

RELAXED SKOLEM MEAN LABELING FOR FIVE STAR

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ABSTRACT

In this paper, we prove $1 \leq m < n$, the five star $K_{1,1} \cup K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a Relaxed skolem mean graph if $|m-n| \leq 9$ for $n = 1,2,3,\dots$ and $m \leq n \leq m+9$

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1. INTRODUCTION

All graphs in this chapter are finite, simple and undirected. Terms not defined here are used in the sense of Harry [10]. In [5], if $\ell \leq m < n$, the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a relaxed skolem mean graph if $|m-n| \leq 6+\ell$ for $\ell = 1, 2, 3, \dots$; $m = 1, 2, 3, \dots, \ell+m \leq n \leq \ell+m+6$. Also, if $\ell \leq m < n$, the four star $k_{1,\ell} \cup k_{1,\ell} \cup k_{1,m} \cup k_{1,n}$ is a relaxed skolem mean graph if $|m-n| \leq 6+2\ell$ for $\ell = 2, 3, 4, \dots$; $m = 2, 3, 4, \dots$ and $2\ell+m \leq n \leq 2\ell+m+6$. In [4], the necessary condition for a graph to be relaxed skolem mean is that $p \geq q$.

2. RELAXED SKOLEM MEAN LABELING

Definition 2.1: The five stars is the disjoint union of $K_{1,a}, K_{1,b}, K_{1,c}, K_{1,d}, K_{1,e}$ then it is denoted by $K_{1,a} \cup K_{1,b} \cup K_{1,c} \cup K_{1,d} \cup K_{1,e}$

Definition 2.2: A graph $G=(V, E)$ with p vertices and q edges is said to be a relaxed skolem mean graph if there exists a function f from the vertex set of G to $\{1, 2, 3, \dots, p+1\}$ such that the induced map f^* from the edge set of G to $\{2, 3, 4, \dots, p+1\}$ defined by

$$f^*(e=uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd, then} \end{cases}$$

the resulting edges get distinct labels from the set $\{2, 3, 4, \dots, p+1\}$.

Note 2.3: In a Relaxed skolem mean graph, $p \geq q$.

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Theorem 2.4: If $1 \leq m < n$ five star $K_{1,a} \cup K_{1,b} \cup K_{1,c} \cup K_{1,d} \cup K_{1,e}$ is a relaxed skolem mean graph if $|m-n| \leq 9$ for $n = 1, 2, 3, 4, \dots$ and $m \leq n \leq m+9$.

Proof: Let $G = K_{1,a} \cup K_{1,b} \cup K_{1,c} \cup K_{1,d} \cup K_{1,e}$

Without loss of generality assume that $1 \leq m < n$.

Hence $|m-n| \leq 9$ implies $n-m \leq 9$ and it means $1 \leq m \leq n \leq m+9$.

There are ten cases viz : $n = m + 9, n = m + 8, n = m + 7, n = m + 6, n = m + 5, n = m + 4, n = m + 3, n = m + 2, n = m + 1$ and $n = m$.

Let us prove in each of the cases the graph G is a Relaxed skolem mean graph.

Case-(a): When $n = m + 9$

We have to prove that G is a relaxed skolem mean graph $n = m + 9$

$$\begin{aligned} V(G) &= \{u, v, w, x, y\} \cup \{u_i : 1 \leq i \leq \ell\}, \{v\} \cup \{v_j : 1 \leq j \leq \ell\}, \{w\} \cup \{w_k : 1 \leq k \leq \ell\} \\ &\quad \{x\} \cup \{x_h : 1 \leq h \leq m\}, \{y\} \cup \{y_s : 1 \leq s \leq n\} \text{ and} \\ E(G) &= \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq \ell\} \cup \{xx_h : 1 \leq h \leq m\} \cup \\ &\quad \{yy_s : 1 \leq s \leq n\}. \end{aligned}$$

Then G has $m+n+8$ nodes and $m+n+3$ links.

The required node labeling $f : V(G) \rightarrow \{1, 2, 3, 4, \dots, m+n+8\}$ is defined as follows

$$\begin{aligned} f(u) &= 1 \quad f(v) = 6; \quad f(y) = 2; \quad f(w) = 4; \quad f(x) = m+n+8; \\ f(u_1) &= 8 \\ f(v_1) &= 10 \\ f(y_1) &= 12 \\ f(w_k) &= 2k+12 \quad \text{for } 1 \leq k \leq m \\ f(x_h) &= 2h+1 \quad \text{for } 1 \leq h \leq n-2 \\ f(x_{n-1}) &= m+n+7 \\ f(x_n) &= m+n+9 \end{aligned}$$

The corresponding link labels are as follows:

$$\begin{aligned} \text{The link label of } uu_1 &\text{ is } 5; \quad vv_1 \text{ is } 8; \quad yy_1 \text{ is } 7; \quad ww_k \text{ is } k+9 \quad 1 \leq k \leq m; \\ xx_h &\text{ is } \frac{m+n+2h+9}{2} \quad 1 \leq h \leq n-2; \quad xx_{n-1} \text{ is } m+n+8; \quad xx_n \text{ is} \\ &\quad m+n+9. \end{aligned}$$

Hence the induced link labels are distinct.

Hence the graph G is Relaxed skolem mean graph.

Case-(b): When $n = m + 8$

We've to prove that G is a relaxed skolem mean graph $n = m + 8$.

$$\begin{aligned} V(G) &= \{u, v, w, x, y\} \cup \{u_i : 1 \leq i \leq \ell\}, \{v\} \cup \{v_j : 1 \leq j \leq \ell\}, \{w\} \cup \{w_k : 1 \leq k \leq \ell\} \\ &\quad \{x\} \cup \{x_h : 1 \leq h \leq m\}, \{y\} \cup \{y_s : 1 \leq s \leq n\} \text{ and} \end{aligned}$$

$$E(G) = \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq \ell\} \cup \{xx_h : 1 \leq h \leq m\} \cup \{yy_s : 1 \leq s \leq n\}.$$

Then G has $m+n+8$ nodes and $m+n+3$ links.

The required node labeling $f : V(G) \rightarrow \{1, 2, 3, 4, \dots, m+n+8\}$ is defined as follows

$$\begin{aligned} f(u) &= 1 \quad f(v) = 2; \quad f(y) = 3; \quad f(w) = 5; \quad f(x) = m+n+8; \\ f(u_1) &= 9 \\ f(v_1) &= 11 \\ f(y_1) &= 13 \\ f(w_k) &= 2k+13 \quad \text{for } 1 \leq k \leq m \\ f(x_h) &= 2h+2 \quad \text{for } 1 \leq h \leq n-2 \\ f(x_{n-1}) &= m+n+7 \\ f(x_n) &= m+n+9 \end{aligned}$$

The corresponding link labels are as follows:

$$\begin{aligned} \text{The link labels of } uu_1 \text{ is } 5; \quad vv_1 \text{ is } 7; \quad yy_1 \text{ is } 8; \quad ww_k \text{ is } k+9 \quad 1 \leq k \leq m; \\ xx_h \text{ is } \frac{m+n+2h+10}{2} \quad 1 \leq h \leq n-2; \quad xx_{n-1} \text{ is } m+n+8; \quad xx_n \text{ is } \\ m+n+9. \end{aligned}$$

Hence the induced link labels are distinct.

Hence the graph G is Relaxed skolem mean graph.

Case-(c): When $n = m + 7$

We've to prove that G is a relaxed skolem mean graph $n = m + 7$.

$$V(G) = \{u, v, w, x, y\} \cup \{u_i : 1 \leq i \leq \ell\}, \{v\} \cup \{v_j : 1 \leq j \leq \ell\}, \{w\} \cup \{w_k : 1 \leq k \leq \ell\} \\ \{x\} \cup \{x_h : 1 \leq h \leq m\}, \{y\} \cup \{y_s : 1 \leq s \leq n\} \text{ and}$$

$$E(G) = \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq \ell\} \cup \{xx_h : 1 \leq h \leq m\} \cup \{yy_s : 1 \leq s \leq n\}.$$

Then G has $m+n+8$ nodes and $m+n+3$ links.

The required node labeling $f : V(G) \rightarrow \{1, 2, 3, 4, \dots, m+n+8\}$ is defined as follows

$$\begin{aligned} f(u) &= 1 \quad f(v) = 2; \quad f(y) = 4; \quad f(w) = 6; \quad f(x) = m+n+8; \\ f(u_1) &= 8 \\ f(v_1) &= 10 \\ f(y_1) &= 12 \\ f(w_k) &= 2k+12 \quad \text{for } 1 \leq k \leq m \\ f(x_h) &= 2h+3 \quad \text{for } 1 \leq h \leq n-2 \\ f(x_{n-1}) &= m+n+7 \\ f(x_n) &= m+n+9 \end{aligned}$$

The corresponding link labels are as follows:

$$\begin{aligned} & \text{The link labels of } uu_1 \text{ is } 5; vv_1 \text{ is } 6; yy_1 \text{ is } 8; ww_k \text{ is } k+9 \quad 1 \leq k \leq m; \\ & xx_h \text{ is } \frac{m+n+2h+11}{2} \quad 1 \leq h \leq n-2; xx_{n-1} \text{ is } m+n+8; xx_n \text{ is } \\ & m+n+9. \end{aligned}$$

Hence the induced link labels are distinct.

Hence the graph G is Relaxed skolem mean graph.

Case-(d): When $n = m + 6$

We've to prove that G is a relaxed skolem mean graph $n = m + 6$.

$$\begin{aligned} V(G) &= \{u, v, w, x, y\} \cup \{u_i : 1 \leq i \leq \ell\}, \{v\} \cup \{v_j : 1 \leq j \leq \ell\}, \{w\} \cup \{w_k : 1 \leq k \leq \ell\} \\ & \quad \{x\} \cup \{x_h : 1 \leq h \leq m\}, \{y\} \cup \{y_s : 1 \leq s \leq n\} \text{ and} \\ E(G) &= \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq \ell\} \cup \{xx_h : 1 \leq h \leq m\} \cup \\ & \quad \{yy_s : 1 \leq s \leq n\}. \end{aligned}$$

Then G has $m+n+8$ nodes and $m+n+3$ links.

The required node labeling $f: V(G) \rightarrow \{1, 2, 3, 4, \dots, m+n+8\}$ is defined as follows

$$\begin{aligned} f(u) &= 1 \quad f(v) = 2; \quad f(y) = 3; \quad f(w) = 5; \quad f(x) = m+n+8; \\ f(u_1) &= 7 \\ f(v_1) &= 9 \\ f(y_1) &= 11 \\ f(w_k) &= 2k+11 \quad \text{for } 1 \leq k \leq m \\ f(x_h) &= 2h+4 \quad \text{for } 1 \leq h \leq n-2 \\ f(x_{n-1}) &= m+n+7 \\ f(x_n) &= m+n+9 \end{aligned}$$

The corresponding link labels are as follows:

$$\begin{aligned} & \text{The link label of } uu_1 \text{ is } 4; vv_1 \text{ is } 6; yy_1 \text{ is } 7; ww_k \text{ is } k+8 \quad 1 \leq k \leq m; \\ & xx_h \text{ is } \frac{m+n+2h+12}{2} \quad 1 \leq h \leq n-2; xx_{n-1} \text{ is } m+n+8; xx_n \text{ is } \\ & m+n+9. \end{aligned}$$

Hence the induced link labels are distinct.

Hence the graph G is Relaxed skolem mean graph.

Case-(e): When $n = m + 5$

We've to prove that G is a relaxed skolem mean graph $n = m + 5$.

$$\begin{aligned} V(G) &= \{u, v, w, x, y\} \cup \{u_i : 1 \leq i \leq \ell\}, \{v\} \cup \{v_j : 1 \leq j \leq \ell\}, \{w\} \cup \{w_k : 1 \leq k \leq \ell\} \\ & \quad \{x\} \cup \{x_h : 1 \leq h \leq m\}, \{y\} \cup \{y_s : 1 \leq s \leq n\} \text{ and} \\ E(G) &= \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq \ell\} \cup \{xx_h : 1 \leq h \leq m\} \cup \\ & \quad \{yy_s : 1 \leq s \leq n\}. \end{aligned}$$

Then G has $m+n+8$ nodes and $m+n+3$ links.

The required node labeling $f: V(G) \rightarrow \{1, 2, 3, 4, \dots, m+n+8\}$ is defined as follows

$$f(u)=1 \quad f(v)=2; \quad f(y)=3; \quad f(w)=5; \quad f(x)=m+n+8;$$

$$f(u_1) = 6$$

$$f(v_1) = 8$$

$$f(y_1) = 10$$

$$f(w_k) = 2k+10 \quad \text{for } 1 \leq k \leq m$$

$$f(x_h) = 2h+5 \quad \text{for } 1 \leq h \leq n-2$$

$$f(x_{n-1}) = m+n+7$$

$$f(x_n) = m+n+9$$

The corresponding link labels are as follows:

The link labels of uu_1 is 4; vv_1 is 5 ; yy_1 is 7; ww_k is $k+8$ $1 \leq k \leq m$;

xx_h is $\frac{m+n+2h+13}{2}$ $1 \leq h \leq n-2$; xx_{n-1} is $m+n+8$; xx_n is

$m+n+9$.

Hence the induced link labels are distinct.

Hence the graph G is Relaxed skolem mean graph.

Case-(f): When $n = m + 4$

We've to prove that G is a relaxed skolem mean graph $n = m + 4$.

$$V(G) = \{u, v, w, x, y\} \cup \{u_i : 1 \leq i \leq \ell\}, \{v\} \cup \{v_j : 1 \leq j \leq \ell\}, \{w\} \cup \{w_k : 1 \leq k \leq \ell\}$$

$$\{x\} \cup \{x_h : 1 \leq h \leq m\}, \{y\} \cup \{y_s : 1 \leq s \leq n\} \text{ and}$$

$$E(G) = \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq \ell\} \cup \{xx_h : 1 \leq h \leq m\} \cup$$

$$\{yy_s : 1 \leq s \leq n\}.$$

Then G has $m+n+8$ nodes and $m+n+3$ links.

The required node labeling $f: V(G) \rightarrow \{1, 2, 3, 4, \dots, m+n+8\}$ is defined as follows

$$f(u)=1 \quad f(v)=2; \quad f(y)=4; \quad f(w)=6; \quad f(x)=m+n+8;$$

$$f(u_1) = 5$$

$$f(v_1) = 7$$

$$f(y_1) = 9$$

$$f(w_k) = 2k+9 \quad \text{for } 1 \leq k \leq m$$

$$f(x_h) = 2h+6 \quad \text{for } 1 \leq h \leq n-2$$

$$f(x_{n-1}) = m+n+7$$

$$f(x_n) = m+n+9$$

The corresponding link labels are as follows:

The link labels of uu_1 is 3; vv_1 is 5 ; yy_1 is 7; ww_k is $k+8$ $1 \leq k \leq m$;

xx_h is $\frac{m+n+2h+14}{2}$ $1 \leq h \leq n-2$; xx_{n-1} is $m+n+8$; xx_n is $m+n+9$.

Hence the induced link labels are distinct.

Hence the graph G is Relaxed skolem mean graph.

Case – (g): When $n = m+3$

We've to prove that G is a relaxed skolem mean graph $n = m+3$.

$$V(G) = \{u, v, w, x, y\} \cup \{u_i : 1 \leq i \leq \ell\}, \{v\} \cup \{v_j : 1 \leq j \leq \ell\}, \{w\} \cup \{w_k : 1 \leq k \leq \ell\}$$

$$\{x\} \cup \{x_h : 1 \leq h \leq m\}, \{y\} \cup \{y_s : 1 \leq s \leq n\} \text{ and}$$

$$E(G) = \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq \ell\} \cup \{xx_h : 1 \leq h \leq m\} \cup$$

$$\{yy_s : 1 \leq s \leq n\}.$$

Then G has $m+n+8$ nodes and $m+n+3$ links.

The required node labeling $f : V(G) \rightarrow \{1, 2, 3, 4, \dots, m+n+8\}$ is defined as follows

$$f(u) = 1; f(v) = 3; f(y) = 5; f(w) = 7; f(x) = m+n+8;$$

$$f(u_1) = 4$$

$$f(v_1) = 6$$

$$f(y_1) = 8$$

$$f(w_k) = 2k+8 \quad \text{for } 1 \leq k \leq m$$

$$f(x_h) = 2h+7 \quad \text{for } 1 \leq h \leq n-2$$

$$f(x_{n-1}) = m+n+7$$

$$f(x_n) = m+n+9$$

The corresponding link labels are as follows:

The link labels of uu_1 is 3; vv_1 is 5 ; yy_1 is 7; ww_k is $k+8$ $1 \leq k \leq m$;

xx_h is $\frac{m+n+2h+15}{2}$ $1 \leq h \leq n-2$; xx_{n-1} is $m+n+8$; xx_n is $m+n+9$.

Hence the induced link labels are distinct.

Hence the graph G is Relaxed skolem mean graph.

Case – (h): When $n = m+2$

We've to prove that G is a relaxed skolem mean graph $n = m+2$.

$$V(G) = \{u, v, w, x, y\} \cup \{u_i : 1 \leq i \leq \ell\}, \{v\} \cup \{v_j : 1 \leq j \leq \ell\}, \{w\} \cup \{w_k : 1 \leq k \leq \ell\}$$

$$\{x\} \cup \{x_h : 1 \leq h \leq m\}, \{y\} \cup \{y_s : 1 \leq s \leq n\} \text{ and}$$

$$E(G) = \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq \ell\} \cup \{xx_h : 1 \leq h \leq m\} \cup$$

$$\{yy_s : 1 \leq s \leq n\}.$$

Then G has $m+n+8$ nodes and $m+n+3$ links.

The required node labeling $f : V(G) \rightarrow \{1, 2, 3, 4, \dots, m+n+8\}$ is defined as follows

$$\begin{aligned} f(u) &= 1 \quad f(v) = 2; \quad f(y) = 4; \quad f(w) = 8; \quad f(x) = m+n+8; \\ f(u_1) &= 3 \\ f(v_1) &= 5 \\ f(y_1) &= 7 \\ f(w_k) &= 2k+7 \quad \text{for } 1 \leq k \leq m \\ f(x_h) &= 2h+8 \quad \text{for } 1 \leq h \leq n-2 \\ f(x_{n-1}) &= m+n+7 \\ f(x_n) &= m+n+9 \end{aligned}$$

The corresponding link labels are as follows:

The link label of uu_1 is 2; vv_1 is 4; yy_1 is 6; ww_k is $k+8$ $1 \leq k \leq m$;
 xx_h is $\frac{m+n+2h+16}{2}$ $1 \leq h \leq n-2$; xx_{n-1} is $m+n+8$; xx_n is $m+n+9$.

Hence the induced link labels are distinct.

Hence the graph G is Relaxed skolem mean graph.

Case-(i): When $n = m + 1$

We've to prove that G is a relaxed skolem mean graph $n = m + 1$.

$$\begin{aligned} V(G) &= \{u, v, w, x, y\} \cup \{u_i : 1 \leq i \leq \ell\}, \{v_j : 1 \leq j \leq \ell\}, \{w_k : 1 \leq k \leq \ell\} \\ &\quad \{x_h : 1 \leq h \leq m\}, \{y_s : 1 \leq s \leq n\} \text{ and} \\ E(G) &= \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq \ell\} \cup \{xx_h : 1 \leq h \leq m\} \cup \\ &\quad \{yy_s : 1 \leq s \leq n\}. \end{aligned}$$

Then G has $m+n+8$ nodes and $m+n+3$ links.

The required node labeling $f : V(G) \rightarrow \{1, 2, 3, 4, \dots, m+n+8\}$ is defined as follows

$$\begin{aligned} f(u) &= 1 \quad f(v) = 5; \quad f(y) = 7; \quad f(w) = 9; \quad f(x) = m+n+8; \\ f(u_1) &= 2 \\ f(v_1) &= 4 \\ f(y_1) &= 6 \\ f(w_k) &= 2k+6 \quad \text{for } 1 \leq k \leq m \\ f(x_h) &= 2h+9 \quad \text{for } 1 \leq h \leq n-2 \\ f(x_{n-1}) &= m+n+7 \\ f(x_n) &= m+n+9 \end{aligned}$$

The corresponding link labels are as follows:

The link label of uu_1 is 2; vv_1 is 5; yy_1 is 7; ww_k is $k+8$ $1 \leq k \leq m$;
 xx_h is $\frac{m+n+2h+17}{2}$ $1 \leq h \leq n-2$; xx_{n-1} is $m+n+8$; xx_n is $m+n+9$.

Hence the induced link labels are distinct.

Hence the graph G is Relaxed skolem mean graph.

Case-(j): When $n = m$

We've to prove that G is a relaxed skolem mean graph $n = m$.

$$V(G) = \{u, v, w, x, y\} \cup \{u_i : 1 \leq i \leq \ell\}, \{v\} \cup \{v_j : 1 \leq j \leq \ell\}, \{w\} \cup \{w_k : 1 \leq k \leq \ell\}$$

$$\{x\} \cup \{x_h : 1 \leq h \leq m\}, \{y\} \cup \{y_s : 1 \leq s \leq n\} \text{ and}$$

$$E(G) = \{uu_i : 1 \leq i \leq \ell\} \cup \{vv_j : 1 \leq j \leq \ell\} \cup \{ww_k : 1 \leq k \leq \ell\} \cup \{xx_h : 1 \leq h \leq m\} \cup$$

$$\{yy_s : 1 \leq s \leq n\}.$$

Then G has $m+n+8$ nodes and $m+n+3$ links.

The required node labeling $f : V(G) \rightarrow \{1, 2, 3, 4, \dots, m+n+8\}$ is defined as follows

$$f(u) = 1 \quad f(v) = 5; \quad f(y) = 7; \quad f(w) = 9; \quad f(x) = m+n+8;$$

$$f(u_1) = 2$$

$$f(v_1) = 4$$

$$f(y_1) = 6$$

$$f(w_k) = 2k+6 \quad \text{for } 1 \leq k \leq m$$

$$f(x_h) = 2h+9 \quad \text{for } 1 \leq h \leq n-2$$

$$f(x_{n-1}) = m+n+7$$

$$f(x_n) = m+n+9$$

The corresponding link labels are as follows:

$$\text{The link label of } uu_1 \text{ is } 2; \quad vv_1 \text{ is } 5; \quad yy_1 \text{ is } 7; \quad ww_k \text{ is } k+8 \quad 1 \leq k \leq m;$$

$$xx_h \text{ is } \frac{m+n+2h+17}{2} \quad 1 \leq h \leq n-2; \quad xx_{n-1} \text{ is } m+n+8; \quad xx_n \text{ is}$$

$$m+n+9.$$

Hence the induced link labels are distinct.

Hence the graph G is Relaxed skolem mean graph.

3. APPLICATION OF GRAPH LABELING

The skolem mean labeling is applied on a graph (network), such as bus topology, mesh topology and star topology in order to solve the problems in establishing fastness, efficient communication and various issues in that area, in which the following will be taken into account.

1. A protocol, with secured communication can be achieved, provided the graph (network) is sufficiently connected.
2. To find an efficient way for safer transmissions in areas such as Cellular telephony, Wi-Fi, Security systems and many more.
3. Channel labeling can be used to determine the time at which sensor communicate.

CONCLUSION

Researchers may get some information related to graph labeling and its applications in communication field and work on some ideas related to their field of research.

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