International Journal of Mathematical Archive-8(8), 2017, 60-67
Available online through www.ijma.info ISSN 2229 – 5046

FUZZY EOQ MODEL WITH SHORTAGES FOR PRODUCTS WITH CONTROLLABLE DETERIORATION RATE AND TIME DEPENDENT DEMAND AND INVENTORY HOLDING COST

R. PALANI*[1], M. MARAGATHAM[2]

[1]Research Scholar, PG and Research Department of Mathematics,
Periyar E.V.R. College, Tiruchirappalli - 620023, Tamil Nadu, India.

[2]Associate Professor, PG and Research Department of Mathematics,
Periyar E.V.R. College, Tiruchirappalli - 620023, Tamil Nadu, India.

(Received On: 15-07-17; Revised & Accepted On: 07-08-17)

ABSTRACT

In this paper, we have developed a deterministic inventory model for deteriorating items in which demand rate and holding cost are quadratic and linear function of time. During deterioration period, deterioration rate can be controlled using preservation technology (PT). The ordering cost, deterioration cost, shortage cost and purchase cost are assumed as triangular fuzzy number. The purpose of our study is to find an optimal replenishment cycle and order quantity so that the total inventory cost per unit time is minimum. In the model considered here, deterioration rate is constant, backlogging rate is variable and depends on the length of the next replenishment. Shortages are allowed and backlogged. An analytic solution which optimizes the total cost is derived. The derived model is illustrated with a numerical example.

Keywords: *Inventory, deteriorating items, triangular fuzzy number, preservation technology, exponential distribution, quadratic demand, shortages, backlogged, time varying holding cost.*

1. INTRODUCTION

Inventory may be considered as an accumulation of a product that would be used to satisfy future demands for that product. An optimal replenishment policy is dependent on ordering cost, inventory carrying cost and shortage cost. An important problem confronting a supply manager in any modern organization is the control and maintenance of inventories of deteriorating items. Fortunately, the rate of deterioration is too small for items like steel, toys, glassware, hardware, etc. There is little requirement for considering deterioration in the determination of economic lot size. So in this paper, an inventory model is developed for deteriorating items by considering the fact that using the preservation technology the retailer can reduce the deterioration rate by which he can reduce the economic losses, improve the customer service level and increase business competitiveness.

In reality, the demand and holding cost for physical goods may be time dependent. Time also plays and important role in the inventory system. So, in this paper we consider that demand and holding cost are time dependent.

Recently, Mishra and Singh [11] developed a deteriorating inventory model with partial backlogging when demand and deterioration rate is constant. Vinod kumar Mishra [17] developed an inventory model of instantaneous deteriorating items with controllable deterioration rate for time dependent demand and holding cost. Vinod kumar Mishra [18] developed deteriorating inventory model with controllable deterioration rate for time-dependent demand and time-varying holding cost. Parmar Kirtan and U. B. Gothi [13] developed EOQ model with constant deterioration rate and time dependent demand and IHC.

Leea and Dye [9] formulated a deteriorating inventory model with stock-dependent demand by allowing preservation technology cost as a decision variable in conjunction with replacement policy. Dye and Hsieh [2] presented an extended model of Hsu et al. [4] by assuming that the preservation technology cost is a function of the length of replenishment cycle.

Corresponding Author: R. Palani[1],*
[1]Research Scholar, PG and Research Department of Mathematics,
Periyar E.V.R. College, Tiruchirappalli - 620023, Tamil Nadu, India.

J. Jagadeeswari and P. K. Chenniappan [5] developed an order level inventory model for deteriorating items with time-quadratic demand and partial backlogging. Sarala Pareek and Garima Sharma [14] developed an inventory model with Weibull distribution deteriorating item with exponential declining demand and partial backlogging. R. Amutha and Dr. E. Chandrasekaran [1] developed an inventory model for deteriorating Products with Weibull Distribution Deterioration, Time-Varying Demand and Partial Backlogging. Kirtan Parmar and U. B. Gothi [7] developed a deterministic inventory model for deteriorating items where time to deterioration has Exponential distribution and with time-dependent quadratic demand. Also, U. B. Gothi and Kirtan Parmar [3] have extended above deterministic inventory model by taking two parameter Weibull distributions to represent the distribution of time to deterioration and shortages are allowed and partially backlogged. Kirtan Parmar and U. B. Gothi [8] developed an economic production model for deteriorating items using three parameter Weibull distributions with constant production rate and time varying holding cost.

The consideration of PT is important due to rapid social changes, and the fact that PT can reduce the deterioration rate significantly. By the efforts of investing in preservation technology, we can reduce the deterioration rate. So in this paper, we made the model of Mishra and Singh [11] more realistic by considering the fact that use of preservation technology can reduce the deterioration rate significantly, which help the retailers to reduce their economic losses.

In many inventory models uncertainty is due to fuzziness and fuzziness is the closed possible approach to reality. In recent years some researchers gave their attention towards a time dependent rate because the demand of newly launched products such as fashionable garments, electronic items and mobiles etc. increases with time and later it becomes constant. Deterioration is defined as damage, decay or spoilage of the items that are stored for future use always loose part of their value with passage of time, so deterioration cannot be avoided in any business scenarios.

Syed [15] developed a fuzzy inventory model without shortages using Signed distance method. They used fuzzy triangular number for both ordering cost and holding cost. Maragatham. M [10], discussed the A fuzzy model with changing deterioration rate. Jhuma Bhowmick [6] discussed the fuzzy inventory model for deteriorating items with time-varying demand and shortages. Umap [16] formed a fuzzy EOQ model for deteriorating items with two warehouses.

Palani. R and Maragatham. M [12], developed EOQ model for controllable deterioration rate and time dependent demand and Inventory holding cost. In this paper, the same inventory model used in fuzzy environment in real life for uncertainty. The triangular fuzzy numbers are used in this paper. This model is solved by analytically to determine the optimal cycle time and the derived model is illustrated by a numerical example. This paper, some of the parameters are considered as triangular fuzzy numbers for fuzzy cases. For defuzzification of the total cost function and optimum order quantity graded mean representation method is used.

2. PRELIMINARIES

2.1 Basic Definitions

2.1.1 Fuzzy Set
If X is a collection of objects denoted generically by x, then a fuzzy set $\tilde{A}$ in X is defined as a set of ordered pairs $\tilde{A} = \{(x; \mu_{\tilde{A}}(x))/x \epsilon X\}$, where $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set $\tilde{A}$. The membership function maps each element of X to a membership grade between o and 1 (included).

2.1.2 α - Cut
The set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree of α is called the α level set or α - cut $(i.e)$, $\tilde{A}^{(\alpha)} = \{x \epsilon X: \mu_{\tilde{A}}(x) \geq \alpha\}$.

2.2 Fuzzy Number
Fuzzy numbers are of great important in fuzzy systems.

2.2.1 Fuzzy Number
A fuzzy subset $\tilde{A}$ of the real line R with membership function $\mu_{\tilde{A}}: R \to [0, 1]$ is called a fuzzy number if

i. $\tilde{A}$ is normal, $(i.e)$, there exit an element x_0 such that $\mu_{\tilde{A}}(x_0) = 1$.
ii. $\tilde{A}$ is fuzzy convex,
   $(i.e)$, $\mu_{\tilde{A}}[\lambda x_1 + (1-\lambda)x_2] \geq \mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2)\ x_1, x_2 \epsilon R, \forall \lambda \epsilon\ [0, 1]$.
iii. $\mu_{\tilde{A}}$ is upper continuous.
iv. $supp\ \tilde{A}$ is bounded,
   where $supp\ \tilde{A} = \{x \epsilon R: \mu_{\tilde{A}}(x) > 0\}$.

2.2.2 Generalized Fuzzy Number
Any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions, is a generalized fuzzy number

i. $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval $[0, 1]$.
ii. $\mu_{\tilde{A}}(x) = 0, \quad -\infty < x \le a_1$.
iii. $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$.
iv. $\mu_{\tilde{A}}(x) = 1, a_2 \le x \le a_3$.
v. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$.
vi. $\mu_{\tilde{A}}(x) = 0, a_4 \le x \le \infty$, where a_1, a_2, a_3, a_4 are real numbers.

2.2.3 Triangular Fuzzy Number
The fuzzy set $\tilde{A} = (a_1, a_2, a_3)$, where $a_1 \le a_2 \le a_3$ and defined on R, is called triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}$$

2.2.4 Operations of Triangular Fuzzy Number
Consider two triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$, $\tilde{B} = (b_1, b_2, b_3)$.

i. The addition of $\tilde{A}$ and $\tilde{B}$ is
$$\tilde{A} + \tilde{B} = (a_1, a_2, a_3) + (b_1, b_2, b_3)$$
$$= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$
where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

ii. The multiplication of $\tilde{A}$ and $\tilde{B}$ is
$$\tilde{A} \times \tilde{B} = (c_1, c_2, c_3),$$
where $T = \{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$, $c_1 = \min T$, $c_2 = a_2b_2$, $c_3 = \max T$
If $a_1, a_2, a_3, b_1, b_2, b_3$ are all non zero positive real numbers, then $\tilde{A} \times \tilde{B} = (a_1b_1, a_2b_2, a_3b_3)$

iii. $[-\tilde{B}] = -(b_1, b_2, b_3) = (-b_3, -b_2, -b_1)$ then the subtraction of $\tilde{B}$ from $\tilde{A}$ is
$$\tilde{A} + \tilde{B} = (a_1, a_2, a_3) - (b_1, b_2, b_3)$$
$$= (a_1 - b_3, a_2 - b_2, a_3 - b_1)$$
where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

iv. $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$, where b_1, b_2, b_3 are all non zero positive real numbers, then division of $\tilde{A}$ and $\tilde{B}$ is $\frac{\tilde{A}}{\tilde{B}} = \frac{(a_1, a_2, a_3)}{(b_1, b_2, b_3)} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$

v. For any real number
$$k, k\tilde{A} = \begin{cases} (ka_1, ka_2, ka_3), & if\ k > 0 \\ (ka_3, ka_2, ka_1), & if\ k < 0 \end{cases}$$

2.3 Defuzzification
Defuzzification is the conversion of a fuzzy quantity to a crisp quantity. Defuzzification methods obtain the representative value of a fuzzy set.

2.3.1 Graded Mean Representation Method
Let $\tilde{A}$ be a fuzzy number with left reference function L and right reference function R. Let L^{-1} and R^{-1} be the inverse functions of L and R respectively.

The graded mean integration representation of $(\tilde{A})$ is defined by

$$\rho(\tilde{A}) = \frac{\frac{1}{2}\int_0^1 h[L^{-1}(h) + R^{-1}(h)]dh}{\int_0^1 h\, dh} \text{ with } 0 < h < 1$$

By the above formula, the graded mean representations of triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is given by $(\tilde{A}) = \frac{a_1 + 4a_2 + a_3}{6}$.

3. ASSUMPTIONS AND NOTATIONS

The mathematical model is based on the following notations and assumptions.

© 2017, IJMA. All Rights Reserved

Notations

$R(t)$: Quadratic demand rate.
$\tilde{A}$: Fuzzy ordering cost per order.
$\widetilde{C_d}$: Fuzzy deterioration cost per unit per unit time.
C_h : Inventory holding cost per unit per unit of time.
$m(\xi)$: Reduced deterioration rate due to use of preservation technology.
θ : Deterioration rate.
τ_p : Resultant deterioration rate, $\tau_p = (\theta - m(\xi))$.
$\widetilde{P_c}$: Fuzzy purchase cost per unit.
Q : Order quantity in one cycle.
t_d : The time from which the deterioration start in the inventory.
t_1 : Length of cycle time (decision variable).
TC : Total cost per unit time.
$\widetilde{TC}_{dG}(t_1)$: The defuzzified value of $\widetilde{TC}(t_1)$ by Graded mean representation method.
$Q(t)$: The instantaneous state of the positive inventory level at time t.
$\tilde{Q}_{dG}$: The defuzzified value of $\tilde{Q}$ by Graded mean representation method.

Assumptions

The model is derived under the following assumptions.

1. The inventory system deals with single item.
2. The annual demand rate is a function of time and it is $R(t) = a + bt + ct^2$ $(a, b, c > 0)$.
3. Preservation technology is used for controlling the deterioration rate.
4. Holding cost is linear function of time and it is $C_h = h + rt$ $(h, r > 0)$.
5. The lead time is zero.
6. Time horizon is finite.
7. The deterioration rate is constant.
8. No repair or replacement of the deteriorated items takes place during a given cycle.
9. Total inventory cost is a real, continuous function which is convex to the origin.

During stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment.

4. MATHEMATICAL MODEL AND ANALYSIS

Here, the replenishment policy of a deteriorating item with backlogging is considered. The objective of the inventory problem is to determine the optimal order quantity and the length of ordering cycle so as to keep the total relevant cost as low as possible, where the preservation technology is used to control the deterioration rate. The behavior of inventory system at any time is shown in Figure 1.

Replenishment is made at time t = 0 and the inventory level is at its maximum level S. During the period [0, t_d] the inventory level is decreasing and at time t_1 the inventory reaches zero level.

The pictorial representation is shown in the Figure 1.

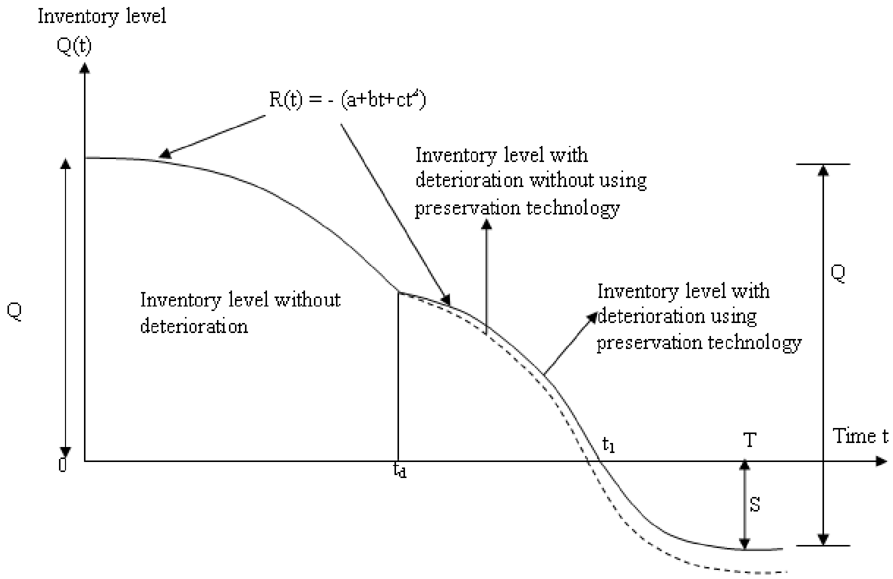

Figure-1: Graphical representation of the inventory system

As described above, the inventory level decreases owing to demand rate as well as deterioration during [0, t_1]. Hence, the differential equation representing the inventory status is given by

$$\frac{dQ(t)}{dt} = -(a+bt+ct^2) \qquad (0 \le t \le t_d) \tag{1}$$

$$\frac{dQ(t)}{dt} + \tau_p Q(t) = -(a+bt+ct^2) \qquad (t_d \le t \le t_1) \tag{2}$$

During the shortage interval [t_1, T] the demand at time t is backlogged at the fraction. Thus, the differential equation governing the amount of demand backlogged is as below.

$$\frac{dQ(t)}{dt} = -(a+bt+ct^2) \qquad (t_1 \le t \le T) \tag{3}$$

The boundary conditions are $Q(0) = Q$ and $Q(t_1) = 0$ (4)

Using the boundary condition $Q(0) = Q$ the solution of the equation (1) is

$$\Rightarrow Q(t) = Q - \left(at + \frac{bt^2}{2} + \frac{ct^3}{3}\right) \qquad (0 \le t \le t_d) \tag{5}$$

Similarly, the solution of equation (2) is given by

$e^{(\theta - m(\xi))t}Q(t) = -\int (a+bt+ct^2)e^{(\theta-m(\xi))t}dt$

$\Rightarrow e^{(\theta-m(\xi))t}Q(t) = \left\{k - \left[at + (a\theta+b)\frac{t^2}{2} + (b\theta+c)\frac{t^3}{3} + c\theta\frac{t^4}{4} - m(\xi)\left\{a\frac{t^2}{2} + b\frac{t^3}{3} + c\frac{t^4}{4}\right\}\right]\right\}$ (Neglecting higher powers of θ) (where $k = at_1 + (a\theta+b)\frac{t_1^2}{2} + (b\theta+c)\frac{t_1^3}{3} + c\theta\frac{t_1^4}{4} - m(\xi)\left\{a\frac{t_1^2}{2} + b\frac{t_1^3}{3} + c\frac{t_1^4}{4}\right\}$ which is obtained using $Q(t_1) = 0$

$$Q(t) = k - k(\theta - m(\xi))t + a(\theta - m(\xi))\frac{t^2}{2} + b(\theta - m(\xi))\frac{t^3}{6} + c(\theta - m(\xi))\frac{t^4}{12} + c(\theta - m(\xi))\frac{t^5}{4} - at - b\frac{t^2}{2} - c\frac{t^3}{3} - m(\xi)\left\{a(2\theta - m(\xi))\frac{t^3}{2} + b(2\theta - m(\xi))\frac{t^4}{3} + c(2\theta - m(\xi))\frac{t^5}{4}\right\} (t_d = t = t_1) \tag{6}$$

In equations (5) and (6) values of $Q(t)$ and $Q(t)$ should coincide at $t = t_d$, which implies that

$$Q - \left(at_d + \frac{bt_d^2}{2} + \frac{ct_d^3}{3}\right) = \Bigg[k - k(\theta - m(\xi))t_d + a(\theta - m(\xi))\frac{t_d^2}{2} + b(\theta - m(\xi))\frac{t_d^3}{6} + c(\theta - m(\xi))\frac{t_d^4}{12} + c(\theta - m(\xi))\frac{t_d^5}{4} - at_d - b\frac{t_d^2}{2} - c\frac{t_d^3}{3} - m(\xi)\left\{a(2\theta - m(\xi))\frac{t_d^3}{2} + b(2\theta - m(\xi))\frac{t_d^4}{3} + c(2\theta - m(\xi))\frac{t_d^5}{4}\right\}\Bigg]$$

$$Q = IM = \Bigg[k - k(\theta - m(\xi))t_d + a(\theta - m(\xi))\frac{t_d^2}{2} + b(\theta - m(\xi))\frac{t_d^3}{6} + c(\theta - m(\xi))\frac{t_d^4}{12} + c(\theta - m(\xi))\frac{t_d^5}{4} - m(\xi)\left\{a(2\theta - m(\xi))\frac{t_d^3}{2} + b(2\theta - m(\xi))\frac{t_d^4}{3} + c(2\theta - m(\xi))\frac{t_d^5}{4}\right\}\Bigg] \tag{7}$$

Solution of equation (3) is given by

$$Q(t) = -\left(at + \frac{bt^2}{2} + \frac{ct^3}{3}\right) + k_1 \tag{8}$$

With boundary condition $Q(t_1) = 0$, we get

$$k_1 = \left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3}\right) \tag{9}$$

Therefore, from (8) and (9)

$$\Rightarrow Q(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{c}{3}(t_1^3 - t^3) \quad (t_1 = t = T) \tag{10}$$

Fuzzy model

In real life situation ordering cost, deterioration cost and purchase cost for uncertainty consider in fuzzy environment system. The total cost followed in proposed inventory model [12].

Here some of the parameters are fuzzy numbers namely $\tilde{A}$, $\widetilde{C_d}$, $\widetilde{C_s}$ and $\widetilde{P_c}$.

Let $\tilde{A} = (A_1, A_2, A_3)$, $\widetilde{C_d} = (C_{d_1}, C_{d_2}, C_{d_3})$, $\widetilde{C_d} = (C_{s_1}, C_{s_2}, C_{s_3})$ and $\widetilde{P_c} = (P_{c_1}, P_{c_2}, P_{c_3})$

© 2017, IJMA. All Rights Reserved

The total cost comprises of following costs

(1) The fuzzy ordering cost $OC = \tilde{A}$ (11)

(2) The fuzzy deterioration cost during the period [t_d, t_1]

$$\widetilde{DC} = \widetilde{C_d}\left\{Q - \left[a(t_1 - t_d) + \frac{b}{2}(t_1^2 - t_d^2) + \frac{c}{3}(t_1^3 - t_d^3)\right]\right\} \tag{12}$$

(3) The inventory holding cost during the period [0, t_1]

$$IHC = \int_0^{t_d}(h+rt)\,Q(t)dt + \int_{t_d}^{t_1}(h+rt)Q(t)dt$$

$$= \int_0^{t_d}\left\{(h+rt)\left[Q - \left(at + \frac{bt^2}{2} + \frac{ct^3}{3}\right)\right]\right\}dt$$

$$+ \int_{t_d}^{t_1}\left\{(h+rt)\left[\begin{array}{c} k - k\big(\theta - m(\xi)\big)t + a\big(\theta - m(\xi)\big)\frac{t^2}{2} \\ +b\big(\theta - m(\xi)\big)\frac{t^3}{6} + c\big(\theta - m(\xi)\big)\frac{t^4}{12} + c\big(\theta - m(\xi)\big)\frac{t^5}{4} - at - b\frac{t^2}{2} \\ -c\frac{t^3}{3} - m(\xi)\left\{a\big(2\theta - m(\xi)\big)\frac{t^3}{2} + b\big(2\theta - m(\xi)\big)\frac{t^4}{3} + c\big(2\theta - m(\xi)\big)\frac{t^5}{4}\right\}\end{array}\right]\right\}dt$$

$$\Rightarrow IHC = \left\{\begin{array}{c} h\left[Qt_d - \left(\frac{at_d^2}{2} + \frac{bt_d^3}{6} + \frac{ct_d^4}{12}\right)\right] + r\left[\frac{Qt_d^2}{2} - \left(\frac{at_d^3}{3} + \frac{bt_d^4}{8} + \frac{ct_d^5}{15}\right)\right] \\ +hk(t_1 - t_d) + \left[rk - hk\big(\theta - m(\xi)\big)\right]\left[\frac{t_1^2 - t_d^2}{2}\right] + ha\big(\theta - m(\xi)\big)\left[\frac{t_1^3 - t_d^3}{6}\right] + hb\big(\theta - m(\xi)\big)\left[\frac{t_1^4 - t_d^4}{24}\right] \\ +hc\big(\theta - m(\xi)\big)\left[\frac{t_1^5 - t_d^5}{60}\right] + hc\big(\theta - m(\xi)\big)\left[\frac{t_1^6 - t_d^6}{24}\right] - m(\xi)\left\{\begin{array}{l} ah\big(2\theta - m(\xi)\big)\left[\frac{t_1^4 - t_d^4}{8}\right] \\ +bh\big(2\theta - m(\xi)\big)\left[\frac{t_1^5 - t_d^5}{15}\right] \\ +ch\big(2\theta - m(\xi)\big)\left[\frac{t_1^6 - t_d^6}{24}\right]\end{array}\right\} \\ +ra\big(\theta - m(\xi)\big)\left[\frac{t_1^4 - t_d^4}{8}\right] + rb\big(\theta - m(\xi)\big)\left[\frac{t_1^5 - t_d^5}{30}\right] \\ +rc\big(\theta - m(\xi)\big)\left[\frac{t_1^6 - t_d^6}{72}\right] + rc\big(\theta - m(\xi)\big)\left[\frac{t_1^7 - t_d^7}{28}\right] - m(\xi)\left\{\begin{array}{l} ra\big(2\theta - m(\xi)\big)\left[\frac{t_1^5 - t_d^5}{10}\right] \\ +rb\big(2\theta - m(\xi)\big)\left[\frac{t_1^6 - t_d^6}{18}\right] \\ +rc\big(2\theta - m(\xi)\big)\left[\frac{t_1^7 - t_d^7}{28}\right]\end{array}\right\} \\ -\left\{ha\left(\frac{t_1^2 - t_d^2}{2}\right) + hb\left(\frac{t_1^3 - t_d^3}{6}\right) + hc\left(\frac{t_1^4 - t_d^4}{12}\right) + ra\left(\frac{t_1^3 - t_d^3}{3}\right) + rb\left(\frac{t_1^4 - t_d^4}{8}\right) + rc\left(\frac{t_1^5 - t_d^5}{15}\right)\right\}\end{array}\right\} \tag{13}$$

Thus, the order size during total interval [0, t_1] is given by

$$Q = IM$$

(4) The shortage cost per cycle

$$\widetilde{SC} = -\widetilde{C_S}\int_{t_1}^{T} Q(t)dt$$

$$\Rightarrow \widetilde{SC} = \widetilde{C_S}\left\{\frac{a}{2}\left[(T^2 - t_1^2) - 2t_1(T - t_1)\right] + \frac{b}{6}\left[2(T^3 - t_1^3) - 3t_1^2(T - t_1)\right] + \frac{c}{12}\left[3(T^4 - t_1^4) - 4t_1^3(T - t_1)\right]\right\} \tag{14}$$

The maximum backordered inventory is obtained at $t = T$ and it is denoted by S. Then from equation (10),

$$S = -Q(T)$$

$$\Rightarrow S = a(T - t_1) + \frac{b}{2}(T^2 - t_1^2) + \frac{c}{3}(T^3 - t_1^3) \tag{15}$$

Thus, the order size during total interval [0, T] is given by

$$Q = IM + S$$

(5) Purchase cost per cycle

$$\widetilde{PC} = \widetilde{P_c}Q$$

$$\widetilde{PC} = \widetilde{P_c}\left\{\left[\begin{array}{c} k - k\big(\theta - m(\xi)\big)t_d + a\big(\theta - m(\xi)\big)\frac{t_d^2}{2} + b\big(\theta - m(\xi)\big)\frac{t_d^3}{6} + c\big(\theta - m(\xi)\big)\frac{t_d^4}{12} \\ +c\big(\theta - m(\xi)\big)\frac{t_d^5}{4} - m(\xi)\left\{a\big(2\theta - m(\xi)\big)\frac{t_d^3}{2} + b\big(2\theta - m(\xi)\big)\frac{t_d^4}{3} + c\big(2\theta - m(\xi)\big)\frac{t_d^5}{4}\right\}\end{array}\right]\right\} \tag{16}$$

Hence the fuzzy total cost per unit time is given by

$$\widetilde{TC}(t_1, T) = \frac{1}{t_1}\left(\widetilde{OC} + \widetilde{DC} + IHC + \widetilde{SC} + \widetilde{PC}\right)$$

$$\widetilde{TC}(t_1,T)=\frac{1}{T}\left\{\begin{array}{c}
\tilde{A}+\widetilde{C_d}\left\{S-\left[a(t_1-t_d)+\frac{b}{2}(t_1^2-t_d^2)+\frac{c}{3}(t_1^3-t_d^3)\right]\right\}\\
+\\
+\left\{\begin{array}{c}
h\left[St_d-\left(\frac{at_d^2}{2}+\frac{bt_d^3}{6}+\frac{ct_d^4}{12}\right)\right]+r\left[\frac{St_d^2}{2}-\left(\frac{at_d^3}{3}+\frac{bt_d^4}{8}+\frac{ct_d^5}{15}\right)\right]\\
+hk(t_1-t_d)+\left[rk-hk\big(\theta-m(\xi)\big)\right]\left[\frac{t_1^2-t_d^2}{2}\right]+ha\big(\theta-m(\xi)\big)\left[\frac{t_1^3-t_d^3}{6}\right]\\
+hb\big(\theta-m(\xi)\big)\left[\frac{t_1^4-t_d^4}{24}\right]\\
+hc\big(\theta-m(\xi)\big)\left[\frac{t_1^5-t_d^5}{60}\right]+hc\big(\theta-m(\xi)\big)\left[\frac{t_1^6-t_d^6}{24}\right]\\
-m(\xi)\left\{\begin{array}{l}
ah\big(2\theta-m(\xi)\big)\left[\frac{t_1^4-t_d^4}{8}\right]\\
+bh\big(2\theta-m(\xi)\big)\left[\frac{t_1^5-t_d^5}{15}\right]\\
+ch\big(2\theta-m(\xi)\big)\left[\frac{t_1^6-t_d^6}{24}\right]
\end{array}\right\}\\
+ra\big(\theta-m(\xi)\big)\left[\frac{t_1^4-t_d^4}{8}\right]+rb\big(\theta-m(\xi)\big)\left[\frac{t_1^5-t_d^5}{30}\right]\\
+rc\big(\theta-m(\xi)\big)\left[\frac{t_1^6-t_d^6}{72}\right]+rc\big(\theta-m(\xi)\big)\left[\frac{t_1^7-t_d^7}{28}\right]\\
-m(\xi)\left\{\begin{array}{l}
ra\big(2\theta-m(\xi)\big)\left[\frac{t_1^5-t_d^5}{10}\right]\\
+rb\big(2\theta-m(\xi)\big)\left[\frac{t_1^6-t_d^6}{18}\right]\\
+rc\big(2\theta-m(\xi)\big)\left[\frac{t_1^7-t_d^7}{28}\right]
\end{array}\right\}\\
-\left\{\begin{array}{c}
ha\left(\frac{t_1^2-t_d^2}{2}\right)+hb\left(\frac{t_1^3-t_d^3}{6}\right)+hc\left(\frac{t_1^4-t_d^4}{12}\right)+ra\left(\frac{t_1^3-t_d^3}{3}\right)\\
+rb\left(\frac{t_1^4-t_d^4}{8}\right)+rc\left(\frac{t_1^5-t_d^5}{15}\right)
\end{array}\right\}
\end{array}\right\}\\
+\widetilde{C_S}\left\{\frac{a}{2}[(T^2-t_1^2)-2t_1(T-t_1)]+\frac{b}{6}[2(T^3-t_1^3)-3t_1^2(T-t_1)]+\frac{c}{12}[3(T^4-t_1^4)-4t_1^3(T-t_1)]\right\}\\
+\widetilde{P_c}\left\{\begin{array}{c}
k-k\big(\theta-m(\xi)\big)t_d+a\big(\theta-m(\xi)\big)\frac{t_d^2}{2}+b\big(\theta-m(\xi)\big)\frac{t_d^3}{6}+c\big(\theta-m(\xi)\big)\frac{t_d^4}{12}+c\big(\theta-m(\xi)\big)\frac{t_d^5}{4}\\
-m(\xi)\left\{a\big(2\theta-m(\xi)\big)\frac{t_d^3}{2}+b\big(2\theta-m(\xi)\big)\frac{t_d^4}{3}+c\big(2\theta-m(\xi)\big)\frac{t_d^5}{4}\right\}+a(T-t_1)+\frac{b}{2}(T^2-t_1^2)\\
+\frac{c}{3}(T^3-t_1^3)
\end{array}\right\}
\end{array}\right\} \quad (17)$$

Then $\widetilde{TC}(t_1)=[TC_1(t_1),\ TC_2(t_2),TC_3(t_3)]$

The fuzzy total cost is defuzzified by graded mean representation method.

$$TC_{dG}(t_1)=\frac{1}{6}[TC_1(t_1)+4\,TC_2(t_2)+\ TC_3(t_3)]$$

Our objective is to determine optimum value of t_1 and T so that TC (t_1, T) is minimum. The values of t_1 and T for which $\frac{\partial TC_{dG}(t_1,T)}{\partial t_1}=0$ and $\frac{\partial TC_{dG}(t_1,T)}{\partial T}=0$ satisfying the condition

$$\left\{\left(\frac{\partial^2 TC_{dG}(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC_{dG}(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 TC_{dG}(t_1,T)}{\partial t_1\partial T}\right)^2\right\}>0$$

The optimal solution of the equation (17) is obtained using Mathematica software. This has been illustrated by the following numerical example.

5. NUMERICAL EXAMPLE

Crisp Model [12]

We consider the following parametric values for $\mathrm{A}=120,\ \mathrm{a}=10,\ \mathrm{b}=8,\ \mathrm{c}=5,\ \mathrm{h}=1,\ \mathrm{r}=\ 0.5,\ \mathrm{t_d}=0.5,$ $\theta=\ 0.87,\ \mathrm{m}(\xi)=0.05,\ \mathrm{C_d}=5,\ \mathrm{C_s}=2,\mathrm{P_c}=15$.

We obtain the optimal value of $t_1=2.82824$ units, T= 3.46282 units $Q\ =\ 199.999$ and optimal total cost (TC) $=\ 343.688$

Fuzzy Model

$\tilde{A}=(117,\ 120,\ 125)$, $\widetilde{C_d}=(4,\ 5,\ 6)$, $\widetilde{C_s}=(1,\ 2,\ 3)$and $\widetilde{P_c}=(12,\ 15,\ 20)$.

We obtain the optimal value of $t_1=2.81859$ units, $T=3.456$ units $Q=198.666$ and minimum total cost $TC_{dG}(t_1)=343.585$.

© 2017, IJMA. All Rights Reserved

6. CONCLUSION

The products with high deterioration rate are always crucible to the retailer's business. In real markets, the retailer can reduce the deterioration rate of a product by making effective capital investment in storehouse equipment. In this study, to reduce the deterioration rate during deterioration period of deteriorating items, we use the preservation technology. A solution procedure is given to find an optimal replenishment cycle, shortage period, order quantity and preservation technology that the total inventory cost per unit time is minimum. For fuzzy model the ordering cost, deterioration cost, shortage cost and purchasing cost are represented by triangular fuzzy numbers. Graded mean representation method is used for defuzzification. A numerical example has been presented to illustrate the model. This model can further be extended by taking more realistic assumptions such as finite replenishment rate, Probabilistic demand rate etc.

7. REFERENCES

1. Amutha R, and Dr. E. Chandrasekaran (2012), "An inventory model for deteriorating products with Weibull distribution deterioration, time-varying demand and partial backlogging", *International Journal of Scientific & Engineering Research*, Volume 3, Issue 10.
2. Dye, C.Y., and Hsieh, T.P., (2012) "An optimal replenishment policy for deteriorating items with effective investment in preservation technology", *European Journal of Operational Research*, 218(1) 106–112.
3. Gothi U. B. and Kirtan Parmar (2015), "Order level lot size inventory model for deteriorating items under quadratic demand with time dependent IHC and partial backlogging", *Research Hub - International Multidisciplinary Research Journal IMRRHJ)*, Vol 2, Issue 2.
4. Hsu, P.H., Wee, H.M., and H.M., Teng, (2010), "Preservation technology investment for deteriorating inventory", *International Journal of Production Economics*, 124(2) 388–394.
5. J. Jagadeeswari and P. K. Chenniappan (2014), "An order level inventory model for deteriorating items with time – quadratic demand and partial backlogging", *Journal of Business and Management Sciences*, Vol. 2, No. 3A, 17 – 20.
6. Jhuma Bhowmick and G.P.Samauta (2011), "A deterministic inventory model of deteriorating items with two rates of production, shortages and variable production cycle", *International Scholarly Research Network, ISRN Applied Mathematics*, article ID 657464, 1- 16.
7. Kirtan Parmar and Gothi U. B. (2014), "Order level inventory model for deteriorating items under quadratic demand with time dependent IHC", *Sankhaya Vignan*, NSV 10, No. 2, pp. 1 – 12.
8. Kirtan Parmar and Gothi U. B. (2015), "An EPQ model of deterioration items using three parameter Weibull distribution with constant production rate and time varying holding cost", International Journal of Science, *Engineering and Technology Research (IJSETR)*, Vol. 4, Issue 2, pp. 0409 – 0416.
9. Leea Y. P., and Dye C. Y., (2012), "An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate", *Computers & Industrial Engineering*, 63(2) 474–482.
10. Maragatham. M and Lakshmidevi. P.K, (2016), "A Fuzzy Model with Changing Deterioration Rate", *International Journal of Engineering Research*, 5 (11), 851-855.
11. Mishra, V.K., and Singh, L.S. (2011),"Deteriorating inventory model for time dependent demand and holding cost with partial backlogging", *International Journal of Management Science and Engineering Management*, 6(4), pp. 267 –271.
12. Palani. R and Maragatham. M (2016), "EOQ model for controllable deterioration rate and time dependent demand and Inventory holding cost", *International Journal of Mathematics Trends and Technology (IJMTT)*, 39 (4), 245-251,
13. Parmar Kirtan and U. B. Gothi (2015), "EOQ model with constant deterioration rate and time dependent demand and IHC", *Pacific Business Review International*, 7(12), pp-01-11.
14. Sarala Pareek and Garima Sharma (2014),"An inventory model with Weibull distribution deteriorating item with exponential declining demand and partial backlogging", *ZENITH International Journal of Multidisciplinary Research*, Vol.4 (7).
15. Syed J.K, and Aziz.L.A (2007), "Fuzzy Inventory Model without Shortages Using Signed Distance Method", *Applied Mathematics & Information Sciences - An International Journal, Dixie W Publishing Corporation*, U.S.A, 1(2), 203-203.
16. Umap H.P (2010), "Fuzzy EOQ model for deteriorating items with two warehouses", *Journal of Statistics and Mathematics*, 1(2), 01-06.
17. Vinod kumar Mishra, (2013), "An inventory model of instantaneous deteriorating items with controllable deterioration rate for time dependent demand and holding cost", *Journal of Industrial Engineering and Management*, 6(2), 495 – 506.
18. Vinod kumarMishra, (2014), "Deteriorating inventory model with controllable deterioration rate for time dependent demand and time varying holding cost", *Yugoslav Journal of Operations Research*, 24, 87-98.

Source of support: Nil, Conflict of interest: None Declared.