling

Proof: Let P_n be a path of length n-1 with n vertices $u_1, u_2, \ldots, u_n$. The vertices are labeled with 0, 1, 2, ... n as $f(u_i) = i-1$, $1 \leq i \leq n$ and the induced edge labeling $f^*: E(G) \rightarrow \{1\}$ is given by $f^*(u\,v) = |[f^*(u)]^3 - [f^*(v)]^3|$ (mod 6) with edge weights one. Since the path satisfies the cube difference prime labeling, the paths are called the cube difference prime graphs.

Theorem 2.2: The paths P_n are cube difference prime 2- equitable labeling.

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of a path P_n. The vertex set of the path graph is $\{u_i \,/\, 1 \leq i \leq n\}$ and the edge set is $\{u_i\, u_{i+1} \,/\, 1 \leq i \leq n$.The order of Pn, is n and the size is n-1, n≥2. The vertex labeling $f: V(G) \rightarrow \{0,1,2,\ldots. n-1\}$ is define by $f(u_i) = i-1$, $1 \leq i \leq n$ and the induced edge labeling $f^*:E(G) \rightarrow \{1, 2\}$ is given by $f^*(u\,v) = |[f^*(u)]^3 - [f^*(v)]^3|$ (mod4), $f(e_i) = 1$, i = 1,4,5,8,9, . . . and $f(e_i) = 3$, i = 2,3,6,7, . . . with $|e_j(1) - e_j(3)| \leq 1$. Hence the paths Pn are cube difference prime 2- equitable graphs.

Theorem 2.3: The comb graphs (Pn , K_1) are cube difference prime 2- equitable labeling.

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of a path P_n. Let $v_1, v_2, \ldots, v_n$ be the corresponding pendent vertices. The vertex set of the comb graph is $\{u_i, v_i \,/\, 1 \leq i \leq n\}$ and the edge set is $\{u_i\, u_{i+1} \,/\, 1 \leq i \leq n\} \cup \{v_i\, v_{i+1} \,/\, 1 \leq i \leq n\} \cup \{u_i\, v_i \,/\, 1 \leq i \leq n\}$. The order of (Pn, K_1) is 2n and the size is 2n-1, n ≥ 3. The vertex labeling $f: V(G) \rightarrow \{0, 1, 2,\ldots. n-1\}$ is define by $f(u_i) = i-1$, $1 \leq i \leq n$ and $f(v_i) = i$, $n \leq i \leq 2n-1$ and the induced edge labeling $f^*: E(G) \rightarrow \{1, 2\}$ is given by $f^*(u\,v) = |[f^*(u)]^3 - [f^*(v)]^3|$ (mod4), $f(e_i) = 1$, i = 1,4,5,8,9, . . . and $f(e_i) = 3$, i = 2, 3, 6, 7, . . . with $|e_j(1) - e_j(3)| \leq 1$. Hence the comb graphs (Pn, K_1) are cube difference prime 2- equitable graphs.

Theorem 2.2: The graphs $P_n{}^2$ are cube difference cordial graphs.

Proof: Let P_n be a path of length n-1 with vertices $u_1, u_2, \ldots u_n$. Here $|V(G)| = |V(P_n{}^2)| = n$ and $|E(G)| = |E(Pn^2)| = 2n-3$, n≥3. The vertex labeling $f: V(G) \rightarrow \{0,1,2,\ldots. n-1\}$ is define by $f(u_i) = i-1$, $1 \leq i \leq n$ and the induced edge labeling $f^*:E(G) \rightarrow \{0, 1\}$ is given by $f^*(u\,v) = |[f^*(u)]^3 - [f^*(v)]^3|$ is 1 if the edge weight is odd and 0 if the edge weight is even with $|e_j(1) - e_j(0)| \leq 1$. Hence the graphs $P_n{}^2$ are cube difference cordial graphs.

Theorem 2.3: The total graph of cycles T (C_n) are cube difference cordial graphs.

Proof: Let C_n be a cycle of length n with vertices $u_1, u_2, \ldots, u_n$. Here $|V(G)| = |V(M(C_n)| = 2n$ and $|E(G)| = |E(M(C_n)| = 4n$, n≥3 . Here the edge sets are $E_1 = \{u_i u_{i+1} \,/\, 1 \leq i \leq n\}$, $E_2 = \{u_{2i} u_{2i+2} \,/\, 1 \leq 2i \leq n/2\}$, $E_3 = \{u_{2i-1} u_{2i+3} \,/\, 1 \leq 2i \leq n/2\}$, $E_4 = \{u_n u_1\}$, $E_5 = \{u_{2n} u_{n+i}\}$ and $E_6 = \{u_{2n} u_1\}$. The vertex labeling $f: V(G) \rightarrow \{0, 1, 2,\ldots. n-1\}$ is define by f(ui) = i-1, $1 \leq i \leq n$ and the induced edge labeling $f^*:E(G) \rightarrow N$ is given by $f^*(u\,v) = |[f^*(u)]^3 - [f^*(v)]^3|$ is 1 if the edge weight is odd and 0 if the edge weight is even with $|e_j(1) - e_j(0)| \leq 1$ for odd graphs and $|e_j(1) - e_j(0)| = 0$ for even graphs.

Theorem 2.4: The shadow graph of paths admit cube difference cordial labeling.

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of a path P_n then the shadow graph of the path is obtained by adding new vertices $v_1, v_2, \ldots, v_n$ corresponding to each vertices of the path graph. Hence $v_1, v_2, \ldots, v_n$ be the vertices of the new path in D_2 (P_n) .The vertex set of D_2 (P_n) is$\{u_i, v_i \,/\, 1 \leq i \leq n\}$ and the edge set is $\{u_i\, u_{i+1} \,/\, 1 \leq i \leq n\} \cup \{v_i\, v_{i+1} \,/\, 1 \leq i \leq n\} \cup \{u_i\, v_{i+1} \,/\, 1 \leq i \leq n-1\} \cup \{v_i\, u_{i+1} \,/\, 1 \leq i \leq n-1\}$

Here the order of the graph D_2 (P_n) is 2n and the size is 4(n-1).The labeling of the edge sets $\{u_i\, u_{i+1} \,/\, 1 \leq i \leq n\}$ and $\{v_i\, v_{i+1} \,/\, 1 \leq i \leq n\}$ are 1 and the other two sets $\{u_i\, v_{i+1} \,/\, 1 \leq i \leq n-1\}$and $\{v_i\, u_{i+1} \,/\, 1 \leq i \leq n-1\}$ have the edge weights as 0. The graph also satisfies the conditio $|e_j(1) - e_j(0)| \leq 1$. Hence the shadow graph of paths admit cube difference cordial labeling.

Theorem 2.5: The total graph of a path T (P_n) admits cube difference cordial labeling.

Proof: Let P_n be a path of length n-1 with vertices $u_1, u_2, \ldots u_{n-1}$ Here $|V(G)| = |V(T(P_n)| = 2n$ and $|E(G)| = |E(T(P_n))| = 3n-1$, n≥3 , $1 \leq i \leq n$. The vertex labeling $f: V(G) \rightarrow \{0,1,2,\ldots. n-1\}$ is define by $f(u_i) = i-1$, $1 \leq i \leq n$ and the induced edge labeling $f^*:E(G) \rightarrow \{0, 1\}$ is given by $f^*(u\,v) = |[f^*(u)]^3 - [f^*(v)]^3|$ is 1 if the edge weight is odd and 0 if the edge weight is even with $|e_j(1) - e_j(0)| \leq 1$

Theorem-2.6: The stars admit cube difference cordial labeling

Proof: Define the mapping $f : V \rightarrow \{0, 1, 2, \ldots p\text{-}1\}$ by $f(v_i) = i$, $1 \leq i \leq n$ and $f(v) = 0$. Hence the pendent vertices of the star graphs are labeled with 1, 2, 3, . . . p-1 and the apex vertex as 0.

The edge weights of the star graphs are 1 and 0 using the condition $f^*(u\, v) = |[f^*(u)]^3 - [f^*(v)]^3|$ is 1 if the edge weight is odd and 0 if the edge weight is even with $|e_j(1) - e_j(0)| \leq 1$. Here the edges are mapped using the function $f^*: E(G) \rightarrow \{0, 1\}$. For the even stars $|e_j(1) - e_j(0)| = 0$ and for the odd stars $|e_j(1) - e_j(0)| \leq 1$. Hence the stars are cube difference cordial.

Theorem-2.7: The stars admit cube difference 6- equitable labeling.

Proof: Let v be the apex vertex and let $v_1, v_2 \ldots v_n$ are the pendent vertices of the star graph $K_{1,n}$
The vertices are given values using the mapping $f : V \rightarrow \{0, 1, 2, \ldots p\text{-}1\}$ define by $f(v_i) = i$, $1 \leq i \leq n$ and $f(v) = 0$. The edge weights are 0, 1 2, 3, 4, 5 using the condition $f^*(u\, v) = |[f^*(u)]^3 - [f^*(v)]^3| \pmod 6$. Hence the star graphs are cube difference 6- equitable graphs.

CONCLUSION

Different types of cube difference labeling are defined and the graphs which satisfies these labeling are called the corresponding cube difference graphs. Some graphs are investigated. It can also be verified for some other graphs. It is an open problem in the research field.

REFERENCES

1. Frank Harrary, Graph theory, Narosa Publishing House- (2001).
2. J A Gallian, A dynamic survey of graph labeling, The Electronics journal of Combinatories,17(2010) # DS.
3. I.Cahit, Cordial graphs "A weaker version of graceful and harmonious graphs Ars Combinatoria, 236(1987) 201- 207.
4. I.Cahit , On cordial and 3-equitable labeling of graphs ,Utilitas Math, 37(1990) 189- 198.
5. J.Shiama "Cube Difference Labeling Of some Graphs" International Journal of Engineering Science and Innovative Technology (IJESIT) Volume 2, Issue 6, November 2013 ISSN: 2319-5967, ISO 9001:2008 Certified.
6. S. Murugesan, D and J. Shiama "Square Difference 3-Equitable Labeling of Paths and Cycles" International Journal of Computer Applications (0975 - 8887) Volume 123 - No.17, August 2015.
7. S. Murugesan and J. Shiama "Square Difference 3- equitable Labeling of Some graphs" International Journal of Computer Applications (0975-08887) Volume 137- No.13, March 2016.
8. S. Murugesan, D. Jayaraman and J. Shiama, 3-Equitable prime cordial labeling of graphs, International Journal of Applied Information Systems, 5(9), July 2013, 1-4. 102.
9. S.K.Vaidya, G.V.Godhasara, Sweta Srivatsav and V.J Kaneria "Some new cordial graphs" Int. J of Math and Math, Sci 4(2) (2008) 89- 92.
10. S.K.Vaidya and P.L.Vihol "Prime cordial labeling of some graphs", Modern Applied Science 4 (8) (2010) 119-126.
11. M.Z.Youssef "A necessary condition on k- equitable labeling" Utilitas Math, 64 (2003) 193-195.

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