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NUMERICAL SOLUTION OF MHD RADIATIVE FLOW AND HEAT TRANSFER OVER AN EXPONENTIALLY STRETCHING SHEET IN THE PRESENCE OF SUCTION/INJECTION

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ABSTRACT

In the present paper a numerical solution for the steady two dimensional laminar flow over an exponentially stretching sheet in the presence of suction-injection with radiative heat flux has been obtained by solving the governing equations using Runge-Kutta method. The fluid is assumed viscous and incompressible. The effects of various quantities of physical interest including Magnetic and Radiation parameters, Suction-injection parameter, Prandtle number and Eckert number for velocity and temperature distribution are derived. Graphical results for velocity and temperature profiles are presented and discussed.

Key words: *MHD, Exponentially stretching sheet, Heat transfer, Radiation, Suction-Injection.*

1. INTRODUCTION

Flow of an incompressible viscous fluid over a stretching sheet is important problem in fluid dynamics and has numerous applications in various fields such as, glass-fibre production, wire drawing, paper production, plastic sheets, metal and polymer processing industries and many others. The boundary flow over a stretching sheet was first studied by Sakiadis [4]. Later, Crane [8] extended this idea for the two dimensional flow over a stretching sheet problem. Gupta and Gupta [12], Carragher and Crane [11], Dutta *et al.* [5] studied the heat transfer in the flow over a stretching surface taking into account different aspects of the problem. Magyari and Keller [7] investigated the study of boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution. Aboeldahab and Gendy [6] studied the radiation effects on MHD free convective flow of a gas past a semi-infinite vertical plate with variable thermophysical properties for a higher-temperature difference. Many others problems on exponentially stretching surface were discussed by Raptis *et al.* [2], Partha *et al.* [9], and Sajid and Hayat [10]. Jat and Chaudhary [13-16] studied the MHD boundary layer flow over a stretching sheet for stagnation point, heat transfer with or without viscous dissipation and Joule heating. Bidin and Nazar [3] studied the Numerical solution of the boundary layer flow over an exponential stretching sheet with thermal radiation. Recently Jat *et al.* [1] studied the thermal radiation effects on MHD flow and heat transfer due to an exponential stretching sheet with viscous dissipation. Realizing the increasing technical applications of MHD affects the present paper studies the problem of MHD boundary layer flow over an exponentially stretching sheet with viscous dissipation and suction-injection.

FORMULATION OF THE PROBLEM

Consider a steady two dimensional laminar flow of a viscous incompressible electrical conducting fluid over a continuous exponentially stretching surface. The x-axis is taken along the stretching surface in the direction of motion and y-axis is perpendicular to it. A uniform magnetic field of strength B_0 is assumed to be applied normal to the stretching surface. The magnetic Reynolds number is taken to be small and therefore the induced magnetic field is neglected. The surface is assumed to be highly elastic and is stretched in the x-direction with a velocity $U = U_0 e^{\frac{x}{L}}$. All the fluid properties are assumed to be constant throughout the motion. Under the usual boundary layer approximations, the governing boundary layer equations by considering the viscous dissipation and radiation effects are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

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$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_0^2 u^2 \tag{3}$$

Where u and v are the velocities in the x- and y-directions respectively, ρ is the density of fluid, μ is the dynamic viscosity, $\nu=\frac{\mu}{\rho}$ is the kinematic viscosity, C_p is the specific heat at constant pressure, κ is thermal conductivity of the fluid under consideration, q_r is the radiative heat flux and T is the temperature.

The boundary conditions are:

$$y=0: \qquad u=U_w=U_0 e^{\frac{x}{L}}, \qquad v=v_0, \qquad T=T_\infty+T_0 e^{\frac{2x}{L}}$$
$$y\to\infty: \qquad u\to 0, \qquad T\to 0 \tag{4}$$

Where U_0, T_0 and L are the reference velocity, temperature and length respectively. In the optically thick limit, the fluid does not absorb its own emitted radiation in which there is no self-absorption, but it does absorb radiation emitted by the boundaries. Cogley *et al.* [6] showed that in such a case radiative heat flux is given by:

$$\frac{\partial q_r}{\partial y}=4\left(T-T_\infty\right)\int_0^\infty K_{\lambda w}\left(\frac{de_{b\lambda}}{dT}\right)_w d\lambda=4I_1\left(T-T_\infty\right) \tag{5}$$

Using equation (5) in equation (3) we get

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\frac{\partial^2 T}{\partial y^2}-4I_1(T-T_\infty)+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_0^2 u^2 \tag{6}$$

Solution of the problem

The equation of continuity (1) is identically satisfied if we choose the stream function ψ such that

$$u=\frac{\partial\psi}{\partial y}\quad,\quad v=-\frac{\partial\psi}{\partial x} \tag{7}$$

The momentum and energy equations can be transformed into the corresponding ordinary differential equations by introducing the following similarity transformations:

$$\psi\left(x,y\right)=\sqrt{2\nu U_0 L e^{\frac{2x}{L}}}\,f\left(\eta\right)$$
$$\frac{T-T_\infty}{T_0}=e^{\frac{2x}{L}}\theta\left(\eta\right)$$

Where $\eta=\sqrt{\frac{U_0}{2\nu L}e^{\frac{2x}{L}}}\,y$ (8)

Then the momentum and energy equations (2) and (6) are transformed to:

$$f'''-2(f')^2+ff''-Mf'=0 \tag{9}$$

$$\theta''-R\theta+\Pr\left(f\theta'-f'\theta+Ec\left(f''\right)^2+MEc\left(f'\right)^2\right)=0 \tag{10}$$

The corresponding boundary conditions are:

$$\eta=0: \qquad f=S \qquad f'=1 \qquad \theta=1$$
$$\eta\to\infty: \qquad f'\to 0 \qquad \theta\to 0 \tag{11}$$

Where prime denotes the differentiation w.r.t. η and dimensionless parameters are:

$M = \frac{2\sigma B_0^2 L}{\rho U_0 e^{\frac{x}{L}}}$, (Magnetic parameter)

$Ec = \frac{U_0^2}{T_0 c_p}$, (Eckert number)

$\Pr = \frac{\mu c_p}{\kappa}$, (Prandtle number)

$R = \frac{8 I_1 \nu L}{e^{\frac{x}{L}}}$, (Radiation parameter)

$S = \frac{-v_0}{e^{\frac{x}{2L}}}\sqrt{\frac{2L}{\nu U_0}}$, (Suction-Injection parameter) (12)

The physical quantities of interest are the skin-friction coefficient c_f and the heat transfer rates i.e. Nusselt number Nu are:

$$c_f = \frac{\tau_w}{\frac{\rho U_w^2}{2}} = \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\frac{\rho U_w^2}{2}}$$

$$\Rightarrow c_f = \frac{2}{\sqrt{\mathrm{Re}}} f^{//}(0) \tag{13}$$

and

$$Nu = \frac{x\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w - T_\infty} \tag{14}$$

$$\Rightarrow Nu = -\sqrt{\mathrm{Re}}\,\theta^{/}(0)$$

where

$\mathrm{Re} = \frac{U_0 L}{\nu}$ (Reynolds number) (15)

RESULT AND DISCUSSION

The set of nonlinear ordinary differential equations (9) and (10) with boundary conditions (11) were solved numerically using Runge-Kutta forth order algorithm with a systematic guessing of $f^{//}(0)$ and $\theta^{/}(0)$ by the shooting technique until the boundary conditions at infinity are satisfied. The step size $\Delta\eta = 0.001$ is used while obtaining the numerical solution and accuracy up to the seventh decimal place i.e. 10^{-7}, which is very sufficient for convergence. In this method, we choose suitable finite values of $\eta \to \infty$ say η, which depends on the values of the parameter used. The computations were done by a program which uses a symbolic and computational computer language Matlab. The velocity profile for various values of parameters and the corresponding temperature profiles are presented via plots which prescribes the effects of parameters. The variation of velocities and corresponding temperature distributions for applied suction or injection are very important in physical point of view. In confining the generating velocity due to shrinking of sheet and maintaining the boundary layer structure, the mass suction at the sheet is most suitable force. The velocity profiles for various values of suction/injection parameter S are plotted in fig. 1-4. Velocity curve increases as applied suction increases whereas velocity curve decreases as injection increases. Thus mass injection acts to increase the momentum boundary layer thickness but suction acts oppositely. The behaviour of temperature profiles for various values of suction injection parameter is presented in the corresponding figures which also show the significant variation in the trend of temperature due to S.

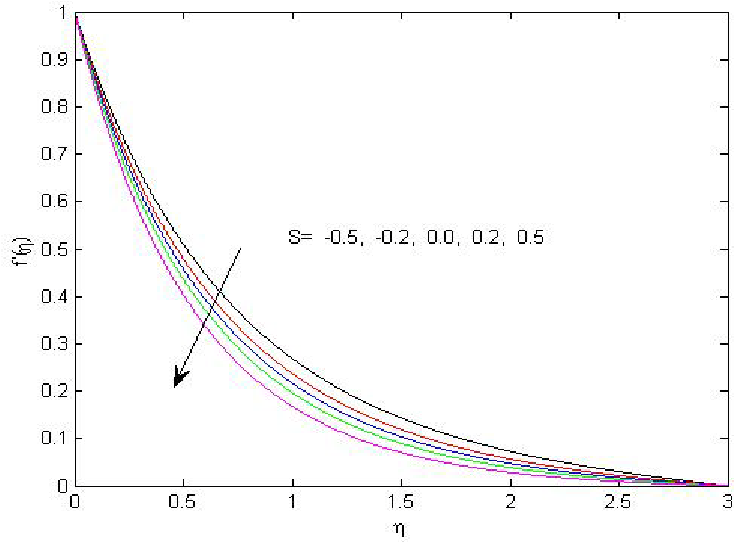

Figure-1: Velocity profile against η for various values of suction-injection parameter S

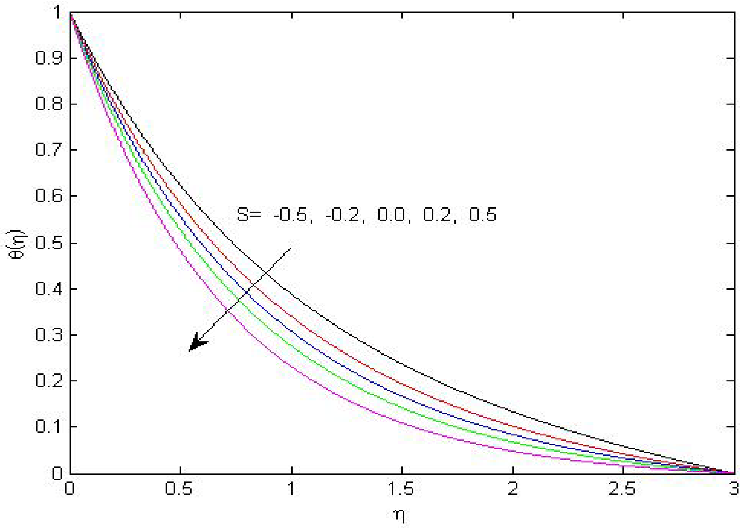

Figure-2: Temperature distribution against η for various values of suction-injection parameter S

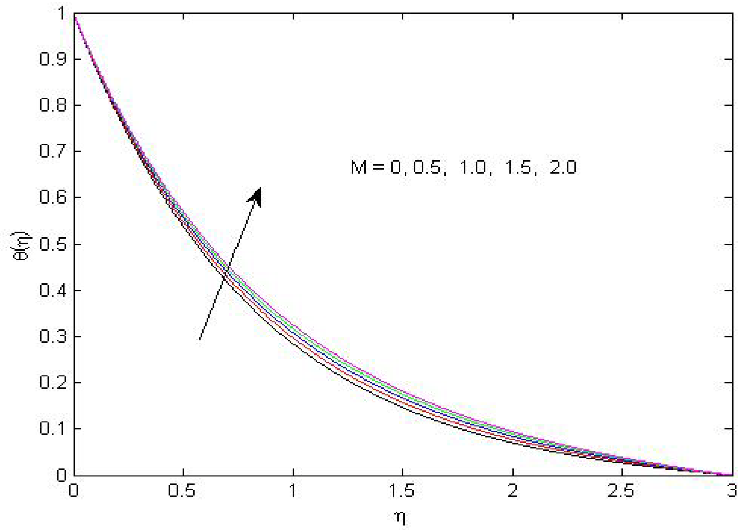

Figure-3: Temperature distribution against η for various values of Magnetic parameter M (S=0)

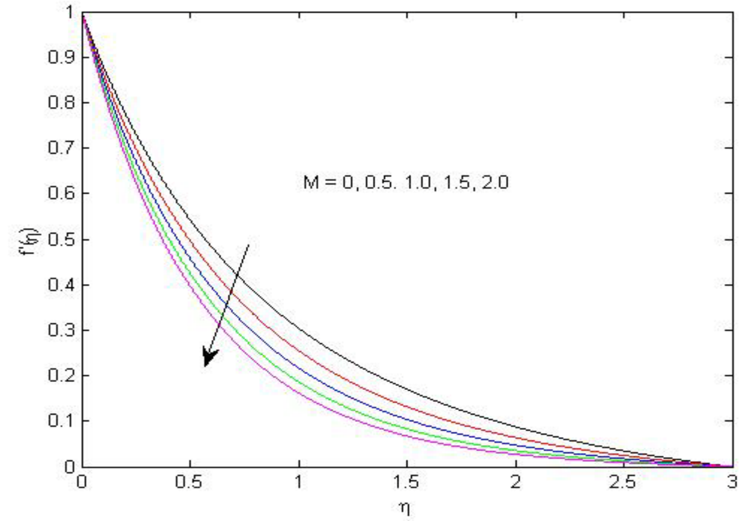

Figure-4: Velocity distribution against η for various values of Magnetic parameter M(S=0)

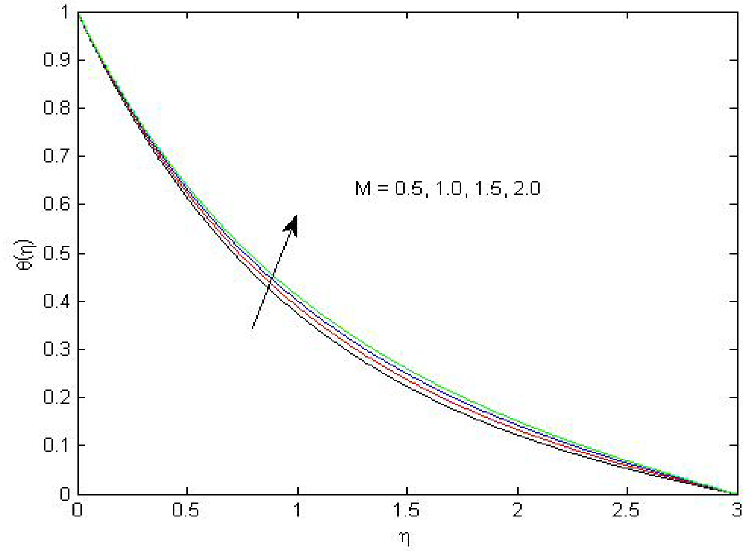

Figure-5: Temperature distribution against η for various values of Magnetic parameter M (S=-0.5)

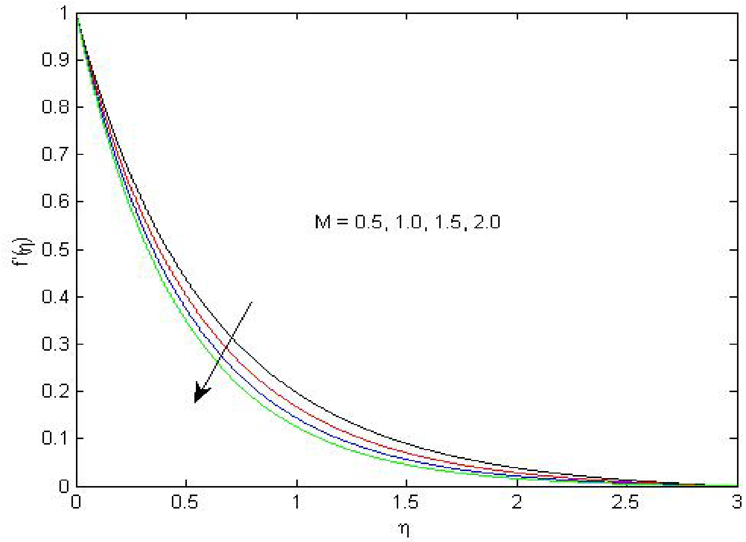

Figure-6: Velocity distribution against η for various values of Magnetic parameter M (S=-0.5)

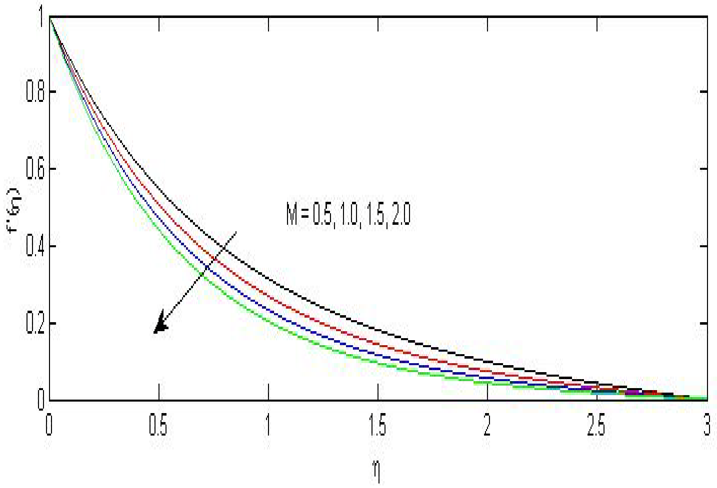

Figure-7: Velocity distribution against η for various values of Magnetic parameter M (S=0.5)

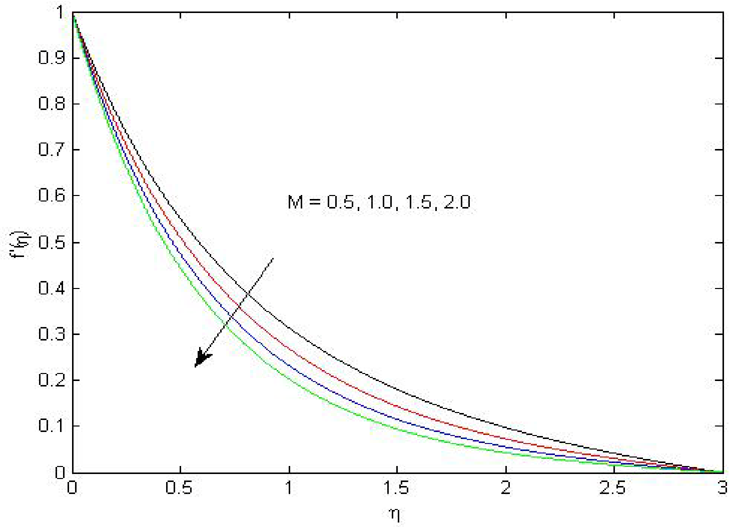

Figure-8: Velocity profiles against η for various values of Magnetic parameter (M) (S=0.5)

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