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ON R# CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, a new class of closed sets called $R^{\#}$ closed sets in topological spaces are introduced and studied. A subset A of a topological space (X,τ) is called $R^{\#}$-closed if U contains generalized closure of A whenever U contains A and U is R open set in (X,τ). This new class of Closed sets lies between the g-closed sets and rg-closed sets in topological spaces. Also some of their properties have been investigated.*

Keywords: *R* -closed sets, w-closed sets, rg-closed sets and $R^{\#}$-closed sets.*

2010 Mathematics Subject Classification: *54A05, 54A10.*

1. INTRODUCTION

In the year 1970, Levine [58] introduced the concept of generalized closed sets in topological spaces, Also N.Palaniapan *et al.* [38], P.sundaram *et al.* [44], S.S.Benchalli *et al.* [7], C.Janaki *et al.* [19] introduced and studied regular generalized closed sets, W-closed sets, RW-closed sets, R*-closed sets in topological spaces respectively. In this paper an attempt is made to study a new class of closed sets called $R^{\#}$-closed sets in topological spaces.

Throughout this paper (X,τ) represent non-empty topological spaces. For a subset A of a topological space (X,τ), cl(A), int(A), scl(A), αcl(A) and spcl(A)) denote the closure of A, the interior of A, the semi-closure of A, the α-closure of A and the semi pre closure of A in a topological space X respectively. We recall the following definitions, which are prerequisites for present study.

2. PRELIMINARIES

Definition 2.1: A subset A of a topological space (X,τ) is called a

1. Generalized closed set (g-closed) [58] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
2. R*-closed set [19] if rcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular semi -open in (X, τ).
3. RW-closed set [7] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular semi -open in (X, τ).
4. Generalized pre regular closed set (gpr-closed) [15] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in (X, τ).
5. Generalized semi pre closed set (gsp-closed) [14] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).
6. Regular generalized closed set (rg-closed) [38] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular-open in (X, τ).
7. Regular weak generalized closed set (rwg-closed) [30] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in (X, τ).
8. w-closed set[44] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi -open in (X, τ).
9. gspr-closed set[35] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular -open in (X, τ).
10. r^g-closed set[43] if gcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular -open in (X, τ).

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3. BASIC PROPERTIES OF $R^{\#}$-CLOSED SETS IN TOPOLOGICAL SPACES

Definition 3.1: A subset A of a space (X,τ) is called $R^{\#}$-closed if gcl(A) ⊆ U whenever A ⊆ U and U is R*open in (X, τ), we use the notation $R^{\#}$-C(X) to denote set of all $R^{\#}$-closed sets in (X, τ).

Example 3.2:

i) Let X = {a, b, c d}, τ = **{**ϕ, X, {a}, {b}, {a, b}, {a, b, c}**}.** Then
Closed sets in (X, τ) are X, ϕ, {d},{c, d},{a, c},{a, c, d}, {b, c, d}
$R^{\#}$-Closed sets in (X, τ) are X, ϕ, {d},{a, c}, {a, d},{b, c}, {b, d},{c, d},{a, b, c}{a, b, d},{a, c, d},{b, c, d}
$R^{\#}$-Open sets in (X, τ) are X, ϕ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}{b, c}, {b, d}{a, b, c}

ii) Let X = {a, b, c, d}, τ = **{**ϕ, X, {a}, {a, b},{a, b, c}**}.** Then
Closed sets in (X, τ) are X, ϕ, {d},{c, d}, {b, c, d}
$R^{\#}$-Closed sets in (X, τ) are X, ϕ, {d}, {a, d}, {b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
$R^{\#}$-Open sets in (X, τ) are X, ϕ, {a}, {b}, {c}, {a, b}, {a, c},{b, c},{a, b, c}

Theorem 3.3: Every g-closed set in X is $R^{\#}$-closed set but not conversely.

Proof: Let A be a g-closed set in topological space X. Let U be any R*-open set in X such that A ⊂ U. Since A is g-closed, we have gcl (A) = A ⊆ U. Therefore gcl(A)=A⊆U .Hence A is $R^{\#}$-closed set in X.

Example 3.4: Let X= {a, b, c, d} and τ={X, ∅, {a},{b},{a, b},{a, b, c}} then the set A={a, c} is $R^{\#}$-closed set but not g-closed in X.

Corollary 3.5:

i. Every closed set is $R^{\#}$-closed set in X.
ii. Every regular closed sets is $R^{\#}$-closed set in X.
iii. Every w-closed set in $R^{\#}$-closed set in X.
iv. Every $\hat{g}$-closed set is $R^{\#}$-closed set in X

Proof:

i. Every closed set is g-closed [58] and follows from theorem 3.3.
ii. Every regular closed set is closed, from stone [57] and then follows corollary 3.5.i).
iii. Every w-closed set is g-closed [44] follows from theorem 3.3.
iv. Every $\hat{g}$-closed set is g-closed [51] follows from theorem 3.3.

Remark 3.6: The converse of the above Corollary 3.5 need not be true as seen from the following example.

Example 3.7: Let X = {a, b, c, d} τ={X, ∅, {a},{b},{a, b},{a, b, c}}, A={a, b, d} is $R^{\#}$-closed set but not a closed (respectively r-closed, w-closed, $\hat{g}$-closed) in X.

Theorem 3.8: Every $R^{\#}$-closed is rg-closed set in X but not conversely.

Proof: Let A be a $R^{\#}$-closed set in X. Let U be any open set in X such that A⊆ U. Since every open set is R* open set and A is $R^{\#}$-closed set, we have gcl(A)⊆U. Thus gcl(A)⊆U, U is open in X. Therefore A is rg-closed in X.

Example 3.9: Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {b},{a, b}, {a, b, c}}. Then the set A={c} is rg-closed set but not $R^{\#}$-closed set in X.

Corollary 3.10: For a topological space (X, τ) the following are hold

i) Every $R^{\#}$-closed set is rgb-closed set
ii) Every $R^{\#}$-closed set is wgrα-closed set
iii) Every $R^{\#}$-closed set is gpr-closed, gspr-closed, and rgβ-closed
iv) Every $R^{\#}$-closed set is r^g-closed and rwg-closed

Proof:

i) Let A be a $R^{\#}$-closed set in X. Let U be any regular open in X such that A⊆U. Since every regular open set is R*-open set in X and A is $R^{\#}$-closed set in X it follows that gcl(A)⊆U. Therefore gcl(A)⊆U, U is open in X. Hence A is rgb-closed set in X.
ii) Let A be a $R^{\#}$-closed set in X. Let U be any rα-open in X such that A⊆U. Since every rα- open set is R*-open set in X and A is $R^{\#}$-closed set in X it follows that gcl(A)⊆U. Therefore gcl(A)⊆U, U is regular open in X. Hence A is wgrα -closed set in X.

iii) Every rg closed and gpr-closed is gspr closed [35], and also gpr-closed is rgβ-closed [43] and follows from Theorem 3.8.
iv) Every rg is r^g-closed [43], rwg –closed [30] and follows from Theorem 3.9(iii)

Remark 3.11: The following examples are shows that the R#-closed sets are independent with some existing closed sets in topological spaces p-closed [32], s-closed [23], α-closed sets [59], semi pre-closed sets [5], b-closed sets [4], rs-closed sets [11], gs-closed sets [1], gα-closed sets [25], αg-closed sets [24], gsp-closed sets [14], gp-closed sets [26], g*-closed [49], swg-closed sets [35], wg-closed sets [31], rgα-closed sets [47], g*p-closed sets [50], wα-closed sets [8], gwα-closed sets [8], R*[19], rgw-closed sets [28], pgpr-closed sets [6], rps-closed [45] sets, gprw-closed sets [29], αrw-closed sets [55], αgp-closed sets [36], βwg^{*}-closed sets [13],$**_{g\alpha}$-closed sets [54], gαb-closed sets [56], sgb-closed sets [18], rg*b-closed sets [17], ps-closed sets [48], αgs-closed sets [41], g#s-closed sets [53], α**g-closed sets [25],g**-closed sets [16], gb-closed sets [2], swg*-closed sets [31], gr-closed sets [9], rps-closed sets [45], βwg $**$-closed sets [46], g#α-closed sets [37], g*s-closed sets [14], #gα-closed sets [12], **s**#gα-closed sets [22], g*p-closed sets [50], gps-closed sets [42], g#p#-closed sets [3].

Example 3.12: Let X = {a, b, c d}, τ = **{**ϕ, X, {a}, {b},{a, b} {a, b, c}**}.** Then

i. α-closed sets in (X,τ)are X,ϕ,{c},{d},{c, d}, {a, c, d},{b, c, d}
ii. rs- closed sets in (X, τ) are X,ϕ,{a},{b},{a, c},{a, d},{b, c},{b, d},{a, c, d},{b, c, d}
iii. gs- closed sets in (X, τ) are X,ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, d},{a, c, d}, {b, c, d}
iv. gα- closed sets in (X,τ) are X,ϕ,{c},{d},{c, d},{a, c, d},{b, c, d}
v. αg- closed sets in (X,τ) are X,ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
vi. gsp- closed sets in (X,τ) are X,ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, d},{a, c, d}, {b, c, d}
vii. gp- closed sets in (X, τ) are X,ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
viii. g*- closed sets in (X, τ) are X, ϕ,{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
ix. wg- closed sets in (X, τ) are X,ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
x. rgα- closed sets in (X, τ) are X,ϕ,{c},{d},{a, b},{c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d}
xi. g*p- closed sets in (X, τ) are X,ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
xii. wα- closed sets in (X, τ) are X,ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
xiii. gwα- closed sets in (X, τ) are X,ϕ,{c},{d},{c, d},{a, c, d},{b, c, d}
xiv. R* closed sets in (X, τ) are X,ϕ{a, b},{c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d}
xv. rgw- closed sets in (X, τ) are X,ϕ,{c},{d},{a, b},{c, d},{a, b, c},{a, b, d},{a, c, d}, {b, c, d}
xvi. pgpr- closed sets in (X, τ) are X,ϕ,{c},{d},{c, d},{a, c, d},{b, c, d}
xvii. rps- closed sets in (X, τ) are X,ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, c, d}, {b, c, d}
xviii. gprw- closed sets in (X, τ) are X,ϕ,{c},{d},{a, b},{c, d},{a, b, c},{a, b, d},{a, c, d}, {b, c, d}
xix. αrw- closed sets in (X, τ) are X,ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
xx. αgp- closed sets in (X, τ) are X,ϕ,{c},{d},{c, d},{a, c, d},{b, c, d}
xxi. ps- closed sets in (X, τ) are X, ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, c}, {a, b, d}, {a, c, d},{b, c, d}
xxii. p- closed sets in (X, τ) are X, ϕ, {c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d}, {b, c, d}
xxiii. αgs- closed sets in (X, τ) are X, ϕ,{c},{d},{c, d},{a, c, d},{b, c, d}
xxiv. g#s- closed sets in (X,τ) are X, ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, c, d}, {b, c, d}}
xxv. α**g- closed sets in (X, τ) are X, ϕ,{a, b},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, c},{a, b, d},{a, c, d}, {b, c, d}
xxvi. gb- closed sets in (X,τ) are X, ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, d}, {a, c, d}, {b, c, d}}
xxvii. βwg**-closed set in (X,τ) are X, ϕ,{c},{d},{a, b},{a, c},{a, d},{b, c},{b, d}, {c, d},{a, b, d},{a, c, d}, {b, c, d}{a, b, c}
xxviii. g#α- closed sets in (X,τ) are X, ϕ,{c},{d},{c, d},{a, c, d},{b, c, d}
xxix. og#-p- closed sets in (X,τ) are X, ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
xxx. g*p- closed sets in (X,τ) are X, ϕ, {c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d}, {b , c, d}
xxxi. g#p#- closed sets in (X, τ) are X, ϕ,{c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d},{a, c, d}, {b, c, d}{a, b, c},{a, b, d}}

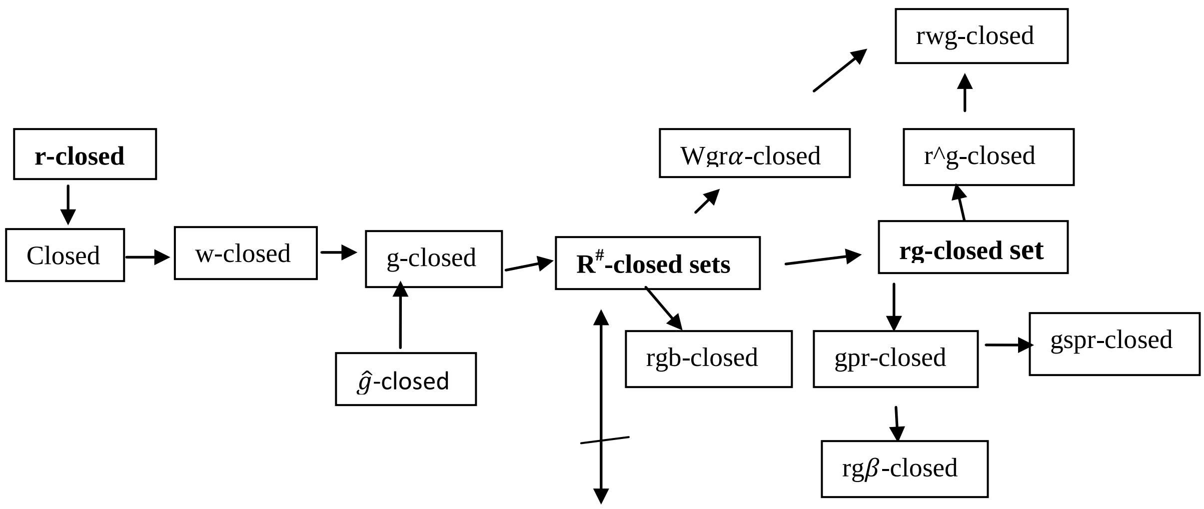
Conclusion: From the above example we proved that the following closed sets are independent to R#-closed sets: α-closed set, rs- closed set, gs- closed set, gα- closed set,αg- closed set, gsp- closed set, gp- closed set, g*- closed set, wg-closed set, rgα- closed set, g*p- closed set, wα- closed set, gw_- closed set, R*closed set, rgw- closed set, pgpr-closed set, rps- closed set, gprw- closed set,αrwclosed set, gp- closed set, ps- closed set, p- closed set, gs- closed set, g#s- closed set, α**g-closed set, gb- closed set, βwg**-closed set, g#α- closed set, og#p- closed set, g*p- closed set, g#p#- closed set.

Example 3.13: Let X = {a, b, c, d}, τ = **{**ϕ, X, {a}, {a, b} {a, b, c}**}.** Then

i. closed sets in (X,τ) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
ii. sp- closed sets in (X,τ) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
iii. b- closed sets in (X, τ) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
iv. swg- closed sets in (X, τ) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
v. gwα- closed sets in (X, τ) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
vi. rgw- closed sets in (X, τ) are X, ϕ, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, c, d}{a, b, d} {b, c, d}
vii. pgpr- closed sets in (X, τ) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
viii. rps- closed sets in (X, τ) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
ix. $\beta wg *$- closed sets in (X, τ) are X, ϕ, {d}, {a, d}, {b, d}, {c, d}, {a, c, d}, {b, c, d}
x. $** g\alpha$- closed sets in (X, τ) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
xi. $g\alpha b$- closed sets in (X, τ) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
xii. sgb- closed sets in (X, τ) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
xiii. rg*b- closed sets in (X, τ) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
xiv. gb- closed sets in (X, τ) are X, ϕ, {b}, {c}, {d}, {a, d}, {b, d}, {c, d}, {a, c, d}, {a, b, d}, {b, c, d}
xv. swg*- closed sets in (X, τ) are X, ϕ, {d}, {a, d}, {b, d}, {c, d}{a, c, d} {a, b, d}, {b, c, d}
xvi. gr- closed sets in (X, τ) are X, ϕ, {d}, {a, d}, {b, d}, {c, d} {a, c, d} {a, b, d}, {b, c, d}
xvii. rps- closed sets in (X, τ) are X, ϕ, {d}, {a, d}, {b, d}, {c, d}{a, c, d}{a, b, d}, {b, c, d}}
xviii. g#α- closed sets in (X, τ) are X, ϕ,
xix. {b}, {c}, {d}, {a, d}, {b, d}, {c, d}, {a, c, d}, {a, b, d}, {b, c, d}
xx. g*s- closed sets in (X, τ) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
xxi. #gα- closed sets in (X, τ) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}

Conclusion: From the above example we proved that the following closed sets are independent to R#-closed sets: s-closed set, sp- closed set, b- closed set, swg-closed set, gwα-closed set,rgw- closed set, pgpr- closed set, rps- closed set, βwg*-closed set,**gα- closed set, gαb- closed set, sgb-closed set, rg*b- closed set, gb-closed set, swg*- closed set, gr- closed set, rps- closed set, g#α - closed set, g*s-closed set, #gα- closed set.

Remark 3.14: From the above results discussion and known results we have the following implications



p-closed, s-closed, α-closed sets, semi pre-closed sets, b-closed sets, rs-closed sets, gs closed sets, gα-closed sets, αg-closed sets, gsp-closed sets, gp-closed sets, g*-closed, swg-closed sets, wg-closed sets, rgα-closed sets, g*p-closed sets, wα-closed sets, gwα-closed sets, R*, rgw-closed sets, pgpr-closed sets, rps-closed sets, gprw-closed sets, αrw-closed sets, αgp-closed sets, βwg^*-closed sets,$**_{g\alpha}$-closed sets, gαb-closed sets, sgb-closed sets, rg*b-closed sets, ps-closed sets, αgs-closed sets, g#s-closed sets, α**g-closed sets, g**-closed sets, gb-closed sets, swg*-closed sets, gr-closed sets, rps-closed sets, $\beta wg **$-closed sets, g#α-closed sets, g*s-closed sets, #gα-closed sets, **s**#gα-closed sets, g*p-closed sets, gps-closed sets, g#p#-closed sets.

Notations:
A ⟶ B means the set A implies the set B but not conversely
A ⟷̸ B means the set A and the set B are independent of each other.

Theorem 3.15: The union of any two $R^{\#}$-closed sets of X is $R^{\#}$-closed s

Proof: Let A and B are the $R^{\#}$-closed sets in topological space (X, τ). Let U be R*-open set in X such that A∪B⊆U, then A⊆U and B⊆U, since A and B are the $R^{\#}$-closed sets, gcl(A)⊆U, gcl(B)⊆U, and we know that gcl(A)∪ gcl(B)= gcl(A∪B)⊆U. Therefore A∪B is $R^{\#}$-closed set in X.

Remark 3.16: The intersection of two $R^{\#}$-closed sets of topological space (X, τ) is generally not a $R^{\#}$-closed set in X.

Example 3.17: Let X = {a, b, c, d} and τ={X, ϕ, {a}, {b}, {a, b}, {a, b, c}} then the set A={a, c} and B={a, d} are $R^{\#}$-closed sets in X, but A∩B={a} is not $R^{\#}$-closed set in X.

Theorem 3.18: If a subset A of topological space (X, τ) is a $R^{\#}$-closed set in X then gcl(A)-A. does not contain any non empty R*-closed set in X.

Proof: Let A be a $R^{\#}$-closed set in X and suppose F be a non empty R*-closed subset of gcl(A)-A F⊆gcl(A)-A⟹ F⊆gcl(A)⋂(X-A) ⟹ F⊆gcl(A) —— (1) & F⊆X-A
⟹ A⊆X-F and X-F is R*-open set and A is an $R^{\#}$-cl~~osed set~~, gcl(A) ⊆X-F
⟹ F⊆X-gcl(A) —— (2) from equations (1) and (2) we gwt F⊆gcl(A)∩(X-gcl(A))=ϕ
⟹ F=Φ thus gcl(A)-A does not contain any non empty R*-closed set in X.

Remark 3.19: The converse of the above Theorem 3.17 need not be true as seen from the following example 3.20.

Example 3.20: Let X = {a, b, c, d} τ = **{** X, ϕ, {a}, {a, b}, {a, b, c}**}** then the set A={a}, gcl{a} = {ac}, gcl{A}-A={a}-{a, c}={c} does not contain any non empty R^{*} closed set in X but A is not $R^{\#}$-closed set.

Theorem 3.21: If A is an $R^{\#}$-closed set in (X, τ) and A⊆B⊆gcl(A) then B is also $R^{\#}$-closed set in X.

Proof: If it is given that A is $R^{\#}$-closed set in X then we have to prove that B is also $R^{\#}$-closed set in X. Let U be an R*-open set of X such that B⊆U.Since A⊆B and A is $R^{\#}$-closed set, gcl(A) ⊆U & A⊆U Now B⊆gcl(A) ⇒gcl(B) ⊆gcl(gcl(A))=gcl(A) ⊆U. Therefore gcl(B) ⊆U Hence B is $R^{\#}$-closed set in X.

Remark 3.22: The converse of the above theorem 3.20 need not to be true as seen from the following example 3.23

Example 3.23: Let X={a, b, c} and τ={x, ϕ, {a}, {b}, {a, b}, {a, b, c}} then the set A={c, d} B={a, c, d} such that A and B are $R^{\#}$-closed sets in X, but A⊆B and B is not a subset of gcl(A) ,because gcl(A) ={c, d}.

Theorem 3.24: Let A be a $R^{\#}$-closed in X. Then A is g-closed if and only if gcl(A)-A is R*-closed.

Proof: Necessity: suppose A be a g-closed set in X then gcl(A)=A that is gcl(A)-A=ϕ which is R*-closed.

Sufficiency: Suppose A is $R^{\#}$-closed in X and gcl(A)-A is R*-closed from theorem 3.18 then gcl(A)-A=ϕ ⇒gcl(A) =A Therefore A is g-closed.

Theorem 3.25: Let$A \subseteq Y \subseteq X$, (X, τ) and (Y, σ) are topological spaces. If A is a $R^{\#}$-closed set in (X,τ). Then A is $R^{\#}$-closed relative to (Y, σ).

Proof: Let $A \subseteq Y \bigcap G$, where G is R*-open since A is $R^{\#}$-closed set in X .Then A⊆X and gcl(A)⊆G this implies that Y∩gcl(A)⊆Y∩G where Y∩gcl(A) is g-closed set of A in Y. Thus A is $R^{\#}$-closed set relative to Y.

Theorem 3.26:

i) If A is regular open and rg-closed set in (X, τ) then A is $R^{\#}$-closed set in (X, τ).
ii) If A is g-open and rg-closed set in (X, τ) then A is $R^{\#}$-closed set in (X, τ).
iii) If A is a regular-open and rwg-closed set in (X, τ) then A is $R^{\#}$-closed in (X, τ).
iv) If A is a regular-open and gpr-closed set in (X, τ) then A is $R^{\#}$-closed in (X, τ).
v) If A is regular open and r^g-closed set in (X, τ) then A is $R^{\#}$-closed set in (X, τ).
vi) If A is regular open and βwg^{**}-closed set in (X, τ) then A is $R^{\#}$-closed set in (X, τ).

Proof:

i. Let A be a regular open and rg-closed set in X. Let U be any R*-open set in X such that A⊆U. Since A is regular open and rg-closed in X by definition gcl(A)⊆(A)⊆U. Hence A is $R^{\#}$-closed set in X.
ii. Let A be a g-open and rg-closed set in X. Let U be any R*-open in X such that A⊆U since A is g-open and rg-closed in X by definition gcl(A)⊆A then gcl(A)⊆A⊆U. Hence A is $R^{\#}$-closed set in X.

iii. Let A be a regular open and rwg-closed set in X. Let U be any R*-open set in X such that A ⊆ U since A is regular open and rwg-closed in X, by definition $cl(\text{int}(A)) \subseteq A$ we know that

$$cl(\text{int}(A)) \subseteq cl(\text{int}(cl(A))) \subseteq A \subseteq U \quad Then \quad gcl(A) \subseteq U.$$ Hence A is $R^{\#}$-closed set in X.

iv. Let A be a regular open and gpr-closed set in X. Let U be any R*-open set in X such that A⊆U.Since A is regular open and gpr-closed in X, by definition pcl(A))⊆A then we know that pcl(A)⊆gcl(A)⊆A hence gcl(A)⊆U. Hence A is $R^{\#}$-closed set in X.

v. Let A be a regular open and r$\hat{g}$-closed set in X. Let U be any R*-open set in X such that A⊆U.Since A is regular open and r^g-closed in X by definition gcl(A)⊆U then A is $R^{\#}$-closed set in X.

vi. Let A is regular open and βwg^{**}-closed set in X. Let U be any R*-open set in X such that A⊆U since A is regular open and βwg^{**}-closed in X by definition gcl(A)⊆U then gcl(A)⊆A⊆U. Hence A is $R^{\#}$-closed set in X.

Remark 3.27: If A is both semi-open and $R^{\#}$-closed set in (X, τ), then A need not be $\hat{g}$-closed in general as seen from the following example 3.28.

Example 3.28: Consider X={a, b, c, d} τ={x, ϕ, {a}, {a, b}, {a, b, c}}, A={a, c, d} is both semi-open and $R^{\#}$-closed but it is not $\hat{g}$- closed set.

Theorem 3.29: If A is both open and g-closed set in X then A is $R^{\#}$ closed set in X.

Proof: Let A be open and g-closed in X. Let U be any R*-open set in X such that A⊆U, by definition cl(A))⊆A ⊆U and gcl(A)=A .This implies that cl(A)⊆gcl(A)⊆A⊆U. Hence gcl(A)⊆U. Therefore A is $R^{\#}$-closed set in X.

Theorem 3.30: If a subset A of a topological space X is both regular open and $R^{\#}$-closed set in X then it is g-closed set in X.

Proof: Suppose a subset A of a topological space X is regular open and $R^{\#}$-closed as every regular open is R*-open now A⊆A then definition of $R^{\#}$-closed , gcl(A)⊆A and also A⊆gcl(A) then gcl(A)=A. Hence A is g-closed in X.

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