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DERIVATION OF NORMAL – EXPONENTIAL MODEL FOR STOCHASTIC COST FRONTIER ANALYSIS

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ABSTRACT

Stochastic Frontier Analysis in term of cost means to produce a specified level of output in a minimal possible manner. This paper provides detailed steps on the estimation of Normal Exponential model for the estimation of stochastic cost frontier. Stochastic Cost Frontier plays a major role in the estimation of Cost Efficiency Scores in the field of production. Here $v_i \sim N(0,\sigma_v^2)$ *and* $u_i \sim \exp onential$ *.The parameters are evaluated using Maximum Likelihood Estimates. The cost efficiency of each producer can be obtained from* $CE_i = \left(e^{-\hat{u}_i}\right)$.

Key words*: Stochastic Frontier Analysis, Stochastic cost frontier, Cost efficiency Scores, Normal Exponential distribution, Cost efficiency scores, Maximum likelihood.*

1. INTRODUCTION

Stochastic Cost Frontier Analysis (SCFA) which began with Farell (1957) has wide range of applications in different fields such as hospitals, banks, education sector etc. SCFA is a mathematical model to estimate cost function parameters using regression techniques. Hence comes the importance of distributional assumption. The SCFA model can be expressed as $y_i = f(x_i, \beta)\, e^{v_i + u_i}$ where y_i is the output, x_i, β are the input and technology parameter respectively. $v_i + u_i$ is the composed error term with v_i as the random shock and u_i captures the cost efficiency. Meeusen and Van den Broeck (1977) assigned exponential distribution to u_i and Aigner, Lovell and Schmidt (1977) considered both exponential and half normal. In this paper v_i follows normal distribution and u_i follows exponential distribution. Alijar (2014) considered truncated and exponential distribution for u_i and proposed that exponential distribution is just like half normal distribution and a special case of truncated normal distribution.In the study related to the estimation of technical efficiency of maize farmers, John Ng'ombe and Thomson Kalinda (2015) assumed exponential distribution for u_i . Maximum likelihood estimation which was developed by Aigner and chu (1968) can be used to maximise the log likelihood function of $f(\varepsilon)$. Normal-Exponential Stochastic Cost Frontier Model (NESCFM) can be estimated once the conditional probability of u_i / ε_i is derived.

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2. THE NORMAL-EXPONENTIAL STOCHASTIC COST FRONTIER MODEL (NESCFM)

In the estimation, of $y_i = f(x_i, \beta)\, e^{v_i + u_i}$

i) $v_i \sim iid\ N(0, \sigma_v^2)$

ii) u_i ~ iid exponential

iii) v_i and u_i are independently and identically distributed of each other and of the regressors.

The probability density function of v is

$$f(v) = \frac{1}{\sqrt{2\pi}\sigma_v} e^{\frac{-v^2}{2\sigma_v^2}} \tag{1}$$

The probability density function of u is

$$f(u) = \frac{1}{\sigma_u} e^{-\frac{u}{\sigma_u}}, \text{ for } u \geq 0 \tag{2}$$

Due to the independence assumption, the joint probability density function of v and u is the product of their individual density functions therefore,

$f(u\,v) = f(u)\, f(v)$

$$f(u,v) = \frac{1}{\sqrt{2\pi}\sigma_u \sigma_v} e^{-\frac{u}{\sigma_u} - \frac{v^2}{2\sigma_v^2}} \tag{3}$$

Taking $\varepsilon = v + u$, the joint probability density function of u and ε is given by

$$f(u,\varepsilon) = \frac{1}{\sqrt{2\pi}\ \sigma_u \sigma_v} e^{-\frac{u}{\sigma_u} - \frac{(\varepsilon - u)^2}{2\sigma_v^2}} \tag{4}$$

$$f(u,\varepsilon) = \frac{1}{\sqrt{2\pi}\ \sigma_u \sigma_v} e^{-\frac{u}{\sigma_u} - \frac{(\varepsilon^2 - 2\varepsilon u + u)^2}{2\sigma_v^2}} \tag{5}$$

$$f(u,\varepsilon) = \frac{1}{\sqrt{2\pi}\ \sigma_u \sigma_v} e^{-\frac{u}{\sigma_u} - \frac{\varepsilon^2}{2\sigma_v^2} + \frac{2\varepsilon u}{2\sigma_v^2} - \frac{u^2}{2\sigma_v^2}} \tag{6}$$

$$f(u,\varepsilon) = \frac{1}{\sqrt{2\pi}\ \sigma_u \sigma_v} e^{-\frac{1}{2}\left[\frac{u^2}{\sigma_v^2} - 2\left(\frac{\varepsilon}{\sigma_v^2} - \frac{1}{\sigma_u}\right)u + \frac{\varepsilon^2}{\sigma_v^2}\right]} \tag{7}$$

$$f(u,\varepsilon) = \frac{1}{\sqrt{2\pi}\ \sigma_u \sigma_v} e^{-\frac{1}{2}\left[\frac{u^2}{\sigma_v^2} - \frac{2u}{\sigma_v}\left(\frac{\varepsilon}{\sigma_v} - \frac{\sigma_v}{\sigma_u}\right) + \frac{\varepsilon^2}{\sigma_v^2}\right]} \tag{8}$$

$$f(u,\varepsilon) = \frac{1}{\sqrt{2\pi}\ \sigma_u \sigma_v} e^{-\frac{1}{2}\left[\frac{u^2}{\sigma_v^2} - \frac{2u}{\sigma_v}\left(\frac{\varepsilon}{\sigma_v} - \frac{1}{\frac{\sigma_u}{\sigma_v}}\right) + \frac{\varepsilon^2}{\sigma_v^2}\right]} \tag{9}$$

Since, $\lambda = \frac{\sigma_u}{\sigma_v}$

$$f(u,\varepsilon) = \frac{1}{\sqrt{2\pi}\ \sigma_u \sigma_v} e^{-\frac{1}{2}\left[\frac{u^2}{\sigma_v^2} - \frac{2u}{\sigma_v}\left(\frac{\varepsilon}{\sigma_v} - \frac{1}{\lambda}\right) + \frac{\varepsilon^2}{\sigma_v^2}\right]} \tag{10}$$

$$f(u,\varepsilon) = \frac{1}{\sqrt{2\pi}\,\sigma_u \sigma_v} e^{-\frac{1}{2}\left[\left(\frac{u}{\sigma_v} - (\frac{\varepsilon}{\sigma_v} - \frac{1}{\lambda})\right)^2 - \left(\frac{\varepsilon}{\sigma_v} - \frac{1}{\lambda}\right)^2 + \frac{\varepsilon^2}{\sigma_v^2}\right]} \tag{11}$$

$$f(u,\varepsilon) = \frac{1}{\sqrt{2\pi}\,\sigma_u \sigma_v} e^{-\frac{1}{2}\left(\frac{u}{\sigma_v} - (\frac{\varepsilon}{\sigma_v} - \frac{1}{\lambda})\right)^2} e^{-\frac{1}{2}\left(-\frac{\varepsilon^2}{\sigma_v^2} + \frac{2\varepsilon}{\sigma_v \lambda} - \frac{1}{\lambda^2} + \frac{\varepsilon^2}{\sigma_v^2}\right)} \tag{12}$$

$$f(u,\varepsilon) = \frac{1}{\sqrt{2\pi}\,\sigma_u \sigma_v} e^{-\frac{1}{2}\left(\frac{u}{\sigma_v} - (\frac{\varepsilon}{\sigma_v} - \frac{1}{\lambda})\right)^2} e^{\frac{1}{2}\left(\frac{1}{\lambda^2} - \frac{2\varepsilon}{\sigma_v \lambda}\right)} \tag{13}$$

The marginal density function of ε is given by

$$f(\varepsilon) = \int_0^\infty f(u,\varepsilon)\, du.$$

$$f(\varepsilon) = \frac{1}{\sqrt{2\pi}\,\sigma_u \sigma_v} e^{\frac{1}{2}\left(\frac{1}{\lambda^2} - \frac{2\varepsilon}{\sigma_v \lambda}\right)} \int_0^\infty e^{-\frac{1}{2}\left(\frac{u}{\sigma_v} - (\frac{\varepsilon}{\sigma_v} - \frac{1}{\lambda})\right)^2} du \tag{14}$$

Define $\frac{u}{\sigma_v} - (\frac{\varepsilon}{\sigma_v} - \frac{1}{\lambda}) = s$; $du = \sigma_v\, ds$ (15)

When $u \to 0$, $s \to \frac{1}{\lambda} - \frac{\varepsilon}{\sigma_v}$; $u \to \infty, s \to \infty$

$$f(\varepsilon) = \frac{1}{\sqrt{2\pi}\,\sigma_u \sigma_v} e^{\frac{1}{2}\left(\frac{1}{\lambda^2} - \frac{2\varepsilon}{\sigma_v \lambda}\right)} \int_{\frac{1}{\lambda} - \frac{\varepsilon}{\sigma_v}}^\infty e^{-\frac{s^2}{2}} \sigma_v ds \tag{16}$$

$$f(\varepsilon) = \frac{1}{\sigma_u} e^{\frac{1}{2}\left(\frac{1}{\lambda^2} - \frac{2\varepsilon}{\sigma_v \lambda}\right)} \frac{1}{\sqrt{2\pi}} \int_{\frac{1}{\lambda} - \frac{\varepsilon}{\sigma_v}}^\infty e^{-\frac{s^2}{2}} \sigma_v ds \tag{17}$$

$$f(\varepsilon) = \frac{1}{\sigma_u} e^{\frac{1}{2}\left(\frac{1}{\lambda^2} - \frac{2\varepsilon}{\sigma_v \lambda}\right)} \left[1 - \Phi\left(\frac{1}{\lambda} - \frac{\varepsilon}{\sigma_v}\right)\right] \tag{18}$$

3. MEAN AND VARIANCE

The marginal density function, which is asymmetrically distributed with mean and variance is given by

$$Mean = E(\varepsilon) = E(v+u) = E(u) = \int_0^\infty u\; f(u)\, du$$

$$E(\varepsilon) = E(u) = \int_0^\infty u\, \frac{1}{\sigma_u}\, e^{-\frac{u}{\sigma_u}}\, du \tag{19}$$

Define, $\frac{u}{\sigma_u} = s$; $du = \sigma_u\; ds$

s varies from 0 to ∞

Therefore, $E(u) = \frac{1}{\sigma_u} \int_0^\infty \sigma_u\; s\, e^{-s} \sigma_u\; ds.$ (20)

$$E(u) = \sigma_u \left[s e^{-s}(-1) - e^{-s}\right]_0^\infty \tag{21}$$

$$E(u) = \sigma_u (1) = \sigma_u \tag{22}$$

To find Variance, $v(\varepsilon) = v(v) + v(u)$

$$V(u) = E(u^2) - \left(E(u)\right)^2 \tag{23}$$

$$E(u^2) = \int_0^\infty u^2 f(u) du$$

$$= \int_0^\infty u^2 \frac{1}{\sigma_u} e^{-\frac{u}{\sigma_u}} du \tag{24}$$

$$Again\ let, \frac{u}{\sigma_u} = s\ ; du = \sigma_u\ ds\ ; u^2 = \sigma_u^2 s^2$$

s varies from 0 to ∞

Hence, $$E(u^2) = \frac{1}{\sigma_u} \int_0^\infty \sigma_u^2\ s^2\ e^{-s} \sigma_u\ ds \tag{25}$$

$$E(u^2) = \frac{\sigma_u^3}{\sigma_u} \int_0^\infty s^2\ e^{-s}\ ds \tag{26}$$

$$E(u^2) = \sigma_u^2 \left[s^2(-e^{-s}) - 2s(e^{-s}) + 2s(-e^{-s})\right]_0^\infty \tag{27}$$

$$E(u^2) = \sigma_u^2 [0 - 2(0-1)] = 2\sigma_u^2 \tag{28}$$

$$v(u) = 2\sigma_u^2 - \sigma_u^2 = \sigma_u^2\ and\ v(v) = \sigma_v^2 \tag{29}$$

$$v(\varepsilon) = \sigma_v^2 + \sigma_u^2 \tag{30}$$

The product of the density function of the individual observations is the likelihood function of the sample, which is given as,

$$L(sample) = \prod_{i=1}^{i=k} f(\varepsilon_i)$$

The log likelihood equation for a sample of K producers is

$$In\ L = -\frac{k}{2}\left(In\ \sigma_u^2\right) + \sum_{i=1}^{k} In\left[1 - \Phi\left(\frac{1}{\lambda} - \frac{\varepsilon_i}{\sigma_v}\right)\right] + \frac{k}{2\lambda^2} - \sum_{i=1}^{k} \frac{\varepsilon_i}{\sigma_v \lambda} \tag{31}$$

Let $\varepsilon_i = y_i - \beta'\ x_i$

$$In\ L = -\frac{K}{2}\left(In\ \sigma_u^2\right) + \sum_{i=1}^{K} In\left[1 - \Phi\left(\frac{1}{\lambda} - \frac{(y_i - \beta'\ x_i)}{\sigma_v}\right)\right] + \frac{K}{2\lambda^2} - \sum_{i=1}^{K} \frac{(y_i - \beta'\ x_i)}{\sigma_v \lambda} \tag{32}$$

Using loglikelihood function, cost efficiency can be measured after the parameter estimation.

4. ESTIMATION OF THE PARAMETERS OF THE NORMAL-EXPONENTIAL STOCHASTIC COST FRONTIER MODEL

Parameters $\sigma_u^2, \sigma_v^2, \lambda^2, \beta$ can be estimated using the first order conditions of the maximization of log-likelihood function of (32) as follows.

$$E^* \sigma_u^2 = \frac{\partial\ InL}{\partial \sigma_u^2} = -\frac{K}{2} \frac{1}{\sigma_u^2} = 0 \tag{33}$$

$$E^* \sigma_v^2 = \frac{\partial\ InL}{\partial \sigma_v^2} = \frac{1}{2\lambda} \sum_{i=1}^{K} \frac{y_i - \beta' x_i}{\sigma_v^3} - \frac{1}{2\lambda \sigma_v^3} \sum_{i=1}^{K} \frac{\phi\left(\frac{1}{\lambda} - \frac{(y_i - \beta' x_i)}{\sigma_v}\right)}{1 - \Phi\left(\frac{1}{\lambda} - \frac{(y_i - \beta' x_i)}{\sigma_v}\right)} \left(y_i - \beta' x_i\right) = 0 \tag{34}$$

$$E^*\beta = \frac{\partial\, InL}{\partial \beta} = \sum_{i=1}^{K} \frac{\phi\left(\frac{1}{\lambda} - \frac{(y_i - \beta' x_i)}{\sigma_v}\right)}{1-\Phi\left(\frac{1}{\lambda} - \frac{(y_i - \beta' x_i)}{\sigma_v}\right)}\left(\frac{x_i}{\sigma_v}\right) + \sum_{i=1}^{K} \frac{x_i}{\sigma_v \lambda} = 0 \qquad (35)$$

$$E^*\lambda^2 = \frac{\partial\, InL}{\partial \lambda^2} = -\frac{k}{2\lambda^4} + \frac{1}{2\lambda^3}\sum_{i=1}^{K} \frac{\phi\left(\frac{1}{\lambda} - \frac{(y_i - \beta' x_i)}{\sigma_v}\right)}{1-\Phi\left(\frac{1}{\lambda} - \frac{(y_i - \beta' x_i)}{\sigma_v}\right)}\left(y_i - \beta' x_i\right) = 0 \qquad (36)$$

Once the parameters are estimated the inefficiency, u_i can be obtained. In order to extract the information that ε_i contains on u_i can be done using the conditional distribution of u_i given ε_i .(Kumbhakar et al. 2003). The conditional distribution of u_i given ε_i when u_i follows half normal distribution is given by Jondrow *et al.*(1982), same condition can be used when u_i is assumed with exponential distribution.

$$f(u/\varepsilon) = \frac{f(u,\varepsilon)}{f(\varepsilon)}$$

$$f(u/\varepsilon) = \frac{\frac{1}{\sqrt{2\pi}\,\sigma_u\sigma_v} e^{-\frac{1}{2}\left(\frac{u}{\sigma_v} - (\frac{\varepsilon}{\sigma_v} - \frac{1}{\lambda})\right)^2} e^{\frac{1}{2}\left(\frac{1}{\lambda^2} - \frac{2\varepsilon}{\sigma_v\lambda}\right)}}{\frac{1}{\sigma_u} e^{\frac{1}{2}\left(\frac{1}{\lambda^2} - \frac{2\varepsilon}{\sigma_v\lambda}\right)}\left[1-\Phi\left(\frac{1}{\lambda} - \frac{\varepsilon}{\sigma_v}\right)\right]} \qquad (37)$$

$$f(u/\varepsilon) = \frac{1}{\sqrt{2\pi}\,\sigma_v}\left[1-\Phi\left(\frac{1}{\lambda} - \frac{\varepsilon}{\sigma_v}\right)\right]^{-1} e^{-\frac{1}{2}\left(\frac{u}{\sigma_v} - (\frac{\varepsilon}{\sigma_v} - \frac{1}{\lambda})\right)^2} \qquad (38)$$

$$f(u/\varepsilon) = \frac{1}{\sqrt{2\pi}\,\sigma_v}\left[1-\Phi\left(\frac{1}{\frac{\sigma_u}{\sigma_v}} - \frac{\varepsilon}{\sigma_v}\right)\right]^{-1} e^{-\frac{1}{2}\left(\frac{u}{\sigma_v} - (\frac{\varepsilon}{\sigma_v} - \frac{1}{\frac{\sigma_u}{\sigma_v}})\right)^2} \qquad (39)$$

$$f(u/\varepsilon) = \frac{1}{\sqrt{2\pi}\,\sigma_v}\left[1-\Phi\left(\frac{\sigma_v^2}{\sigma_u\sigma_v} - \frac{\varepsilon}{\sigma_v}\right)\right]^{-1} e^{-\frac{1}{2}\left(\frac{u}{\sigma_v} - (\frac{\varepsilon}{\sigma_v} - \frac{\sigma_v^2}{\sigma_u\sigma_v})\right)^2} \qquad (40)$$

$$f(u/\varepsilon) = \frac{1}{\sqrt{2\pi}\,\sigma_v}\left[1-\Phi\left(-\left\{\frac{\varepsilon}{\sigma_v} - \frac{\sigma_v^2}{\sigma_u\sigma_v}\right\}\right)\right]^{-1} e^{-\frac{1}{2\sigma_v}\left(u - (\varepsilon - \frac{\sigma_v^2}{\sigma_u})\right)^2} \qquad (41)$$

Let $\hat{\mu} = \varepsilon - \frac{\sigma_v^2}{\sigma_u}$

$$f(u/\varepsilon) = \frac{1}{\sqrt{2\pi}\,\sigma_v}\left[1-\Phi\left(-\frac{\hat{\mu}}{\sigma_v}\right)\right]^{-1} e^{-\frac{1}{2\sigma_v}\left(u - \hat{\mu}\right)^2} \qquad (42)$$

5. MEASURE OF COST EFFICIENCY OF NORMAL-EXPONENTIAL STOCHASTIC COST FRONTIER MODEL

As $f(u/\varepsilon)$ is distributed as $N^+(\hat{\mu}, \sigma_v^2)$, the mean of this distribution can serve as a point estimator of u_i which is given by

$$E(u/\varepsilon) = \int_0^\infty u \; f(u/\varepsilon) du \tag{43}$$

$$E(u/\varepsilon) = \int_0^\infty u \frac{1}{\sqrt{2\pi}\,\sigma_v}\left[1-\Phi\left(-\frac{\hat{\mu}}{\sigma_v}\right)\right]^{-1} e^{-\frac{1}{2\sigma_v}\left(u-\hat{\mu}\right)^2} du \tag{44}$$

$$E(u/\varepsilon) = \frac{1}{\sqrt{2\pi}\,\sigma_v}\left[1-\Phi\left(-\frac{\hat{\mu}}{\sigma_v}\right)\right]^{-1} \int_0^\infty u\, e^{-\frac{1}{2\sigma_v}\left(u-\hat{\mu}\right)^2} du \tag{45}$$

Let $s = \dfrac{u-\hat{\mu}}{\sigma_v}; u = s\sigma_v + \hat{\mu}; du = \sigma_v ds.$

s varies from $-\dfrac{\hat{\mu}}{\sigma_v}$ to ∞

$$E(u/\varepsilon) = \frac{1}{\sqrt{2\pi}\,\sigma_v}\left[1-\Phi\left(-\frac{\hat{\mu}}{\sigma_v}\right)\right]^{-1} \int_{-\frac{\hat{\mu}}{\sigma_v}}^{\infty} \left(s\sigma_v + \hat{\mu}\right) e^{-\frac{s^2}{2}} \sigma_v \, ds \tag{46}$$

$$E(u/\varepsilon) = \frac{1}{\sqrt{2\pi}}\left[1-\Phi\left(-\frac{\hat{\mu}}{\sigma_v}\right)\right]^{-1} \left\{\sigma_v \int_{-\frac{\hat{\mu}}{\sigma_v}}^{\infty} s\, e^{-\frac{s^2}{2}} ds + \hat{\mu} \int_{-\frac{\hat{\mu}}{\sigma_v}}^{\infty} e^{-\frac{s^2}{2}} ds \right\} \tag{47}$$

$$E(u/\varepsilon) = \frac{1}{\sqrt{2\pi}}\left[1-\Phi\left(-\frac{\hat{\mu}}{\sigma_v}\right)\right]^{-1} \sigma_v \left(e^{-\frac{s^2}{2}}(-1)\right)_{\frac{\hat{\mu}}{\sigma_v}}^{\infty} + \hat{\mu}\left[1-\Phi\left(-\frac{\hat{\mu}}{\sigma_v}\right)\right]^{-1} \frac{1}{\sqrt{2\pi}} \int_{-\frac{\hat{\mu}}{\sigma_v}}^{\infty} e^{-\frac{s^2}{2}} ds \tag{48}$$

$$E(u/\varepsilon) = \left[1-\Phi\left(-\frac{\hat{\mu}}{\sigma_v}\right)\right]^{-1} \sigma_v \frac{1}{\sqrt{2\pi}} e^{-\frac{\hat{\mu}^2}{2\sigma_v^2}} + \hat{\mu}\left[1-\Phi\left(-\frac{\hat{\mu}}{\sigma_v}\right)\right]^{-1} \frac{1}{\sqrt{2\pi}} \int_{-\frac{\hat{\mu}}{\sigma_v}}^{\infty} e^{-\frac{s^2}{2}} ds \tag{49}$$

$$E(u/\varepsilon) = \left[1-\Phi\left(-\frac{\hat{\mu}}{\sigma_v}\right)\right]^{-1} \sigma_v\, \phi\left(\frac{\hat{\mu}}{\sigma_v}\right) + \hat{\mu}\left[1-\Phi\left(-\frac{\hat{\mu}}{\sigma_v}\right)\right]^{-1}\left[1-\Phi\left(-\frac{\hat{\mu}}{\sigma_v}\right)\right] \tag{50}$$

$$E(u/\varepsilon) = \hat{\mu} + \frac{\sigma_v\, \phi\left(\frac{\hat{\mu}}{\sigma_v}\right)}{\left[1-\Phi\left(-\frac{\hat{\mu}}{\sigma_v}\right)\right]} \tag{51}$$

6. CONCLUSION

Once the marginal density function of ε_i is calculated, using log likelihood functions, $\sigma_u^2, \sigma_v^2, \lambda^2, \beta$ are estimated. The cost efficiency of each producer can be obtained from $CE_i = \left(e^{-\widehat{u_i}}\right)$,where $\widehat{u_i} = \mathrm{E}(u_i/\varepsilon_i)$. If an organisation uses its resources allocatively and technically efficient then it can be said that it has achieved cost efficiency. We may apply the models to real data sets to get the cost efficiencies for each producers. Similarly, we can estimate the parameters for other distributions and derive cost efficiency. According to the value of the loglikelihood function of each model, we can conclude which model is better. Analysis of the collected data can be done using LIMDEP software.

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