International Journal of Mathematical Archive-8(7), 2017, 230-233
Available online through www.ijma.info ISSN 2229 – 5046

GEOMETRIC ARITHMETIC STATUS INDEX Of GRAPHS

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(Received On: 29-06-17; Revised & Accepted On: 24-07-17)

ABSTRACT

The status of a vertex u in a connected graph G, denoted by $\sigma(u)$ is defined as the sum of the distance between u and all other vertices of a graph G . In this paper we have defined geometric arithmetic status (GAS) index of a graph and obtained the bounds for it. Further geometric arithmetic status index of some graphs are obtained. Also the regression analysis of the boiling point of Paraffin with GAS index is carried out.

Keywords: *Distance, status, diameter, geometric arithmetic status index.*

AMS Subject Classification: *05C12. 05C90.*

1. INTRODUCTION

In theoretical chemistry, the graph theoretic models can be used to study the properties of molecules. One of the known graph parameter is the Wiener index which was used to study the chemical properties of paraffin's [3]. Many distance based indices of a graph, such as status connectivity indices [4], Zagreb indices [7, 8], Wiener index [6] and Geometric arithmetic index [1] have been appeared in the literature. In this paper we introduce and study the new index called geometric arithmetic status (GAS) index. Let G be a connected graph with n vertices and m edges. Let $V(G)$ and $E(G)$ be its vertex and edge sets, respectively. The *distance* between the vertex u and v is the length of the shortest path joining u and v and is denoted by $d_G(u,v)$
.
The *status* of a vertex $u \in V(G)$ denoted by $\sigma_G(u)$ is defined as [2],

$$\sigma_G(u) = \sum_{v\in V(G)} d(u,v).$$

The Wiener index $W(G)$ of a connected graph G is defined as [6].

$$W(G) = \sum_{\{u,v\}\subseteq V(G)} d_G(u,v) = \frac{1}{2}\sum_{v\in V(G)} \sigma_G(u).$$

In [1], Vukiĉeviĉ *et al.* defined a new topological index "geometric arithmetic index" of a graph G denoted by $GA(G)$ and is defined by,

$$GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{d_u\, d_u}}{d_u + d_u}$$

Inspired by this definition, we define here geometric arithmetic status index of a connected graph G as,

$$GAS(G) = \sum_{uv\in E(G)} \frac{2\sqrt{\sigma_u\, \sigma_u}}{\sigma_u + \sigma_u}.$$

2. RESULTS

In this paper, we obtain the bounds for the geometric arithmetic status index. Also we obtain the geometric arithmetic status index of some graphs. Further the correlation between the boiling point of paraffin's and geometric arithmetic status index of the corresponding molecular graph is studied.

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3. BOUNDS FOR THE GEOMETRIC ARITHMETIC STATUS INDEX

Theorem 1: Let G be a connected graph with n vertices and let $diam(G) = D$ Then,

$$\sum_{uv\in E(G)} \frac{2\sqrt{(D(n-1)-(D-1)d(u))(D(n-1)-(D-1)d(v))}}{2D(n-1)-(D-1)[d(u)+d(v)]} \le GAS(G) \le \sum_{uv\in E(G)} \frac{2\sqrt{(2n-2-d(u))(2n-2-d(v))}}{4n-4-[d(u)+d(v)]} \quad (1)$$

Equality holds on both sides if and only if $diam(G) \le 2$.

Proof: Lower bound: For any vertex u of G there are $d(u)$ vertices which are at distance 1 from u and the remaining $n-1-d(u)$ vertices are at distance at most D Therefore,

$$\sigma(u) \le d(u) + D\big(n-1-d(u)\big) = D(n-1)-(D-1)d(u).$$

Therefore,

$$\sigma(u)+\sigma(v) \le 2D(n-1)-(D-1)[d(u)+d(v)]$$

Therefore,

$$GAS(G) = \sum_{uv\in E(G)} \frac{2\sqrt{\sigma_u \sigma_u}}{\sigma_u+\sigma_u} \ge \sum_{uv\in E(G)} \frac{2\sqrt{(D(n-1)-(D-1)d(u))(D(n-1)-(D-1)d(v))}}{2D(n-1)-(D-1)[d(u)+d(v)]}$$

Upper bound: For any vertex u of G, there are $d(u)$ vertices which are at distance 1 from u and the remaining $n-1-d(u)$ vertices are at distance at least 2 Therefore,

$$\sigma(u) \ge d(u) + 2\big(n-1-d(u)\big) = 2n-2-d(u)$$

Therefore,

$$\sigma(u)+\sigma(v) \ge 4n-4-[d(u)+d(v)]$$

Therefore,

$$GAS(G) = \sum_{uv\in E(G)} \frac{2\sqrt{\sigma_u \sigma_u}}{\sigma_u+\sigma_u} \le \sum_{uv\in E(G)} \frac{2\sqrt{(2n-2-d(u))(2n-2-d(v))}}{4n-4-[d(u)+d(v)]}$$

For equality: If the diameter D is 1 or 2 then the equality holds.

4. GEOMETRIC ARITHMETIC STATUS INDEX OF SOME STANDARD GRAPHS

Proposition 1: For a complete graph K_n on n vertices, $GAS(K_n) = \frac{n(n-1)}{2}$.

Proof: For any vertex u in K_n, we have, $\sigma(u) = n-1$. Therefore by definition of geometric arithmetic status index, we get

$$GAS(K_n) = \frac{n(n-1)}{2}.$$

Proposition 2: For a complete bipartite graph $K_{p,q}$,

$$GAS(K_{p,q}) = \frac{2pq\sqrt{2(p^2+q^2)-6(p+q)+5pq+4}}{3(p+q)-4}.$$

Proof: The vertex set $V(K_{p,q})$ can be partitioned into two independent sets V_1 and V_2 such that, the vertex $u \in V_1$ and $v \in V_2$ for all edge uv of $K_{p,q}$. Therefore $d(u) = q$ and $d(v) = p$. And the graph $K_{p,q}$ has $n = p+q$ vertices and $m = pq$ edges. Also $diam(K_{p,q}) \le 2$. Therefore by equality part of Theorem 1, we get

$$GAS(K_{p,q}) = \frac{2pq\sqrt{[2(p+q)-2-q][2(p+q)-2-p]}}{3(p+q)-4}.$$

$$GAS(K_{p,q}) = \frac{2pq\sqrt{2(p^2+q^2)-6(p+q)+5pq+4}}{3(p+q)-4}.$$

Proposition 3: For a cycle C_n on $n \ge 3$ vertices, $GAS(C_n) = n$.

Proof: If n is even number, then for any vertex u of C_n,

$$\sigma(u) = 2\left[1+2+\cdots+\frac{n-1}{2}\right]+\frac{n}{2} = \frac{n^2}{4}.$$

Therefore

$$GAS(C_n) = \sum_{uv\in E(G)} \frac{2\sqrt{\sigma_u\,\sigma_u}}{\sigma_u+\sigma_u} = \frac{2n\sqrt{\frac{n^4}{16}}}{\frac{n^2}{4}+\frac{n^2}{4}} = n$$

If n is odd number, then for any vertex u of C_n,

$$\sigma(u) = 2\left[1+2+\cdots+\frac{n-1}{2}\right] = \frac{n^2-1}{4}.$$

Therefore

$$GAS(C_n) = \sum_{uv\in E(G)} \frac{2\sqrt{\sigma_u\,\sigma_u}}{\sigma_u+\sigma_u} = \frac{2n\sqrt{\frac{(n^2-1)^2}{4}}}{\frac{n^2-1}{4}+\frac{n^2-1}{4}} = n.$$

A *wheel* W_{n+1} is a graph obtained from the cycle C_n, $n \geq 3$ by adding a new vertex and making it adjacent to all the vertices of C_n. The degree of a central vertex of W_{n+1} is n and the degree of all other vertices is 3.

Proposition 4: For a wheel graph W_{n+1}, $n \geq 3$,

$$GAS(W_{n+1}) = \frac{n(3n-7)+2n\sqrt{2n^2-9n+10.}}{3n-7}$$

Proof: Here we partition the edge set $E(W_{n+1})$ into two sets E_1 and E_2, where $E_1 = \{uv|d(u) = n \; and \; d(v) = 3\}$ and $E_2 = \{uv|d(u) = 3 \; and \; d(v) = 3\}$. Also $|E_1| = n$ and $|E_2| = n$ and $diam(W_{n+1}) = 2$. Therefore by equality part of Theorem 1, we get

$$GAS(W_{n+1}) = \sum_{uv\in E_1} \frac{\sqrt{[2n-2-d(u)][2n-2-d(v)]}}{4n-4-[d(u)+d(v)]} + \sum_{uv\in E_2} \frac{\sqrt{[2n-2-d(u)][2n-2-d(v)]}}{4n-4-[d(u)+d(v)]}$$

$$GAS(W_{n+1}) = \frac{2n\sqrt{[2n-2-n][2n-2-3]}}{4n-4-[n+3]} + \frac{2n\sqrt{[2n-2-3][2n-2-3]}}{4n-4-[3+3]}$$

$$GAS(W_{n+1}) = \frac{n(3n-7)+2n\sqrt{2n^2-9n+10}}{3n-7}.$$

5. CORRELATION BETWEEN GEOMETRIC ARITHMETIC STATUS INDEX AND BOILING POINT OF PARAFFIN'S

In this section we study the correlation between the boiling point (BP) of the paraffin hydrocarbons and the geometric arithmetic status index of the corresponding molecular graphs.

Table-1: Boiling point and geometric arithmetic status index of paraffin's

Paraffin	**Boiling point (BP)^0 C**	**Geometric arithmetic status index (GAS)**
3-methylpentane	62.9	4.924
2,2-dimethylbutane	50	4.872
2,3 dimethylbutane	57.9	4.872
2,2 dimethylpentane	79	5.903
3,3 dimethylpentane	86	5.880
n-octane	125	6.968
3-methylheptane	118	6.946
3-ethylhexane	118	6.930
2,2-dimethylhexane	107	6.926
2,4 dimethylhexane	108	6.930
2-methyl, 3-ethylpentane	116	6.908
2,2,4-trimethylpentane	99	6.902
n-dodecane	216.2	10.974

Using the data of Table 1, the scatter plot between the boiling point (BP) and geometric arithmetic status (GAS) index of paraffin's is depicted in Figure 1.

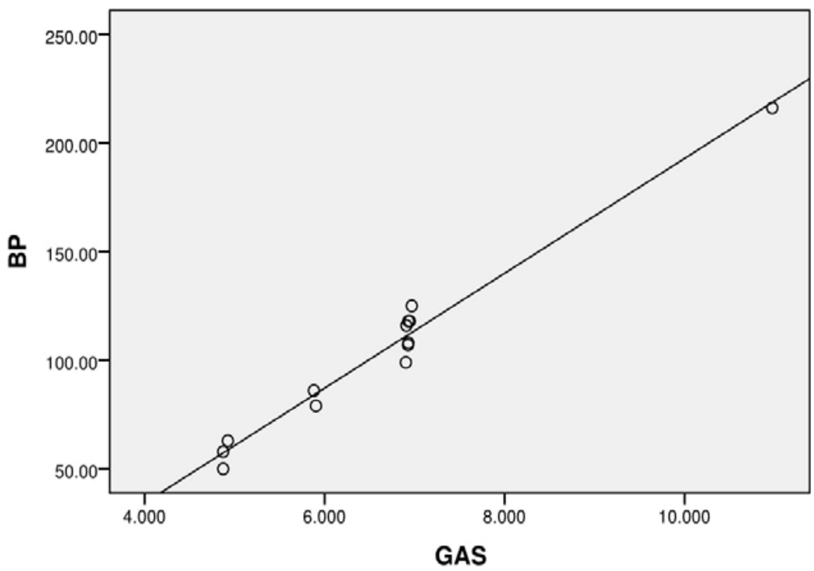

Figure-1: Regression line between the boiling point and geometric arithmetic status index of paraffin's.

The linear regression between BP and GAS is $BP = -71.473(\pm 8.715) + 26.440(\pm 1.285)GAS$.

The correlation of the boiling point of paraffin's with the geometric arithmetic status index is good (R=0.987).

6. CONCLUSION

Recently H S. Ramane et al. defined harmonic status index of graphs [5]. Motivated by their work, in this paper we have introduced geometric arithmetic status index of graphs. And computed this index for some specific graphs. Also the bounds of this index is reported. Further the regression analysis of the boiling points of paraffin's with geometric arithmetic status (GAS) index have been carried out. The GAS index has good correlation (0.987) compared to the harmonic status index (0.9444) with the boiling point of paraffin's.

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Source of support: Nil, Conflict of interest: None Declared.