International Journal of Mathematical Archive-8(8), 2017, 126-133
Available online through www.ijma.info ISSN 2229 – 5046

ON gg-CLOSED SETS IN TOPOLOGICAL SPACES

BASAVARAJ M. ITTANAGI[1] and GOVARDHANA REDDY H G*

**Department of Mathematics,
Siddaganga Institute of Technology, Tumakuru-03,
Affiliated to VTU, Belagavi, Karnataka state, India.**

**[*]Department of Mathematics,
Alliance College of Engineering and Design,
Alliance University, Bangalore, Karnataka state, India.**

(Received On: 26-07-17; Revised & Accepted On: 16-08-17)

ABSTRACT

In this research paper, a new type of closed sets called generalization of generalized closed sets (gg-closed sets) are introduced and studied in a topological space (X, τ). We show that gg-closed sets lies between the classes of all generalized closed sets and regular ^ generalized closed sets in topological spaces. Also some of their properties have been investigated.

2010 Mathematics Classification: *54A05, 54A10.*

Keywords: *Regular closed sets, g-closed sets, r^g-closed sets and gg-closed sets.*

1. INTRODUCTION

The generalization of a closed set was introduced in the year 1970 by N. Levine. Different types of generalization of closed sets were introduced and studied by various mathematicians. N. Levin introduced and studied generalized closed sets [16] and semi open sets [17] in topological spaces. Stone and Cameron introduced regular open sets [31] and regular semi open sets [6] respectively. In this research paper X or (X, τ) represents a non-empty topological space and $A \subseteq X$. For $A \subseteq X$ in (X, τ), cl(A), int(A), scl(A), αcl(A), spcl(A) and gcl(A) represents the closure of A, the interior of A, the semi-closure of A, the α-closure of A, the semi pre closure of A and the g-closure of A in X respectively.

2. PRELIMINARIES

In this section, we recall some existing closed and open sets in (X, τ).

Definition 2.1: A subset A of a topological space (X, τ) is called a

1. Regular open set [31] if A= int(cl(A)) and regular closed if A= cl(int(A))
2. Semi-open set [17] if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.
3. Regular semi open set [6] if there exists a regular open set U such that $U \subseteq A \subseteq cl(U)$
4. Generalized closed set (g-closed) [16] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
5. r^g-closed set [27] if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular -open in (X, τ).
6. RW-closed set [3] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi -open in (X, τ).

The complement of the closed sets mentioned above are their open sets respectively and vice versa.

3. BASIC PROPERTIES OF gg- CLOSED SETS IN TOPOLOGICAL SPACES

Definition 3.1: A subset *A* of a topological space (X,τ) is called generalization of generalized closed (gg-closed) set if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi open in (X, τ). We use the notion GGC(X) to denote the set of all gg-closed sets in (X,τ).

Corresponding Author: Govardhana Reddy H G*
***[*]Department of Mathematics, Alliance College of Engineering and Design,
Alliance University, Bangalore, Karnataka state, India.***

Example 3.2: Let X = {p, q, r, s} and τ = {ϕ, X, {p}, {q}, {p, q}, {p, q, r}} be a topology on X.
Then GGC(X)={X, ϕ, {s}, {p, q}, {p, s}, {q, s}, {r, s}, {p, q, r}, {p, q, s}, {p, r, s}, {q, r, s}}.

First we show that all gg-closed sets lies between the classes of generalized-closed sets and r^g-closed sets.

Theorem 3.3:-Every g-closed set is gg-closed set in X.

Proof: Let A be g-closed set in topological space X and U be any regular semiopen set in X such that A ⊆ U. Since A is g closed, we have gcl(A)=A. Therefore gcl(A) ⊆U. Hence A is gg-closed set in X.

Remark 3.4: The converse part of theorem 3.3 is not true in general as seen from the example 3.5 given below.

Example 3.5: Let X = {p, q, r, s} and τ = **{**ϕ, X, {p}, {q}, {p, q}, {p, q, r}**}** be a topology on X. Then A = {p, q} is gg-closed but not g-closed set in X.

Corollary 3.6:

i) Every closed set is gg-closed set in X.
ii) Every regular closed set is gg-closed set in X.
iii) Every л-closed set [11] is gg-closed set in X.
iv) Every δ-closed set [37] is gg-closed set in X.
v) Every θ-closed set [37] is gg-closed set in X.
vi) Every ω -closed set [29] is gg-closed set in X.
vii) Every θg-closed set [9] is gg-closed set in X.
viii) Every g*-closed set [34] is gg-closed set in X.
ix) Every ∗∗gα-closed set [39] is gg-closed set in X.
x) Every *gα-closed set [38] is gg-closed set in X.
xi) Every gr-closed set [28] is gg-closed set in X.
xii) Every δg-closd set [10] is gg-closed set in X.
xiii) Every rb-closed set [23] is gg-closed set in X.
xiv) Every $g^{\#}$-closed set [35] is gg-closed set in X.
xv) Every swg*-closed set [21] is gg-closed set in X.
xvi) Every βwg*-closed set [8] is gg-closed set in X.

Proof:

i) Every closed set is g-closed from Hameed [14] and follows from Theorem 3.3
ii) Every regular closed set is closed from Stone [31] and then follows from Corollary 3.6 i)
iii) Follow from Dontchev and Noiri, Every л-closed set is closed [11] and then from Corollary 3.6 i)
iv) Follow from Vellicko, Every δ-closed set is closed [37] and then from Corollary 3.6 i)
v) Follow from Vellicko, Every θ-closed set is closed [37] and then from Corollary 3.6 i)
vi) Follow from Sheik John, Every ω -closed set is g-closed [29] and then follow from Theorem 3.3
vii) Follow from Dontchev and Maki, Every θg-closed closed set is g-closed [9] and then follow from Theorem 3.3
viii) Follow from Veera Kumar, Every g*-closed set is g-closed [34] and then follow from Theorem 3.3
ix) Follow from Vigneshwaran, Every ∗∗gα-closed set is g-closed [39] and then follow from Theorem 3.3
x) Follow from Vigneshwaran, Every *gα-closed set is ∗∗gα -closed [38] and then follow from Corollary 3.6 ix)
xi) Follow from Sharmista Bhattacharya, Every gr-closed set is g-closed [28] and then follow from Theorem 3.3
xii) Follow from Dontchev and Ganster, Every δg-closed set is g-closed [10] and then follow from Theorem 3.3
xiii) Follow from Narmadha, Nagaveni and Noiri, Every rb closed set is δg-closed set [23] and then follow from Corollary 3.6 xii)
xiv) Follow from Veera Kumar, Every $g^{\#}$-closed set is g-closed set [35] and then follow from Theorem 3.3
xv) Let A be swg*-closed set in X and U be any regular semi open set in topological space (X, τ) such that A ⊆ U. Since every regular semi open is semiopen in X, by the definition of swg*-closed, gcl(A) ⊆ U. Therefore A is gg-closed set in X.
xvi) Proof is similar to corollary 3.6 xv)

Remark 3.7: The converse part of Corollary 3.6 is not true in general as seen from the example 3.8 given below.

Example 3.8: Let X = {p, q, r, s} and τ = **{**ϕ, X, {p}, {q}, {p, q}, {p, q, r}**}** be a topology on X**.** Then A= {p, q} is gg-closed but not a closed (respectively regular-closed, л-closed, θ-closed, δ-closed, w-closed, θg-closed, δg-closed, g*-closed, ∗∗gα -closed, *gα-closed, gr-closed, rb-closed, $g^{\#}$-closed, swg*-closed and βwg*-closed) set in X.

Theorem 3.9: Every gg- closed set in X is r^g-closed set in X.

Proof: Let A be gg-closed set in X. Let U be regular open in X such that A ⊆ U. Since every regular open set is regular semi open [13], U is regular semi open in X. Since A is gg-closed set we have gcl(A) ⊆ U. Therefore A is r^g-closed set in X.

Remark 3.10: The converse part of theorem 3.9 is not true in general as seen from the example 3.11 given below.

Example 3.11: Let X = {p, q, r, s} and τ = **{**ϕ, X, {p}, {q}, {p, q}, {p, q, r}**}** be a topology on X. Then A={r} is r^g-closed but not a gg-closed set.

Corollary 3.12:
- i) Every gg-closed set is rwg- closed [25] set in X.
- ii) Every gg-closed set is βwg**-closed set [32] in X.

Proof:
- i) Follow from Savithri and Janaki. Every r^g-closed set is rwg-closed set [27] and then follows from Theorem 3.9
- ii) Follow from Subashini Jesu Rajan ,Every rwg-closed set is βwg**-closed set [32] and then from corollary 3.12

Remark 3.13: The converse part of Corollary 3.12 is not true in general as seen from the example 3.14 given below.

Example 3.14: Let X = {p, q, r, s} and τ = **{**ϕ, X, {p}, {q}, {p, q}, {p, q, r}**}** be a topology on X. Then A={r} is a ***r***wg-closed and also βwg**-closed set in X but not gg-closed.

Remark 3.15: The following example shows that the gg-closed sets are independent with some existing closed sets in topological spaces.

Example 3.16: Let X = {p, q, r, s} and τ = **{**ϕ, X, {p}, {q}, {p, q}, {p, q, r}**}** be a topology on X**.** Then
1) α- closed sets [24], gα- closed sets [19] and swg- closed sets [22] in (X, τ) are X, ϕ, {r},{s}, {r, s}, {p, r, s}, {q, r, s}
2) αg- closed sets [18], gp- closed sets [20], wg- closed sets [22], g*p- closed sets [36], wα- closed sets [4], αrw-closed sets [40] and ρ- closed sets [7] in (X, τ) are X, ϕ, {r}, {s}, {p, s}, {q, s}, {r, s}, {p, q, s}, {p, r, s}, {q, r, s}
3) rgα- closed sets [33], rgw- closed sets [25], wgrα- closed sets [15] and gprw- closed sets [26] (X, τ) are X, ϕ, {r}, {s}, {p, q}, {r, s}, {p, q, r}, {p, q, s}, {p, r, s}, {q, r, s}
4) 4) semi- closed sets [17], semi pre- closed sets [1], sg- closed sets [5] and rps- closed sets [30] in (X, τ) are X, ϕ, {p}, {q}, {r}, {s}, {p, r}, {p, s}, {q, r}, {q, s}, {r, s}, {p, r, s}, {q, r, s}
5) gs- closed sets [2] and gsp- closed sets [12] in (X, τ) are X, ϕ, {p}, {q}, {r}, {s}, {p, r}, {p, s}, {q, r}, {q, s}, {r, s}, {p, q, s}, {p, r, s}, {q, r, s}.

Remark 3.17: The following two examples show that gg-closed sets and rw-closed sets are independent of each other.

Example 3.18: Let X = {p, q, r} and τ = {ϕ, X, {p}, {q}, {p, q}, {p, r}} be a topology on X. Then
GGC(X) ={X, ϕ, {q}, {r},{p, q},{p, r},{q, r}}
RWC(X) ={X, ϕ, {p},{q},{r},{p, q},{p, r}, {q, r}}

Example 3.19: Let X = {p, q, r, s} and τ = {ϕ, X, {p}, {q}, {p, q}, {p, q, r}} be a topology on X. Then
GGC(X) ={X, ϕ,{s},{p, q},{p, s},{q, s},{r, s},{p, q, r},{p, q, s},{p, r, s},{q, r, s}}
RWC(X) ={ X,ϕ,{s},{p, q},{r, s}, {p, q, r},{p, q, s},{p, r, s}, {q, r, s}}.

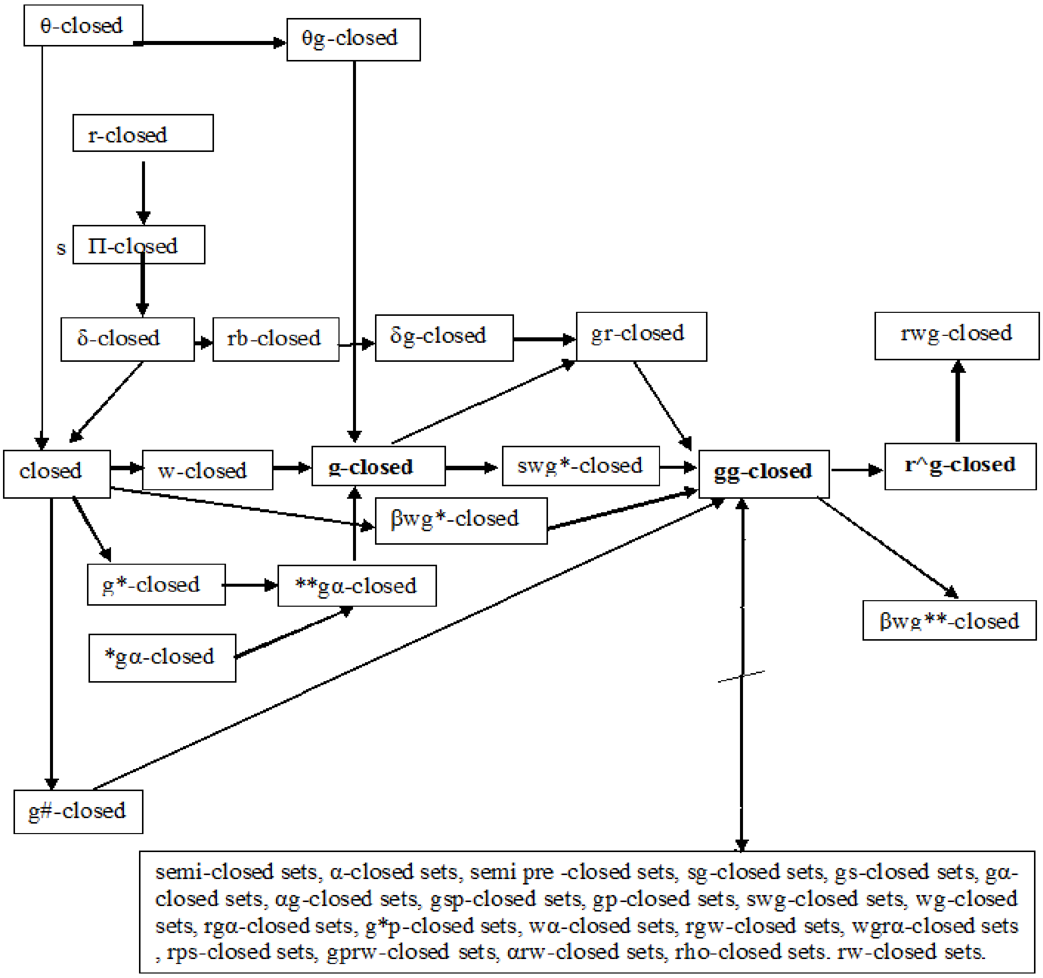
Theorem 3.20: The union of any two gg-closed sets of (X, τ) is gg-closed set in (X, τ).

Proof: Let A and B be gg-closed sets in topological space (X, τ) and U be any regular semiopen set in topological space (X, τ) such that (A∪B)⊆U that is A⊆U and B⊆U. Since A and B are gg-closed sets we have gcl(A) ⊆ U and gcl(B)⊆U and we know that (gcl(A)∪gcl(B))=gcl(A∪B)⊆U. Therefore A∪B is gg-closed set in X.

Remark 3.21: The intersection of two gg-closed sets in topological space (X, τ) is not a gg-closed in general as seen from the example 3.22 given below.

Example 3.22: Let X = {p, q, r} and τ = {ϕ, X, {p}, {q}, {p, q}} be a topology on X. Then A = {p, q} and B = {p, r} are gg-closed sets in X but A∩B = {p} is not a gg-closed set in topological space (X, τ).

Remark 3.23: From the results discussed above and the known facts, the relation between gg-closed sets and some existing closed sets in topological spaces is established in figure 1.



Notations:
A ⟶ B ⟶ C means the set A implies the set B but not conversely and the set B implies set C but not conversely and hence the set A implies set C but not conversely.

A ⟷ B means the sets A and B are independent of each other.

Figure-1

Theorem 3.24: If a subset A of a topological space (X, τ) is gg-closed then gcl(A) - A does not contain any non-empty regular semiopen set in (X, τ).

Proof: Let A be a gg-closed set in X and suppose F be a non-empty regular semiopen subset of gcl(A) - A that is F ⊆ (gcl(A)-A) ⟹ F⊆(gcl(A)∩(X-A)) ⟹ F⊆gcl(A) →(1) and F⊆(X-A) ⟹ A⊆(X-F) and (X-F) is regular semi open set , gcl(A) ⊆ (X-F). This implies F ⊆ X - gcl(A) →(2). From (1) and (2) we get F⊆(gcl(A)∩(X-gcl(A))= φ. Thus gcl(A)-A does not contain any non-empty regular semiopen in X.

Remark 3.25: The converse part of theorem 3.24 is not true in general as seen from the example 3.26 given below.

Example 3.26: Let X = {p, q, r, s} and τ = {ϕ, X, {p}, {q}, {p, q}, {p, q, r}} be a topology on X. Then A={q, r}, gcl(A)-A={q, r, s}-{q, r}={s} does not contain non empty regular semiopen set in topological space (X, τ) but A is not gg-closed set in (X, τ).

Corollary 3.27: If a subset A of a topological space (X, τ) is gg-closed then gcl(A)-A does not contain any non-empty regular open set in (X, τ) but converse part is not true in general.

Proof: Follows from Theorem 3.24 and the fact that every regular open set is regular semiopen set.

Corollary 3.28: If a subset A of a topological space (X, τ) is gg-closed then gcl(A)-A does not contain any non-empty regular closed set in (X, τ) but converse part is not true in general.

Proof: Follows from Theorem 3.24 and the fact that every regular closed set is regular semiopen set.

Theorem 3.29: If A is gg-closed set in topological space (X, τ) and A ⊆ B ⊆ gcl(A) then B is gg-closed set in (X, τ).

Proof: Let A be gg-closed set in topological space (X, τ) such that B⊆gcl(A). Let U be a regular semiopen set of topological space (X, τ) such that B ⊆ U then A ⊆ U. Since A is gg-closed set, gcl(A) ⊆ U and A⊆U. Now B⊆gcl(A) ⟹ gcl(B) ⊆ gcl(gcl(A)) = gcl(A) ⊆ U. That is gcl(B) ⊆ U. Therefore B is a gg-closed set in X.

Remark 3.30: The converse part of theorem 3.29 is not true in general as seen from the example 3.31 given below.

Example 3.31: Let X = {p, q, r, s} and τ = {ϕ, X, {p}, {q}, {p, q}, {p, q, r}} be a topology on X. Then A={s}, B={r, s} such that A and B are gg-closed sets in X but A⊆B and B is not a subset of gcl(A) because gcl(A)={s}.

Theorem 3.32: Let (X, τ) be a topological space then for each x∈X the set X-{x} is gg-closed or regular semiopen in X.

Proof: Let x∈X. Suppose X-{x} is not a regular semiopen set. Then X is the only regular semiopen set containing X-{x}, that is X-{x} ⊆X ⟹ gcl(X-{x}) ⊆gcl(X) ⟹ gcl(X-{x})⊆X. Therefore X-{x} is gg-closed set in (X, τ).

Theorem 3.33: Let A be gg-closed set in topological space (X, τ). Then A is g-closed iff gcl(A)-A is regular semiopen set in (X, τ).

Proof: Necessity: Suppose A is g-closed set in topological space (X, τ). Then gcl(A)=A that is gcl(A)-A=ϕ which is regular semiopen set in X.

Sufficiency: Suppose A is gg-closed set in topological space (X, τ) and gcl(A)-A is regular semiopen, By the Theorem 3.24 gcl(A)-A=ϕ ⟹ gcl(A)=A. Therefore A is g-closed.

Theorem 3.34: Let (X, τ) and (Y,σ) are two topological spaces and A⊆Y⊆X. If A is gg-closed set in (X, τ) then A is gg-closed set relative to (Y,σ).

Proof: Let A⊆Y∩G where G is regular semiopen. Since A is gg-closed set in X, then A⊆G and gcl(A)⊆G. This implies that Y ∩gcl(A) ⊆Y∩G where Y ∩gcl(A) is closed set of A in Y. Hence A is gg-closed set relative to a topological space (Y, σ).

Theorem 3.35: Let (X, τ) be a topological space and if RSO(X) = {X, ϕ} then for each and every subset of X is a gg-closed set.

Proof: Let A be any subset of a topological space (X, τ) and the set of all regular semi open sets in X is RSO(X) = {X, ϕ}.Suppose A=ϕ.Then ϕ is gg-closed set. Suppose A≠ ϕ. Then X is one and only regular semiopen set containing A and therefore gcl(A)⊆X. Hence A is gg-closed set in X.

Remark 3.36: The converse part of theorem 3.35 is not true in general as seen from the example 3.37 given below.

Example 3.37: Let X = {p, q, r} and τ = {ϕ, X, {p}, {q, r}} be a topology on X. Then every subset of (X,τ) is a gg-closed set in X but RSO = {ϕ, X, {p}, {q, r}}.

Theorem 3.38: If A is semiopen and swg*-closed set in a topological space (X, τ) then A is gg-closed set in (X, τ).

Proof: Let A be semiopen and swg*-closed of a topological space (X, τ). Let U be any regular semiopen set in (X, τ) such that A⊆U. By the definition, we have gcl(A)⊆A then gcl(A)⊆A⊆U. Hence A is gg-closed set in (X, τ).

Remark 3.39: If A is both semiopen and gg-closed set of (X, τ) then A is not a swg*-closed in general as seen from the example 3.40 given below.

Example 3.40: Let X = {p, q, r} and τ = {ϕ, X, {p}, {q}, {p, q}} be a topology on X. Then A = {p, q} is semiopen and gg-closed but not a swg*-closed set in X.

Theorem 3.41: If A is β-open and βwg*-closed of a topological space X then A is gg-closed set in X.

Proof: Let A be β-open and βwg*-closed of a topological space X. Let U be any regular semiopen set in X such that A⊆U. By the definition, we have gcl(A)⊆A then gcl(A)⊆A⊆U. Hence A is gg-closed set in X.

Remark 3.42: If A is β-open and gg-closed of a topological space X then A is not a βwg*-closed in general as seen from the example 3.43 given below.

Example 3.43: Let X = {p, q, r} and τ = {ϕ, X, {p}, {q}, {p, q}} be a topology on X. Then A = {p, q} is both β-open and gg-closed but not a βwg*-closed in X.

Theorem 3.44: Let (X, τ) be a topological space and if A is open and g-closed then A is gg-closed set in (X, τ)**.**

Proof: Let A be an open and g-closed set in (X, τ). Let U be any regular semi open set in (X, τ) such that A⊆U. By definition, we have cl(A)⊆A⊆U and gcl(A)=A. This implies that cl(A)⊆gcl(A) ⊆A⊆U. That is gcl(A)⊆U. Hence A is gg-closed set in (X, τ).

Remark 3.45: If A is both open and gg-closed of a topological space (X, τ) then A is not a g-closed in general as seen from the example 3.46 given below.

Example 3.46: Let X = {p, q, r} and τ = {ϕ, X, {p}, {q}, {p, q}} be a topology on X. Then A = {p, q} is both open and gg-closed but not a g-closed in X.

Theorem 3.47: If A is an open and gr-closed in a topological space (X, τ) then A is gg-closed set in (X, τ).

Proof: Let A be an open and gr-closed in a topological space (X, τ). Let U be any regular semiopen set in (X, τ) such that A⊆U. Now A⊆A. By definition, we have rcl(A)⊆A and the fact that rcl(A)⊆gcl(A) ⊆A. Hence gcl(A)⊆U. Therefore A is gg-closed set in X.

Remark 3.48: If A is an open and gg-closed in a topological space (X, τ) then it is not a gr-closed as seen in the example 3.49 given below.

Example 3.49: Let X = {p, q, r} and τ = {ϕ, X, {p}, {q}, {p, q}, {p, r}} be a topology on X. Then A= {p, q} is both open and gg-closed but not a gr-closed in X.

Theorem 3.50: If A is regular semiopen and rw-closed in a topological space (X, τ) then A is gg-closed set in (X, τ).

Proof: Let A be regular semiopen and rw-closed set in (X, τ). Let U be any regular semiopen set in (X, τ) such that A⊆U. Now A⊆A. By definition, we have cl(A)⊆A and fact that cl(A) ⊆gcl(A)⊆A. Hence gcl(A)⊆U. Therefore A is gg-closed set in X.

Remark 3.51: If A is both regular semi open and gg-closed set in (X, τ) then A is not a rw-closed in general as seen from the example 3.52 given below.

Example 3.52: Let X = {p, q, r, s} and τ = {ϕ, X, {p}, {q}, {p, q}, {p, q, r}} be a topology on X. Then A= {p, s} is both regular semi open and gg-closed but not a rw-closed in (X, τ).

Theorem 3.53: If A is semiopen and w-closed in a topological space (X, τ) then A is gg-closed set in (X, τ).

Proof: Let A be semiopen and w-closed set in (X, τ). Let U be any regular semiopen set in (X, τ) such that A⊆U. Now A⊆A. By definition, we have cl(A)⊆A and fact that cl(A)⊆gcl(A) ⊆A. Hence gcl(A)⊆U and therefore A is gg-closed set in X.

Remark 3.54: If A is both semi open and gg-closed set in a topological space (X, τ) then A is not a w-closed set in general as seen from the example 3.58 given below.

Example 3.55: Let X = {p, q, r} and τ = {ϕ, X, {p}, {q}, {p, q}} be a topology on X. Then A= {p, q} is both semi open and gg-closed but not a w-closed in (X, τ).

Theorem 3.56: In a topological space (X, τ), if A is regular open and gg-closed then A is regular closed and g-clopen.

Proof: Suppose A is regular open and gg-closed set in X. Since every regular open set of a topological space (X, τ) is regular semiopen and A⊆A, we have gcl(A)⊆A and A⊆gcl(A). Therefore gcl(A)=A that is A is g-closed. Since A is regular open then A is open that implies g-open. Now cl(int(A))=cl(A)=A. Hence the proof.

Theorem 3.57: If A is both regular open and gg-closed set of a topological space (X, τ) then it is g-closed.

Proof: let A be regular open and gg-closed of a topological space (X, τ), Since every regular open set in a topological space (X, τ) is regular semiopen and A⊆A, then by the definition of gg-closed we have gcl(A) ⊆A and also A⊆gcl(A). Therefore gcl(A)=A. Hence A is g-closed.

Corollary 3.58: In a topological space (X, τ), if A is regular open, gg-closed and U is g-closed then (A∩U) is gg-closed set in (X, τ).

Proof: Let A be regular open and gg-closed set in (X, τ) and U be g-closed in (X, τ). By Theorem 3.56, A is g-closed and hence the proof.

4. REFERENCES

1. D.Andrijevic, Semi-preopen sets, Mat. Vesnik, 38(1) (1986), 24-32.
2. S.P.Arya and T.Nour, Characterizations of s-normal spaces, Indian J. Pure. Appl. Math, 12 (8) (1990), 717-719.
3. S.S. Benchalli and R.S. Wali, On RW-closed sets in topological spaces, Malays. Math. sci. Soc. 30 (2007), 99-110.
4. S. S. Benchalli, P. G. Patil and T. D. Rayanagaudar, ωα-Closed sets is Topological Spaces, The global j. Appl. Math. Sci. 2, 53-63.
5. P.Bhattacharyya and B.K.Lahiri, Semi-generalized closed sets in topology, Indian J. Math., 29 (1987)376-382.
6. D.E.Cameron**,** Properties of s-closed spaces Proc. Amer. Math. Soc., 72 (1978), 581-586.
7. C. Deva manoharan, S.Pious Missier and S. Jafari, ρ-Closed Sets in Topological Spaces. European journal of pure and applied mathematics, Vol. 5, No. 4, 2012, 554-566.
8. C. Dhanapakyam, J. Subashini and K. Indirani, ON - βwg* set and continuity in topological spaces, IJMA-5(5), 2014, 222-233
9. J.Dontchev and H.Maki, On θ-generalized closed sets,International J. Math and Math. Sci. Vol 22, No 2, 239-249
10. J.Dontchev and M.Ganster, On δ-generalized set $T_{3/4}$ spaces, Mem. Fac. Sci. Kochi Uni. Ser. A. Math., 17(1996), 15-31.
11. J.Dontchev and T.Nori, Quasi-normal spaces and πg-closed sets, Acta Math. Hungar 89(3) (2000), 211-219.
12. J.Dontchev, On generalized semi-preopen sets, Mem. Fac. Sci. Kochi Univ. Ser.A. Math., 16(1995), 35-48.
13. G.L. Garg and D. Sivaraj, On Sc-compact and S-Closed Spaces, Bolletino U.M.I (6) 3-B (1984), 321-332.
14. Najiac Falah Hameed, Certain types of generalized closed sets, Journal of Al-Nahrain University Vol.12 (3), September, 2009, pp.1 -150 Science 144.
15. A. Jayalakshmi & C. Janaki, On wgrα-Closed Sets in Topological Spaces, International Journal of Mathematical Archive-3(6), 2012, 2386-2392.
16. N.Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(2) (1970), 89-96.
17. N.Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.
18. H.Maki, R.Devi and K.Balachandran, Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 15 (1994), 51-63.
19. H.Maki, R.Devi and K.Balachandran, Generalized α-closed sets in topology, Bull. Fukuoka Univ. Ed. Part III, 42(1993), 13- 21.
20. H.Maki, J.Umehara and T.Noiri, Every topological space is pre-$T_{1/2}$, Mem. Fac. Sci. Kochi Univ.Ser. A. Math. 17(1966) 33-42.
21. C. Mukundhan and N. Nagaveni, A Weaker Form of a Generalized Closed Set, Int. J. Contemp. Math. Sciences, Vol. 6, 2011, no. 20, 949 – 961
22. N.Nagaveni, Studies on generalizations of homeomorphisms in topological spaces, Ph.D., Thesis, Bharathiar University, Coimbatore (1999).
23. A. Narmadha, N. Nagaveni and T. Noiri, On Regular b-Open Sets in Topological Spaces Int. Journal of Math. Analysis, Vol. 7, 2013, no. 19, 937 – 948
24. O.Njåstad, On some classes of nearly open sets, Pacific J. Math., 15(1965), 961-970.
25. Sanjay Mishra, Nitin Bhardwaj and Varun Joshi, On Regular Generalized weakly (rgw)-Closed Sets in Topological Spaces, Int. Journal of Math. Analysis, Vol. 6, 2012, no. 39, 1939 – 1952.

26. Sanjay Mishra, Nitin Bhardwaj and Varun Joshi, On Generalized Pre Regular Weakly (gprw)-Closed Sets in Topological Spaces, International Mathematical Forum, Vol. 7, 2012, no. 40, 1981 – 1992.
27. Savithiri D and Janaki C, .On Regular ^Generalized closed sets in topological spaces, International Journal of Mathematical Archive-4(4), 2013, 162-169.
28. Sharmistha Bhattacharya, On Generalized Regular Closed Sets, Int. J. Contemp. Math. Sciences, Vol. 6, 2011, no. 3, 145 – 152.
29. M.Sheik John, On ω-closed sets in Topology, Acta Ciencia Indica, 4 (2000), 389-392.
30. T. Shyla Isac Mary and P. Thangavelu, On Regular Pre-Semiclosed sets in topological spaces, KBM J. Of Math. Sci and Comp. Applications (1), 9-17.
31. M.Stone, Application of the theory of Boolean rings to general topology, Trans. Amer. Math Soci, 41 (1937), 374-481.
32. Subashini jesu rajan, On βwg** set and Continuity in Topological Spaces, International journal of computing, Vol 4 Issue 3, July 2014.
33. A. Vadivel and K. Vairamanickam, rgα-Closed Sets and rgα-Open Sets in Topological Spaces, Int. Journal of Math. Analysis, Vol. 3, 2009, no. 37, 1803 – 1819.
34. M.K.R.S. Veera Kumar, Between closed sets and g-closed sets, Mem. Fac. Sci. Kochi Univ., (Math.) 21(2000), 1-19.
35. M.K.R.S. Veera kumar, g#-closed sets in topological spaces, Mem. Fac. Sci. Kochi Univ. Ser. A. Math 24(2003), 1-13.
36. M. K. R. S. Veerakumar, g*-pre closed sets Acta Ciencia Indica (Mathematics) Meerut.
37. N.V.Velicko, H-closed topological spaces. Amer. Math. Soc. Transl. 78 (1968), 103-118.
38. M. Vigneshwaran and R. Devi, On GαO-kernel in the digital plane, International Journal of Mathematical Archive-3(6), 2012, 2358-2373.
39. M. Vigneshwaran and A. Singaravelan, Applications of **gα-closed sets in Topological Spaces, IJMA-5(10), 2014, 139-150.
40. R. S. Wali and Prabhavati S. Mandalgeri, on α regular ω-closed sets in topological spaces.' Int. Journal of Mathematical Archive-5(10), 2014, 68-76.

Source of support: Nil, Conflict of interest: None Declared.