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A STUDY ON INTUITIONISTIC FUZZY GRAPHS OF SECOND TYPE

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ABSTRACT

In this paper, we define the degree of Intuitionistic Fuzzy Graphs of Second Type and Complete Intuitionistic Fuzzy Graphs of Second Type. Also establish some of their properties.

Key words: *Intuitionistic fuzzy set, Intuitionistic fuzzy sets of second type, Intuitionistic fuzzy graphs, Intuitionistic fuzzy graphs of second type, degree, complete.*

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1. INTRODUCTION

Fuzzy sets were introduced by Lotfi. A. Zadeh [10] in 1965 as a generalisation of classical (crisp) sets. Further the fuzzy sets are generalised by Krassimir.T. Atanassov [1] in which he has taken non-membership values also into consideration and introduced Intuitionistic Fuzzy sets [IFS] and their extensions like Intuitionistic Fuzzy Sets of Second Type [IFSST], Intuitionistic L-Fuzzy Sets [ILFS] and Temporal Intuitionistic Fuzzy Sets [TIFS] and the concept of intuitionistic fuzzy relations. R. Parvathi and M. G. Karunambigai [5] introduced Intuitionistic Fuzzy Graphs [IFG] and Complete IFG and analyzed their components. Further A. Nagoor Gani and S. Shajitha Begum [4] introduced the degree of Intuitionistic Fuzzy Graphs. In section 2, we give some basic definitions and in section 3, we define the degree of Intuitionistic Fuzzy Graphs of Second Type [IFGST], Complete Intuitionistic Fuzzy Graphs of Second Type. Also establish some of their properties. The paper is concluded in section 4 .

2. PRELIMINARIES

In this section, we give some basic definitions.

Definition 2.1 [1]: An Intuitionistic fuzzy set A in a universal set E is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\},$$

Where $\mu_A: E \to [0,1]$ and $\nu_A: E \to [0,1]$ denote the degree of membership and the degree of non-membership of the element x in E respectively, satisfying $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Definition 2.2 [1]: An Intuitionistic fuzzy sets of second type A in a universal set E is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\},$$

Where $\mu_A: E \to [0,1]$ and $\nu_A: E \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in E$ respectively, satisfying $0 \le \mu_A(x)^2 + \nu_A(x)^2 \le 1$.

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Definition 2.3 [5]: An Intuitionistic fuzzy graph is of the form $G = [V, E]$ where

(i) $V = \{v_1, v_2, \dots v_n\}$ such that $\mu_1: V \to [0,1]$ and $\nu_1: V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and
$0 \le \mu_1(v_i) + \nu_1(v_i) \le 1$ for every $v_i \in V,\ (i = 1,2, \dots n)$

(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$ and $\nu_2: V \times V \to [0,1]$ are such that
$\mu_2(v_i, v_j) \le \min[\mu_1(v_i), \mu_1(v_j)]$,
$\nu_2(v_i, v_j) \le \max[\nu_1(v_i), \nu_1(v_j)]$
and $0 \le \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in E,\ (i, j = 1,2, \dots n)$

Definition 2.4 [4]: Let $G = [V, E]$ be an IFG then the degree of a vertex v is defined by,
$d(v) = (d_\mu(v), d_\nu(v))$ where $d_\mu(v) = \sum_{u \ne v} \mu_2(v, u)$ and $d_\nu(v) = \sum_{u \ne v} \nu_2(v, u)$ for all $u, v \in V$

Definition 2.5 [4]: The minimum degree of G is $\delta(G) = [\delta_\mu(G), \delta_\nu(G)]$ where
$\delta_\mu(G) = \wedge \{d_\mu(v): v \in V\}$ and $\delta_\nu(G) = \wedge \{d_\nu(v): v \in V\}$

Definition 2.6 [4]: The maximum degree of G is $\Delta(G) = [\Delta_\mu(G), \Delta_\nu(G)]$ where
$\Delta_\mu(G) = \vee \{d_\mu(v): v \in V\}$ and $\Delta_\nu(G) = \vee \{d_\nu(v): v \in V\}$

Definition 2.7 [4]: Let $G = [V, E]$ be an IFG. The total degree of a vertex $v \in V$ is defined as
$td(v) = \left[\sum_{v_1 v_2 \in E} d_{\mu_2}(v) + \mu_1(v),\ \sum_{v_1 v_2 \in E} d_{\nu_2}(v) + \nu_1(v),\right]$

Definition 2.8 [6]: An IFG, is called the complete if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\nu_{2ij} = \max(\nu_{1i}, \nu_{1j})$ for all $(v_i, v_j) \in V$

Definition 2.9 [9]: An Intuitionistic fuzzy graphs of second type is of the form $G = [V, E]$ where

(i) $V = \{v_1, v_2, \dots v_n\}$ such that $\mu_1: V \to [0,1]$ and $\nu_1: V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and
$0 \le \mu_1(v_i)^2 + \nu_1(v_i)^2 \le 1$ for every $v_i \in V,\ (i = 1,2, \dots n)$

(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$ and $\nu_2: V \times V \to [0,1]$ are such that
$\mu_2(v_i, v_j) \le \min[\mu_1(v_i)^2, \mu_1(v_j)^2]$,
$\nu_2(v_i, v_j) \le \max\left[\nu_1(v_i)^2, \nu_1(v_j)^2\right]$
and $0 \le \mu_2(v_i, v_j)^2 + \nu_2(v_i, v_j)^2 \le 1$ for every $(v_i, v_j) \in E,\ (i, j = 1,2, \dots n)$

3. INTUITIONISTIC FUZZY GRAPHS OF SECOND TYPE

In this section, we define the degree of intuitionistic fuzzy graphs of second type and the concept of complete intuitionistic fuzzy graphs of second type. Also establish some of their properties.

Definition 3.1: Let $G = [V, E]$ be an IFGST then the degree of a vertex v is denoted by $d(v)$ and defined as,
$d(v) = (d_\mu(v), d_\nu(v))$ where $d_\mu(v) = \sum_{u \ne v} \mu_2(v, u)$ and $d_\nu(v) = \sum_{u \ne v} \nu_2(v, u)$ for all $u, v \in V$

Definition 3.2: The minimum degree of G is denoted by $\delta(G)$ and defined as,
$\delta(G) = [\delta_\mu(G), \delta_\nu(G)]$ where $\delta_\mu(G) = \min\{d_\mu(v): v \in V\}$ and $\delta_\nu(G) = \min\{d_\nu(v): v \in V\}$

Definition 3.3: The maximum degree of G is denoted by $\Delta(G)$ and defined as,
$\Delta(G) = [\Delta_\mu(G), \Delta_\nu(G)]$ where $\Delta_\mu(G) = \max\{d_\mu(v): v \in V\}$ and $\Delta_\nu(G) = \max\{d_\nu(v): v \in V\}$

Definition 3.4: Let $G = [V, E]$ be an IFGST. The total degree of a vertex $v \in V$ is defined as
$td(v) = \left[\sum_{v_1 v_2 \in E} d_{\mu_2}(v) + \mu_1(v),\ \sum_{v_1 v_2 \in E} d_{\nu_2}(v) + \nu_1(v),\right]$

Example 3.1:

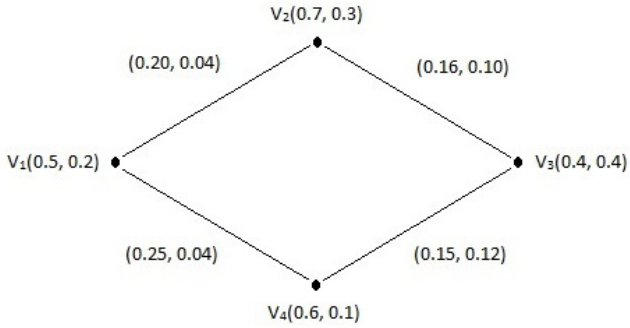

fig. 1

In the above fig. 1. We have,

The degree of a vertices of G are $d(v_1) = [0.45, 0.08]$, $d(v_2) = [0.36, 0.14]$, $d(v_3) = [0.31, 0.22]$, $d(v_4) = [0.40, 0.16]$, the minimum degree of G is $\delta[G] = [0.31, 0.08]$ and the maximum degree of G is $\Delta[G] = [0.45, 0.22]$

Definition 3.5: An IFGST, $G = [V, E]$ is called the complete IFGST if for every $v_i, v_j \in V$ such that $\mu_{2ij} = \min(\mu_{1i}^2, \mu_{1j}^2)$ and $\nu_{2ij} = \max(\nu_{1i}^2, \nu_{1j}^2)$

Example 3.2:

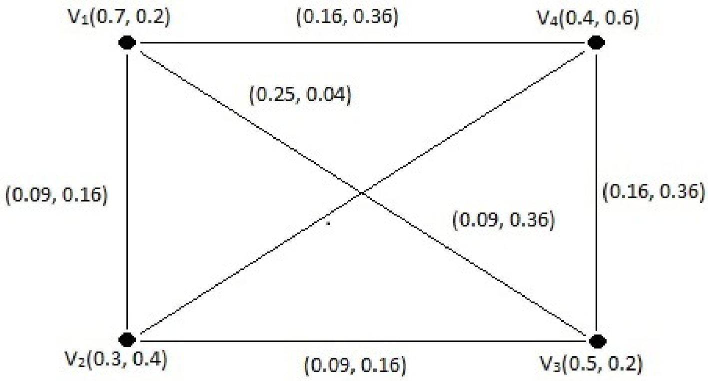

fig. 2

Theorem 3.1: For an IFGST, $G = [V, E]$ we have the following

(i) The sum of degree of membership value of all the vertices is equal to twice the sum of the membership values of all edges $\sum d_\mu(v_i) = 2\sum \mu_2(v, u)$

(ii) The sum of degree of non-membership value of all the vertices is equal to twice the sum of the non-membership values of all edges $\sum d_\nu(v_i) = 2\sum \nu_2(v, u)$

Proof:

Let $G = [V, E]$ be an IFGST where $V = \{v_1, v_2 \ldots v_n\}$

Consider, $\sum d(v_i) = \left[\sum d_\mu(v_i), \sum d_\nu(v_i)\right]$

$$= \left[d_\mu(v_1), d_\nu(v_1) + d_\mu(v_2), d_\nu(v_2) + \cdots + d_\mu(v_n), d_\nu(v_n)\right]$$

$$= \begin{bmatrix} \mu_2(v_1, v_2), \nu_2(v_1, v_2) + \mu_2(v_1, v_3), \nu_2(v_1, v_3) + \cdots \mu_2(v_1, v_n), \nu_2(v_1, v_n) \\ +\mu_2(v_2, v_1), \nu_2(v_2, v_1) + \mu_2(v_2, v_3), \nu_2(v_2, v_3) + \cdots \mu_2(v_2, v_n), \nu_2(v_2, v_n) \\ + \cdots + \\ +\mu_2(v_n, v_1), \nu_2(v_n, v_1) + \mu_2(v_n, v_2), \nu_2(v_n, v_2) + \cdots \mu_2(v_{n-1}, v_n), \nu_2(v_{n-1}, v_n) \end{bmatrix}$$

$$= \begin{bmatrix} \mu_2(v_1, v_2), \nu_2(v_1, v_2) + \mu_2(v_2, v_1), \nu_2(v_2, v_1) + \mu_2(v_1, v_3), \nu_2(v_1, v_3) + \\ \mu_2(v_3, v_1), \nu_2(v_3, v_1) + \cdots + \mu_2(v_1, v_n), \nu_2(v_1, v_n) + \mu_2(v_n, v_1), \nu_2(v_n, v_1) \end{bmatrix}$$

$$= \left[2\left(\mu_2(v_1, v_2), \nu_2(v_1, v_2)\right) + 2\left(\mu_2(v_1, v_3), \nu_2(v_1, v_3)\right) + \cdots + 2(\mu_2(v_1, v_n), \nu_2(v_1, v_n))\right]$$

$$= \left[2\sum \mu_2(v, u), 2\sum \nu_2(v, u)\right]$$

This completes the proof.

Theorem 3.2: The maximum degree of any vertex in an IFGST, G with n vertices is $n-1$

Proof: Let $G=[V,E]$ be an IFGST with n vertices.

In G the maximum degree of membership or non-membership value given to an edge is 1 and the number of edges incident on a vertex can be at most $n-1$

Therefore the maximum degree of any vertex in G is $n-1$

This completes the proof.

Theorem 3.3: A Complete IFGST, $G=[V,E]$ must have at least one pair of vertices whose membership degrees are equal and at least one pair of vertices whose non-membership degrees are equal.

Proof: Let $G=[V,E]$ be the Complete IFGST with n vertices.

Consider $\mu_1(v_i)$ and $\nu_1(v_i)$ are equal for all $v_i \in V$ then at least one pair of vertices whose membership degrees are equal and at least one pair of vertices whose non-membership degrees are equal.

Suppose $\mu_1(v_i)$ and $\nu_1(v_i)$ are distinct for all $v_i \in V$ then we have,

$d_\mu(v_i) = \sum_{v_i \neq v_j} \mu_2(v_i, v_j)$ and $d_\nu(v_i) = \sum_{v_i \neq v_j} \nu_2(v_i, v_j)$

Since $v_i$are adjacent to v_j, we have at least one pair of vertices whose membership degrees are equal and at least one pair of vertices whose non-membership degrees are equal.
This completes the proof.

4. CONCLUSION

In this paper, we have defined the degree of intuitionistic fuzzy graphs of second type and the concept of complete intuitionistic fuzzy graphs of second type. Also we established some of their properties. In future we will study some more properties and applications of IFGST.

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