

DIFFERENT TYPES OF CUBE DIFFERENCE LABELING

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ABSTRACT

In this paper different types of cube difference labeling are defined. Cube difference labeling was defined and proved that different types of graphs are cube difference graphs in [5], in continuation to this and [5], [6], [7], [8] the cube difference prime labeling, cube difference prime 2- equitable labeling, cube difference 6- equitable labeling are defined and some graphs which satisfies these labeling are investigated.

Key words: Cube difference prime labeling, cube difference prime 2- equitable labeling and cube difference 6- equitable labeling.

INTRODUCTION

Throughout this paper by a graph we mean a finite simple graph $G = (V, E)$ with p vertices q edges. A detailed survey of graph labeling can be found in [2]. Cube difference labeling of some graphs are proved in [5]. In this paper we introduce different types of cube difference labeling namely cube difference prime labeling, cube difference prime 2- equitable labeling, cube difference 6- equitable labeling and cube difference cordial labeling. In a similar way square difference 3- equitable labeling was defined and proved some graphs are square difference 3- equitable graphs.

Definition 1.1: Let $G = (V(G), E(G))$ be a graph. G is said to have cube difference prime labeling if there exist a bijection $f: V(G) \rightarrow \{0, 1, 2, \dots, p-1\}$ such that the induced function $f^*: E(G) \rightarrow \{1\}$ given by $f^*(u v) = |[f^*(u)]^3 - [f^*(v)]^3| \pmod{6}$ are unity.

Definition 1.2: The graphs which satisfies the cube difference prime labeling is called the cube difference prime graphs.

Definition 1.3: Let $G = (V(G), E(G))$ be a graph. G is said to have cube difference prime 2- equitable labeling if there exist a bijection $f: V(G) \rightarrow \{0, 1, 2, \dots, p-1\}$ such that the induced function $f^*: E(G) \rightarrow \{1, 3\}$ given by $f^*(u v) = |[f^*(u)]^3 - [f^*(v)]^3| \pmod{4}$ are the primes 1 and 3 with $|e_j(1) - e_j(3)| \leq 1$

Definition 1.4: The graphs which satisfies the cube difference prime 2- equitable labeling is called the cube difference prime 2- equitable graphs

Definition 1.5: Let $G = (V(G), E(G))$ be a graph. G is said to have cube difference 6- equitable labeling if there exist a bijection $f: V(G) \rightarrow \{0, 1, 2, \dots, p-1\}$ such that the induced function $f^*: E(G) \rightarrow N$ given by $f^*(u v) = |[f^*(u)]^3 - [f^*(v)]^3| \pmod{6}$ are the numbers 0, 1, 2, 3, 4, 5

Definition 1.6: The graphs which satisfies the cube difference 6- equitable labeling is called the cube difference 6- equitable graphs

Definition 1.7: Let $G = (V(G), E(G))$ be a graph. G is said to have cube difference cordial labeling if there exist a bijection $f: V(G) \rightarrow \{0, 1, 2, \dots, p-1\}$ such that the induced function $f^*: E(G) \rightarrow \{0, 1\}$ given by $f^*(u v) = |[f^*(u)]^3 - [f^*(v)]^3|$ is 1 if the edge weight is odd and 0 if the edge weight is even with $|e_j(1) - e_j(0)| \leq 1$

Definition 1.8: The graphs which satisfies the cube difference cordial labeling is called the cube difference cordial graphs.

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2. THE GRAPHS WITH DIFFERENT CD LABELING

Theorem 2.1: The path P_n admits cube difference prime labeling

Proof: Let P_n be a path of length $n-1$ with n vertices u_1, u_2, \dots, u_n . The vertices are labeled with $0, 1, 2, \dots, n$ as $f(u_i) = i-1, 1 \leq i \leq n$ and the induced edge labeling $f^*: E(G) \rightarrow \{1\}$ is given by $f^*(u v) = |[f^*(u)]^3 - [f^*(v)]^3| \pmod{6}$ with edge weights one. Since the path satisfies the cube difference prime labeling, the paths are called the cube difference prime graphs.

Theorem 2.2: The paths P_n are cube difference prime 2- equitable labeling.

Proof: Let u_1, u_2, \dots, u_n be the vertices of a path P_n . The vertex set of the path graph is $\{u_i / 1 \leq i \leq n\}$ and the edge set is $\{u_i u_{i+1} / 1 \leq i \leq n-1\}$. The order of P_n is n and the size is $n-1, n \geq 2$. The vertex labeling $f: V(G) \rightarrow \{0, 1, 2, \dots, n-1\}$ is define by $f(u_i) = i-1, 1 \leq i \leq n$ and the induced edge labeling $f^*: E(G) \rightarrow \{1, 2\}$ is given by $f^*(u v) = |[f^*(u)]^3 - [f^*(v)]^3| \pmod{4}$, $f(e_i) = 1, i = 1, 4, 5, 8, 9, \dots$ and $f(e_i) = 3, i = 2, 3, 6, 7, \dots$ with $|e_j(1) - e_j(3)| \leq 1$. Hence the paths P_n are cube difference prime 2- equitable graphs.

Theorem 2.3: The comb graphs (P_n, K_1) are cube difference prime 2- equitable labeling.

Proof: Let u_1, u_2, \dots, u_n be the vertices of a path P_n . Let v_1, v_2, \dots, v_n be the corresponding pendent vertices. The vertex set of the comb graph is $\{u_i, v_i / 1 \leq i \leq n\}$ and the edge set is $\{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i v_i / 1 \leq i \leq n\}$. The order of (P_n, K_1) is $2n$ and the size is $2n-1, n \geq 3$. The vertex labeling $f: V(G) \rightarrow \{0, 1, 2, \dots, n-1\}$ is define by $f(u_i) = i-1, 1 \leq i \leq n$ and $f(v_i) = i, n \leq i \leq 2n-1$ and the induced edge labeling $f^*: E(G) \rightarrow \{1, 2\}$ is given by $f^*(u v) = |[f^*(u)]^3 - [f^*(v)]^3| \pmod{4}$, $f(e_i) = 1, i = 1, 4, 5, 8, 9, \dots$ and $f(e_i) = 3, i = 2, 3, 6, 7, \dots$ with $|e_j(1) - e_j(3)| \leq 1$. Hence the comb graphs (P_n, K_1) are cube difference prime 2- equitable graphs.

Theorem 2.2: The graphs P_n^2 are cube difference cordial graphs.

Proof: Let P_n be a path of length $n-1$ with vertices u_1, u_2, \dots, u_n . Here $|V(G)| = |V(P_n^2)| = n$ and $|E(G)| = |E(P_n^2)| = 2n-3, n \geq 3$. The vertex labeling $f: V(G) \rightarrow \{0, 1, 2, \dots, n-1\}$ is define by $f(u_i) = i-1, 1 \leq i \leq n$ and the induced edge labeling $f^*: E(G) \rightarrow \{0, 1\}$ is given by $f^*(u v) = |[f^*(u)]^3 - [f^*(v)]^3|$ is 1 if the edge weight is odd and 0 if the edge weight is even with $|e_j(1) - e_j(0)| \leq 1$. Hence the graphs P_n^2 are cube difference cordial graphs.

Theorem 2.3: The total graph of cycles $T(C_n)$ are cube difference cordial graphs.

Proof: Let C_n be a cycle of length n with vertices u_1, u_2, \dots, u_n . Here $|V(G)| = |V(M(C_n))| = 2n$ and $|E(G)| = |E(M(C_n))| = 4n, n \geq 3$. Here the edge sets are $E_1 = \{u_i u_{i+1} / 1 \leq i \leq n\}$, $E_2 = \{u_{2i} u_{2i+2} / 1 \leq 2i \leq n/2\}$, $E_3 = \{u_{2i-1} u_{2i+3} / 1 \leq 2i \leq n/2\}$, $E_4 = \{u_n u_1\}$, $E_5 = \{u_{2n} u_{n+1}\}$ and $E_6 = \{u_{2n} u_1\}$. The vertex labeling $f: V(G) \rightarrow \{0, 1, 2, \dots, n-1\}$ is define by $f(u_i) = i-1, 1 \leq i \leq n$ and the induced edge labeling $f^*: E(G) \rightarrow N$ is given by $f^*(u v) = |[f^*(u)]^3 - [f^*(v)]^3|$ is 1 if the edge weight is odd and 0 if the edge weight is even with $|e_j(1) - e_j(0)| \leq 1$ for odd graphs and $|e_j(1) - e_j(0)| = 0$ for even graphs.

Theorem 2.4: The shadow graph of paths admit cube difference cordial labeling.

Proof: Let u_1, u_2, \dots, u_n be the vertices of a path P_n then the shadow graph of the path is obtained by adding new vertices v_1, v_2, \dots, v_n corresponding to each vertices of the path graph. Hence v_1, v_2, \dots, v_n be the vertices of the new path in $D_2(P_n)$. The vertex set of $D_2(P_n)$ is $\{u_i, v_i / 1 \leq i \leq n\}$ and the edge set is $\{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_i u_{i+1} / 1 \leq i \leq n-1\}$

Here the order of the graph $D_2(P_n)$ is $2n$ and the size is $4(n-1)$. The labeling of the edge sets $\{u_i u_{i+1} / 1 \leq i \leq n-1\}$ and $\{v_i v_{i+1} / 1 \leq i \leq n-1\}$ are 1 and the other two sets $\{u_i v_{i+1} / 1 \leq i \leq n-1\}$ and $\{v_i u_{i+1} / 1 \leq i \leq n-1\}$ have the edge weights as 0. The graph also satisfies the condition $|e_j(1) - e_j(0)| \leq 1$. Hence the shadow graph of paths admit cube difference cordial labeling.

Theorem 2.5: The total graph of a path $T(P_n)$ admits cube difference cordial labeling.

Proof: Let P_n be a path of length $n-1$ with vertices u_1, u_2, \dots, u_{n-1} . Here $|V(G)| = |V(T(P_n))| = 2n$ and $|E(G)| = |E(T(P_n))| = 3n-1, n \geq 3, 1 \leq i \leq n$. The vertex labeling $f: V(G) \rightarrow \{0, 1, 2, \dots, n-1\}$ is define by $f(u_i) = i-1, 1 \leq i \leq n$ and the induced edge labeling $f^*: E(G) \rightarrow \{0, 1\}$ is given by $f^*(u v) = |[f^*(u)]^3 - [f^*(v)]^3|$ is 1 if the edge weight is odd and 0 if the edge weight is even with $|e_j(1) - e_j(0)| \leq 1$

Theorem-2.6: The stars admit cube difference cordial labeling

Proof: Define the mapping $f : V \rightarrow \{0, 1, 2, \dots, p-1\}$ by $f(v_i) = i$, $1 \leq i \leq n$ and $f(v) = 0$. Hence the pendent vertices of the star graphs are labeled with $1, 2, 3, \dots, p-1$ and the apex vertex as 0 .

The edge weights of the star graphs are 1 and 0 using the condition $f^*(u v) = |[f^*(u)]^3 - [f^*(v)]^3|$ is 1 if the edge weight is odd and 0 if the edge weight is even with $|e_j(1) - e_j(0)| \leq 1$. Here the edges are mapped using the function $f^*: E(G) \rightarrow \{0, 1\}$. For the even stars $|e_j(1) - e_j(0)| = 0$ and for the odd stars $|e_j(1) - e_j(0)| \leq 1$. Hence the stars are cube difference cordial.

Theorem-2.7: The stars admit cube difference 6- equitable labeling.

Proof: Let v be the apex vertex and let v_1, v_2, \dots, v_n are the pendent vertices of the star graph $K_{1,n}$

The vertices are given values using the mapping $f : V \rightarrow \{0, 1, 2, \dots, p-1\}$ define by $f(v_i) = i$, $1 \leq i \leq n$ and $f(v) = 0$. The edge weights are $0, 1, 2, 3, 4, 5$ using the condition $f^*(u v) = |[f^*(u)]^3 - [f^*(v)]^3| \pmod{6}$. Hence the star graphs are cube difference 6- equitable graphs.

CONCLUSION

Different types of cube difference labeling are defined and the graphs which satisfies these labeling are called the corresponding cube difference graphs. Some graphs are investigated. It can also be verified for some other graphs. It is an open problem in the research field.

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