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COMMUTATIVE THEOREM ON A NEAR-FIELD SPACES AND SUB NEAR-FIELD SPACES OVER A NEAR-FIELD

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ABSTRACT

The development of the general structure theory for near-field spaces and sub near-field spaces over a near-field, a great deal of work was done that showed under certain types of hypothesis, near-field spaces had to be commutative or almost commutative. For a good cross section of the kind of result that was obtained, one can look and in the bibliographies given in these.

Of these type of questions studied, one outstanding one remained open, It asked Suppose N is a near-field space in which, for any a, b $\in$ N, there are integers $m = m(a, b) \geq 1$, $n = n(a, b) \geq 1$ such that $a^m b^n = b^n a^m$. must the Commutator sub near-field space over a near-field N then be a nil sub near-field space? Equivalently, if N is as above and has no non zero nil sub near-field spaces, must N be commutative?

Keywords: *near-field spaces, sub near-field space, near-field space, semi simple near-field space.*

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INTRODUCTION

Some in depth study and generalization, main results cum progress on this was made. In a fairly recent articles Dr. N. V. Nagendram showed. Let N be a near-field space, M be a commutative sub near-field space over a near-field N and suppose that given $s \in N$, $s^n \in M$ for some $n = n(s) \geq 1$. Then the commutative sub near-field space is nil sub near-field space over a near-field N. Dr. N. V. Nagendram's situation is a very special case of the question asked at the beginning, for, if $a^n \in M$ and $b^m \in M$, then $a^n b^m = b^m a^n$, since M is commutative sub near-field space over a near-field N.

In a recent article, Dr. N. V. Nagendram introduced the concept of the Hypercenter of a near-field space over a near-field. The Hypercenter, S, of the near-field space over a near-field N is defined by $S = \{s \in N / sx^n = x^n s, n = n(x,s) \geq 1$, for all $x \in N\}$. Dr. N V Nagendram showed that if N has no non-zero nil sub near-field spaces, then S = Z, the center of N.

As pointed out, Dr. N.V. Nagendram's result followed from this theorem that identifies the center and Hypercenter. We cite the result here because we make much use of it in this paper.

The result we prove here settles the open question, mentioned at the outset, in the affirmative. We prove the

Theorem: Let N be a near-field space in which, given a, b $\in$ N, there exist integers $m = m(a, b) \geq 1$, $n = n(a, b) \geq 1$ such that $a^m b^n = b^n a^m$. Then, the Commutator sub near-field space N of a near-field is nil sub near-field space. In particular, if N has no non-zero sub near-field spaces, then N must be commutative near-field space over a near-field N.

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Proof: This theorem will be proved as a consequence of a series of lemmas, and reductions we shall make. Note that in the hypothesis of the theorem we may assume without any loss of generality that m = n, for, if $a^m b^n = b^n a^m$ then $a^{mn} b^{mn} = b^{mn} a^{mn}$.

In all that follows N will be a near-field space in which, for any pair a, b ∈ N, there is an integer n = n(a, b) ≥ 1 such that $a^n b^n = b^n a^n$. we begin with a result that is known, we include it and its proof for the sake of completeness.

Lemma 1: If N is a division near-field space, then N is commutative.

Proof: we prove this lemma negation method. For that suppose the result is false. By result on the Hypercenter quoted earlier, there must be elements a and b in N such that commutes with na positive power of b. Let $C_m = \{x \in N / x\, b^{m!} = b^{m!} x\}$ and let $B = \bigcup_{m \geq 1} C_m$. Clearly, B is a subdivision near-field space of N and, since $a\, b^{m!} \neq b^{m!} a$ for all m ≥ 1, a ∉ B. Thus B ≠ N. However, given x ∈ N, $x^m b^{m!} = b^{m!} x^m$ for some appropriate m ≥ 1, and so $x^m b^{m!} = b^{m!} x^m$. therefore, $x^m \in B\ \forall\ x \in N$. Hence N must be a near-field space. But N is not a division near-field space. With this contradiction, the lemma is proved.

Lemma 2: If N is semi-simple near-field space then N is commutative near-field space.

Proof: To settle the semi-simple near-field space case, it is enough to handle the situation in which N is primitive near-field space. Suppose then N is primitive near-field space. If N is a division n ear-field space it must be commutative by lemma 1. If N is not a division near-field space by the density theorem of the near-field of 2 x 2 matrices, D_2 over a division near-field space is a homomorphic image of a sub near-field space of N. But, then, D_2 inherits the hypothesis $a^n b^n = b^n a^n$. This however, is patently false for $a = e_{11}$ and $b = e_{11} + e_{12}$. Thus, N is a division near-field space and so is commutative near-field space.

To prove the theorem it is enough to prove that if N has no nonzero nil sub near-field spaces, then N must be commutative near-field space. We proceed by assuming this to be false. We now make a series of reductions, based on the falsity of the theorem, then will eventually lead us to a contradiction.

Since N has no nil sub near-field spaces, N is a sub direct product of prime near-field spaces of N_x, having no nonzero nil sub near-field spaces, in which there is a non nilpotent element c_α with the following property; given a nonzero sub near-field space V_x of N_x, then $a_\alpha^{t(V_\alpha)} \in Vx$ for some integer $t(V_\alpha) \geq 1$. Since each N_α inherits the hypothesis $a^n b^n = b^n a^n$, it is enough to prove each N_α is commutative.

Thus, we may assume, henceforth, that N is a prime near-field space, having no nonzero nil sub near-fields and containing a nilpotent element c such that $c^{t(V)} \in V$ for any nonzero sub near-field V of N, where t(V) ≥ 1. Here we assume that J(N) ≠ 0, where J(N) is the Jacobson radical sub near-field space of N.

Now Jacobson radical near-field space (N) is itself a prime near-field space, and, of course, $a^n b^n = b^n a^n$ for all a, b ∈ J(N). Also since J(N) ≠ 0 is a sub near-field space of N, $d = c^i \in$ (N), some i, if U ≠ 0 is a sub near-field space of J(N), then U ⊃ V= J(N) U J(N) ≠ 0 is a sub near-field space of N. Hence, $c^k \in V$ for some k, where $c^{ki} = d^i$ is in V and so, is in U. In short, (N) has all the properties of N. If J(N) is commutative near-field space, then N is commutative near-field space. Thus, from now on, we may assume without loss of generality that N = J(N), that is, N is its own radical sub near-field space and near-field space.

Since N = J(N), given x ∈ N ∃ x' ∈ N ∋ x +x' + xx' = x + x' +x'x = 0. The mapping φ : N → N given by φ(y) = (1 + x) y (1+ x') = y + xy +y'x + yx' + xyx' is an automorphism of N. we write it formally as $\varphi(y) = (1 + x)\, y\, (1+ x)^{-1}$. This completes the proof of the lemma.

Lemma 3: If N = J(N) has no zero-divisors, then N must be commutative near-field space.

Proof: Let Z be the center of near-field space N and suppose that x ∉ Z. Since the Hypercenter of N coincides with Z in our situation, x is not in the Hypercenter of N. Thus, there is an element a ∈ N such that $xa^m \neq a^m x$ for all m > 0. By our basic hypothesis on N, we can find an integer n such that both $[(1 + x)a(1 + x)^{-1}]^n$ and $[(1 + ax)a(1 + ax)^{-1}]^n$ commute with a^n. Thus, both $a_1 = [(1 + x)a^n(1 + x)^{-1}]$ and $a_2 = [(1 + ax)a^n (1 + xa)^{-1}]$ commute with a^n.

Now $(1+x)a^n = a_1(1 + x)$, $(1 + ax)a^n = a_2(1 + ax)$ (1)

Multiply the first equation from the left by a and subtract the second equation from this, we get
$a^n(a – 1) = aa_1 – a_2 + (aa_1 – a_2a)x$ (2)

Since the left side of (2) commutes with a^n and a, a_1 and a_2 commute with a^n, we get from (2), on commuting it with a^n, that
$(aa_1 – a_2a)(xa^n – a^nx) = 0$ (3)

Since N has no zero divisors and since $xa^n \neq a^nx$, we must have, from (3), that $aa_1 = a_2a$.

Since $aa_1 = a_2a$, (2) reduces to $a^n(a – 1) = aa_1 – a_1 = a_2a – a_2 = a_2(a – 1)$ and since a is the radical, a – 1 is formally invertible, hence, $a^n = a_2$. But then $aa_1 = a_2a = a^na = a^{n+1}$, which gives us $a_1 = a^n$. Using $a_1 = a^n$ we get from (1) the contradiction that $xa^n = a^nx$. This completes the proof of the lemma.

Note: Thus we may assume that N has zero-divisor sub near-field spaces. But a prime near-field space that has nontrivial zero-devisors must have nonzero nil potent elements. Thus, we have an element $a \neq 0$ in N such that $a^2 = 0$.

Lemma 4: If $a^2= 0$, $a \neq 0$ then aNa is a nil right sub near-field space of N.

Proof: By our basic hypothesis on N, there exists an integer n≥1 such that $[(1 + a)(ax)^n(1 + a)^{-1}] = [(1 + a)(ax)(1 + a)^{-1}]^n$ and $(ax)^n$ commute. Since $a^2 = 0$, $(1 + a)^{-1} = 1 – a$ thus $[(1+a)(ax)^n(1 - a) (ax)^n] = (ax)^n [(1 - a)(ax)^n(1 - a)]$. Using $a^2 = 0$ this reduces to $(ax)^{2n} = (ax)^{2n}(1 – a)$, hence $(ax)^{2n}a = 0$, and so $(ax)^{2n+1} = 0$. Thus indeed, aN is nil sub near-field space of N. A near-field space has a nonzero nil right sub near-field space, it have a nonzero nil two sided sub near-field space. This completes the proof of the lemma.

Lemma 5: Every zero-divisor near-field space in N is nilpotent sub near-field space.

Proof: First, we recall exactly what hypothesis N carries, in addition to the basic one that $a^nb^n = b^na^n$. we have that N is prime near-field space, N = J(N), and that there is an element $c \in N$, which is not nilpotent, such that $c^{t(V)} \in V$ for any sub near-field space $V \neq 0$ of N so N has no nil sub near-field spaces. In addition, N has zero-divisors.

Suppose that ab = 0 where $a \neq 0$, $b \neq 0$. Let $\lambda = \{x \in N / xb^m = 0$ for some integer m$\}$, and let $\rho = \{ x \in N / b^mx = 0$ for some integer m$\}$. Clearly, ρ is a right sub near-field space and λ is a left sub near-field space, of N. we claim that ρ = λ. For if $r \in \lambda$, then $rb^m = 0$ for some m. If r' is the quasi-inverse of r, that is, if r + r’ + r’r = 0, then $r'b^m = 0$. Now for some integer n, $(1 + r)b^{mn}(1 + r')b^{mn} = b^{mn}(1 + r)b^{mn}(1 + r')$.

Using $rb^m = r'b^m = 0$, we get from this last relation that that $b^{2mn} = b^{2mn} (1+r')$, hence, $b^{2mn}r' = 0$. But then, $b^{2mn}r = 0$ and so $r \in \rho$. Hence, $\lambda \subset \rho$. Similarly, $\rho \subset \lambda$; hence, ρ = λ is a two sided sub near-field space of N.

Since ab = 0, $a \neq 0$, we have that $\rho = \lambda \neq 0$. Thus $c^k \in \lambda$ for some k. Hence, $c^kb^t = 0$ for some t. Let $V = \{x \in N / c^mx = 0$ for some m$\}$. As we did for λ and ρ above, we have that V is a sub near-field space of N. If $V \neq 0$ we would have that $c^r \in V$ for some r, giving us the contradiction $0 = c^mc^r = c^{m+r}$, since c is not nilpotent. Thus, V = 0. But $b^t \in V$. Hence, bt = 0. In other words, we have shown that every zero-divisor in N is nilpotent. This completes the proof of the lemma.

Lemma 6: If N is not commutative near-field space then N must be torsion-free near-field space.

Proof: If N is not commutative it must have an element $a \neq 0$ such that $a^2 = 0$. If $x \in N$, there is an integer n ∋ $(1 + a)(x)^n(1 + a)^{-1}x^n = x^n(1 + a)x^n(1 + a)^{-1}$ since $(1+a)^{-1} = 1 – a$, we get from this relation that
$ax^{2n} – 2x^nax^n + x^{2n}a = ax^nax^n – x^nax^na$ (4)

If char $N \neq 2$, we can find an integer n so that both (4) holds and $(1 - a)(x)^n(1 - a)^{-1} x)^n = x^n (1 - a)x^n(1 - a)^{-1}$. This gives us , as above, that
$ax^{2n} – 2x^nax^n + x^{2n}a = x^nax^na – ax^nax^n$ (4)

Adding (4), (5) and using that char $N \neq 2$ gives us that
$ax^{2n} – 2x^nax^n + x^{2n}a = 0$, that is, that $(ax^n - x^na)a^n = x^n(ax^n – x^na)$. If N is not torsion free sub near-field space, then char $N = \rho \neq 0$ and from $(ax^n – x^na) x^n = x^n(ax^n - x^na)$ we get that $ax^{pn} – x^{pn}a = px^{n(p-1)}(ax^n – x^na) = 0$. This says that a commutes with some power of every element; hence, a must be in the Hypercenter of N. Since N has no nil sub near-field spaces, its Hypercenter is its center. Thus, $a \in Z$, the center of N. But the center of a prime near-field space has no nilpotent elements, hence this is not possible. To show that N is torsion free sub near – field space, therefore, we must merely rule out the possibility that char N = 2.

If char N = 2, then (4) reduces to $ax^{2n} + x^{2n}a = (ax^n)^2 + (x^na)^2$. Let $y = x^n$. Hence, $ay^2 + y^2a = (ay)^2 + (ya)^2$, whence, multiplying by a, $ay^2a = ayaya$.

Now, the relation $ay^2 + y^2a = (ay)^2 + (ya)^2$ comes from the fact that $(1+a)y(1+a)^{-1}$ commutes with y. But $(1+a)y'(1+a)^{-1}$ then also commutes with y; hence, $ay^{2r} + y^{2r}a = (ay^r)^2 + (y^ra)^2$ for all r. Thus, $ay^{2r}a = ay^ray^ra$.

aN is nil sub near-field space, hence, $(ay)^{2m} = 0$ for some m. But then $ay^{2m}a = ay^{2m-1}a = ... = (ay)^{2m}a = 0$. Since $ay^{2(m+1)} + y^{2(m+1)}a = 0$, we get that $ay^{2m+1} = y^{2m+1}a$. Recalling that $y = x^n$, we have that a commutes with a power of x for every $x \in N$. Thus again, a is in the Hypercenter of N, hence, in the center of N. This can not be, since a is nilpotent. Thus, char N = 2 is not possible. In short, the only way out is that N is torsion free sub-near-field space.

Note: N is torsion-free sub near-field space, we also have that if $a^2 = 0$ and $x \in N$, then $(ax^n - x^na)x^n = x^n(ax^n - x^na)$ for some integer $n \geq 1$.

Lemma 7: If $a \neq 0$, $a^2 = 0$ and if $x \in N$, then for some integer $n \geq 1$, $(ax^n - x^na)x^n = x^n(ax^n - x^na)$.

Lemma 8: Let N be a 2- torsion free near-field space and let a, b$\in$N. Suppose that $a^n = 0$ and that
$(ab - ba)b = b(ab - ba)$. Then, for any $n \geq 3$,

$$(a+b)^n = b^n + nb^{n-1}a + \frac{n(n-1)}{2}b^{n-2}(ab-ba) + \frac{n(n-1)(n-2)}{6}b^{n-3}aba.$$ Also $abab^i = b^iaba$ for any $i \geq 1$.

Proof: Since $(ab - ba)b = b(ab - ba)$, $ab^2 + b^2a = 2bab$. Multiplying from the left by *a* yields $ab^2a = 2abab$ and multiplying from the right by *a* yields $aba^2 = 2baba$. Since N is 2-torsion free sub near-field space, we get that abab = baba. Because aba commutes with b it commutes with all b^i. The proof is a straightforward method of induction. We can now finish the proof of the theorem.

Proof of the theorem: Assuming that the theorem was false, we have reduced down to the following situation. N is prime near-field space, torsion free without nil sub near-field spaces, has zero divisors, and all its zero divisors are nilpotent. Furthermore, if $a \neq 0$, $a^2 = 0$, then for any $x \in N$, there is an integer *n* such that $(ax^n - x^na)x^n = x^n(ax^n - x^na)$. we show that there lead us to a contradiction.

We claim that, given $x \in N$ and $a^2 = 0$, then $ax^r = x^ra$ for some $r \geq 1$, depending on x and a. If x is a zero divisor, then it is certainly correct, for x must be nilpotent, hence, $x^r = 0$ for some r. Thus, we may assume that x is regular that is non zero divisor. Then for some $n \geq 1$, $(ax^n - x^na)x^n = x^n(ax^n - x^na)$. Let $b = x^n$. by our basic assumption on N, there is an integer $m \geq 1$ such that $(a+b)^n b^m = b^m(a+b)^m$. Clearly, we can pick $m \geq 3$ then

$$(a+b)^m = b^m + nb^{m-1}a + \frac{m(m-1)}{2}b^{m-2}(ab-ba) + \frac{m(m-1)(m-2)}{6}b^{m-3}aba..$$

On the right-hand side, b^m, $b^{m-2}(ab - ba)$, and $b^{m-3}aba$ all commute with b, hence with b^m, since the left hand-side commutes with b^m, we end up with $mb^{m-1}ab^m = b^m(mb^{m-1}a)$. This gives us that $mb^{m-1}(ab^m - b^ma) = 0$, since N is torsion free near-field space, we have that $b^{m-1}(ab^m - b^ma)$. But $b = x^n$ and since x is regular, b, and so b^{m-1}, must be regular. The upshot of this is that $ab^m = b^ma$, which is to say that $ax^{mn} = x^{mn}a$.

Thus, *a* commutes with some power of every element in N and so *a* is in the Hypercenter of N. Since N has no nil sub near-field spaces, the Hypercenter of N is merely the center of N. Hence, the element $a \neq 0$, which is nilpotent, is in the center of the prime near-field space N. This is a contradiction ⊗. This completes the proof of the theorem.

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