

ON SEMI α - REGULAR WEAKLY CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In topological space, a new class of closed sets is introduced and named as semi α -regular weekly closed sets ($s\alpha rw$ closed) is studied in this paper. Semi α -regular weekly closed sets ($s\alpha rw$ closed) that lies in-between the class of α closed sets and generalised semi closed sets whose basic properties are to be studied.

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Key words: Topological space, semi closed sets, α closed sets, rw closed, αrw closed sets and gs closed sets and $s\alpha rw$ closed set

1. INTRODUCTION

Levis S (19) was first to introduce generalized closed (g - closed) sets in 1970. Later he defined the class of set in which he pursued his further investigations. Stone introduced another class termed regular open sets (31) in which he investigated further, whereas Bencheachi introduced and investigated rw -open sets (5). Few others also introduced new class of sets in which investigations were made. In recent times, R. S. Wali and Mendalgeri (37) introduced and studied α -regular weekly closed (αrw closed). A new class of closed set named **semi α - regular weakly closed ($s\alpha rw$ closed)** that lies between α closed & gs closed is introduced in this paper which is studied with its basic property.

2. PRELIMINARIES

In this research paper X or (X, τ) represents a non-empty topological space and $A \subseteq X$. For $A \subseteq X$ in (X, τ) , $cl(A)$, $int(A)$, $scl(A)$, $\alpha cl(A)$, $spcl(A)$ and $gcl(A)$ represents the closure of A , the interior of A , the semi-closure of A , the α -closure of A , the semi pre closure of A and the g -closure of A in (X, τ) respectively.

The below stated are definitions of closed sets in topological space

Let (X, τ) be a topological space and A be a subset of X , and then A is defined as

- (i) semi (20) whenever $A \subseteq cl(int(A))$ and semi closed whenever $int(cl(A)) \subseteq A$
- (ii) Pre-open (24) whenever $A \subseteq int(cl(A))$ and pre closed whenever $cl(int(P)) \subseteq A$.
- (iii) α -open set (16) whenever $A \subseteq int(cl(int(A)))$ and α closed whenever $cl(int(cl(A))) \subseteq A$
- (iv) Semi pre-open (2) (β -open) whenever $A \subseteq cl(int(cl(A)))$ and semi-pre closed (β -closed) whenever $int(cl(int(A))) \subseteq A$.
- (v) Regular open (31) whenever $A = int(cl(A))$ and regular closed whenever $A = cl(int(A))$.
- (vi) Regular α -open (33) (ra -open) whenever there is a regular open U such that $U \subseteq A \subseteq \alpha cl(U)$.
- (vii) Regular semi open (11) whenever there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.
- (viii) generalized closed (g closed) (19) whenever $cl(A) \subseteq U$ and U be open in X .
- (ix) Semi generalized closed (sg closed) (8) whenever $scl(A) \subseteq U$ and U be semi-open in X .
- (x) generalized semi closed (gs closed) (4) whenever $scl(A) \subseteq U$ and U be open in X .

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- (xi) α -generalized closed (αg closed)(21) whenever $\alpha\text{-cl}(A) \subseteq U$ and U be open in X .
- (xii) regular generalized closed (rg closed)(28) whenever $\text{cl}(A) \subseteq U$ and U be regular open in X .
- (xiii) generalized pre regular closed (gpr closed)(14) whenever $\text{pcl}(A) \subseteq U$ and U be regular open in X .
- (xiv) generalized semi pre closed (gsp closed)(13) whenever $\text{spcl}(A) \subseteq U$ and U be open in X .
- (xv) ω closed (32) whenever $\text{cl}(A) \subseteq U$ and U be semi-open in X .
- (xvi) strongly generalized closed (g^* closed) (29) whenever $\text{cl}(A) \subseteq U$ and U be g -Open in X .
- (xvii) Weakly generalized closed (wg closed) (27) whenever $\text{cl}(\text{Int}(A)) \subseteq U$ and U be open in X .
- (xviii) π generalized closed (πg closed)(12) whenever $\text{cl}(A) \subseteq U$ and U be π -open in X .
- (xix) Semi weakly generalized closed (swg closed) (27) whenever $\text{cl}(\text{Int}(A)) \subseteq U$ and U be semi open in X .
- (xx) Regular weakly generalized closed (rwg closed) (27) whenever $\text{cl}(\text{Int}(A)) \subseteq U$ and U be regular open in X .
- (xxi) Regular generalized α closed set ($rg\alpha$ closed)(33) whenever $\alpha\text{cl}(A) \subseteq U$ and U be regular α -open in X .
- (xxii) g^* pre closed (g^*p closed) (34) whenever $\text{pcl}(A) \subseteq U$ and U be g -open in X .
- (xxiii) α generalized regular closed (agr closed) (35) whenever $\alpha\text{cl}(A) \subseteq U$ and U be regular-open in X .
- (xxiv) $\omega\alpha$ closed (7) whenever $\alpha\text{-cl}(A) \subseteq U$ and U be ω -open in X .
- (xxv) generalized $\omega\alpha$ closed ($g\omega\alpha$ closed) (6) whenever $\alpha\text{-cl}(A) \subseteq U$ and U be $\omega\alpha$ -open in X .
- (xxvi) regular ω closed($r\omega$ closed) (5) whenever $\text{cl}(A) \subseteq U$ and U be regular semi- open in X .
- (xxvii) generalized regular closed (gr -closed) (9) whenever $\text{rcl}(A) \subseteq U$ and U be open in X .
- (xxviii) R^* closed (R^* closed) (15) whenever $\text{rcl}(A) \subseteq U$ and U be regular semi- open in X .
- (xxix) regular generalized weak (rgw closed) (26) whenever $\text{cl}(\text{int}(A)) \subseteq U$ and U be regular semi open in X .
- (xxx) Weak generalized regular α closed ($wgr\alpha$ closed) (17) whenever $\text{cl}(\text{int}(A)) \subseteq U$ and U be regular α -open in X .
- (xxxii) pre generalized pre regular closed ($pgpr$ closed) (3) whenever $\text{pcl}(A) \subseteq U$ and U be rg - open in X .
- (xxxiii) regular pre semi closed (rps closed) (30) whenever it is regular and U be a rg - open in X .
- (xxxiii) generalized pre regular weakly closed ($gprw$ closed)(18) whenever $\text{pcl}(A) \subseteq U$ and U be regular semi-open in X .
- (xxxiv) generalized pre closed (gp closed) (23) whenever $\text{pcl}(A) \subseteq U$ and U be open in X .
- (xxxv) α regular weakly closed(arw closed) (37) whenever $\alpha\text{-cl}(A) \subseteq U$ and U be $r\omega$ -open in X .

Above is list of closed sets whose compliments are their open sets respectively.

3. PROPERTIES OF SEMI α -REGULAR WEAKLY CLOSED SETS IN TOPOLOGICAL SPACES

Definition 3.1: Let A be subset of topological space (X, τ) , A defined semi α -regular weakly closed set ($sarw$ closed set) whenever $\text{scl}(A) \subseteq U$ and $A \subseteq U$ where U be arw -open in X . The family of all semi α -regular weakly closed sets in (X, τ) , is denoted by $sarw C(X)$.

The compliment of semi α -regular weakly closed sets in (X, τ) , is semi α -regular weakly open sets in (X, τ) , The family of all semi α -regular weakly open sets in (X, τ) , is denoted by $sarw O(X)$.

Example 3.2: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology then $sarw$ closed sets are $X, \phi, \{1\}, \{4\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$ $sarw$ open sets are $X, \phi, \{1\}, \{2\}, \{3\}, \{1, 4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{1, 2, 3\}, \{2, 3, 4\}$

First we have to be proving the collection of semi α regular weakly closed sets is lies between α -closed and generalised semi-closed.

Theorem 3.3: Each α closed is $sarw$ closed in X , its converse is untrue

Proof: Let A be α closed set in X . and U is any arw -open in X such that $A \subseteq U$. We write $\text{scl}(A) = A \subseteq U$, that is, $\text{scl}(A) \subseteq U$. Therefore A is $sarw$ closed in X .

The example 3.4 proves each $sarw$ closed is not α closed is in X .

Example 3.4: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ then the set $A = \{2, 4\}$ is $sarw$ closed set but not α closed in X .

Corollary 3.5:

- (i) Each closed set is $sarw$ closed in X , and its converse is untrue.
- (ii) Each regular closed set is $sarw$ closed in X , and its converse is untrue.
- (iii) Each π -closed set is $sarw$ closed in X , and its converse is untrue.
- (iv) Each δ -closed set is $sarw$ closed in X , and its converse is untrue.

Proof:

- (i) Following from Theorem 3.2. Each closed is α closed.
- (ii) From Stone (31) & corollary 3.4 (i). Each regular closed set is closed.
- (iii) Following Dontchev & Noiri (12), and corollary 3.4 (i). Each π - closed set is closed.
- (iv) From Velicko. Each δ - closed set is closed (36) and corollary 3.4.

Example 3.6: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology.

- (i) Let $A = \{1\}$ is $s\alpha r w$ closed set and is not closed in X .
- (ii) Let $A = \{1, 2, 4\}$ is $s\alpha r w$ closed set and is not regular closed in X .
- (iii) Let $A = \{1, 2, 4\}$ is $s\alpha r w$ closed set and is not π closed in X .
- (iv) Let $A = \{2, 3\}$ is $s\alpha r w$ closed set and is not δ closed in X .

Theorem 3.7: Each $s\alpha r w$ closed set is $g s$ closed set in X and converse is untrue.

Proof: Assume A is $s\alpha r w$ closed set in X and U is open set in X such that $A \subseteq U$. We know that each open set is $r w$ -open set (5), we write, $scl(A) \subseteq U$ and since U is open in X . A is $g s$ closed set in X .

The example 3.8 shows that the converse of the above theorem need not be true.

Example 3.8: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ then the set $A = \{2\}$ is $g s$ closed set but it is not $s\alpha r w$ closed set in X .

Theorem 3.9:

- (i) Each $s\alpha r w$ closed set are $g s p$ closed in X , and its converse is untrue.
- (ii) Each $s\alpha r w$ closed set are $r p s$ closed in X , and its converse is untrue.
- (iii) Each $s\alpha r w$ closed set are $g s$ closed in X , and its converse is untrue.
- (iv) Each $s\alpha r w$ closed set are $g s p r$ closed in X and its converse is untrue.

Proof:

- (i) Assume A is $s\alpha r w$ closed set and an open set U in X , such that $A \subseteq U$. We know that each open set is regular ω - open set in X (5) and A is $s\alpha r w$ closed set in X it follows $scl(A) \subseteq U$, where $spcl(A) \subseteq scl(A)$. Then $spcl(A) \subseteq U$, U is open in X . Thus A is a $g s p$ closed set in X .
- (ii) Assume A is $s\alpha r w$ closed set and a rg -open set U in X such that $A \subseteq U$. We know that each rg -open set is regular ω - open set in X (5) and A be $s\alpha r w$ closed set in X it follows $scl(A) \subseteq U$, where $spcl(A) \subseteq scl(A)$. Then $spcl(A) \subseteq U$, U is rg -open in X . Thus A is a $r p s$ - closed set in X .
- (iii) Assume A is $s\alpha r w$ closed set and semi open set U in X such that $A \subseteq U$. We know that each open set is regular ω - open set in X (5) and A be $s\alpha r w$ - closed set in X it follows that $scl(A) \subseteq U$ and $scl(A) \subseteq scl(A)$ is always, $scl(A) \subseteq scl(A) \subseteq U$, $scl(A) \subseteq U$, U is semi open in X . Thus A is a $g s$ -closed set in X .
- (iv) Assume A is $s\alpha r w$ closed set and regular open set U in X such that $A \subseteq U$. We know that each regular open set is regular ω - open set in X (5) and A be $s\alpha r w$ - closed set in M it follows $scl(A) \subseteq U$, where $spcl(A) \subseteq scl(A)$. Then $spcl(A) \subseteq U$, U is regular open set. Thus A is a $g s p r$ closed set in X .

Example 3.10: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology

- (i) Let $A = \{1, 2\}$ is $g s p$ closed which is not $s\alpha r w$ closed in X .
- (ii) Let $A = \{1, 3\}$ is $r p s$ closed which is not $s\alpha r w$ closed in X .
- (iii) Let $A = \{2\}$ is $g s$ closed which is not $s\alpha r w$ closed in X .
- (iv) Let $A = \{1, 2, 3\}$ is $g s p r$ closed which is not $s\alpha r w$ closed in X .

Remark 3.11: In general, union of two $s\alpha r w$ closed subsets need not be $s\alpha r w$ closed set in X .

Example 3.12 Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology. Assume two $s\alpha r w$ closed subsets of X say, $A = \{1\}$ and $B = \{2, 3\}$, then $A \cup B = \{1, 2, 3\}$ which is not contained in $s\alpha r w$ closed set in X . Thus union of two $s\alpha r w$ closed sets isn't $s\alpha r w$ closed set in X .

Remark 3.13: In general, intersection of two $s\alpha r w$ closed sets is not $s\alpha r w$ closed set in X .

Example 3.14: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology. Assume two $s\alpha r w$ closed subsets of X , say, $A = \{2, 3\}$ and $B = \{2, 4\}$, then $A \cap B = \{2\}$ which is not contained in $s\alpha r w$ closed set in X . Therefore intersection of two $s\alpha r w$ closed sets isn't $s\alpha r w$ closed set in X .

Remark 3.15: Shown below are examples to prove $s\alpha r w$ closed sets are independent of g - closed, $g p r w$ - closed, $r p s$ - closed, R^* - closed, $g r$ - closed, $w g r \alpha$ - closed, $r \omega$ -closed sets, $s w g$ - closed, $r g w$ - closed, $g p r$ - closed, $g^* p$ - closed sets, β - closed, semi closed, ω - closed, $g \omega \alpha$ - closed, pre closed, $\alpha r \omega$ -closed.

Example 3.16: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$. Then

- (i) closed sets are $X, \phi, \{4\}, \{1, 4\}, \{2, 3, 4\}$
- (ii) sarw closed sets are $X, \phi, \{1\}, \{4\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (iii) g closed sets are $X, \phi, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (iv) sg closed sets are $X, \phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (v) gs closed sets are $X, \phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (vi) ga closed sets are $X, \phi, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (vii) ag closed sets are $X, \phi, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (viii) rw closed sets are $X, \phi, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (ix) arw closed sets are $X, \phi, \{4\}, \{1, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (x) gsp closed sets are $X, \phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xi) gp closed sets are $X, \phi, \{2\}, \{3\}, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xii) gpr closed sets are $X, \phi, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xiii) g^* closed sets are $X, \phi, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xiv) πg closed sets are $X, \phi, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xv) w - closed sets are $X, \phi, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xvi) swg closed sets are $X, \phi, \{2\}, \{3\}, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xvii) rg closed sets are $X, \phi, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xviii) rwg closed sets are $X, \phi, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xix) gspr closed sets are $X, \phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xx) wg closed sets are $X, \phi, \{2\}, \{3\}, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xxi) agr closed sets are $X, \phi, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xxii) wa- closed sets are $X, \phi, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xxiii) gprw closed sets are $X, \phi, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xxiv) rps closed sets are $X, \phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xxv) R^* closed sets are $X, \phi, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}$
- (xxvi) gr closed sets are $X, \phi, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xxvii) rgw closed sets are $X, \phi, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}$
- (xxviii) β closed sets are $X, \phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xxix) semi closed sets are $X, \phi, \{1\}, \{4\}, \{1, 4\}, \{2, 3\}, \{2, 3, 4\}$
- (xxx) pgpr closed sets are $X, \phi, \{2\}, \{3\}, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xxxii) g^*p closed sets are $X, \phi, \{2\}, \{3\}, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xxxiii) $g\omega\alpha$ closed sets are $X, \phi, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xxxiv) pre closed sets are $X, \phi, \{2\}, \{3\}, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$
- (xxxv) π closed sets are $X, \phi, \{4\}, \{1, 4\}, \{2, 3, 4\}$
- (xxxvi) δ closed sets are $X, \phi, \{4\}, \{1, 4\}, \{2, 3, 4\}$

Remark 3.17: From the results discussed above and with the known facts, the relation between sarw-closed sets and some existing closed sets in topological spaces is established in Figure 1.

$A \longrightarrow B$ denotes A implies B, and its converse is untrue.

$A \longleftrightarrow B$ denotes A and B are independent to each.

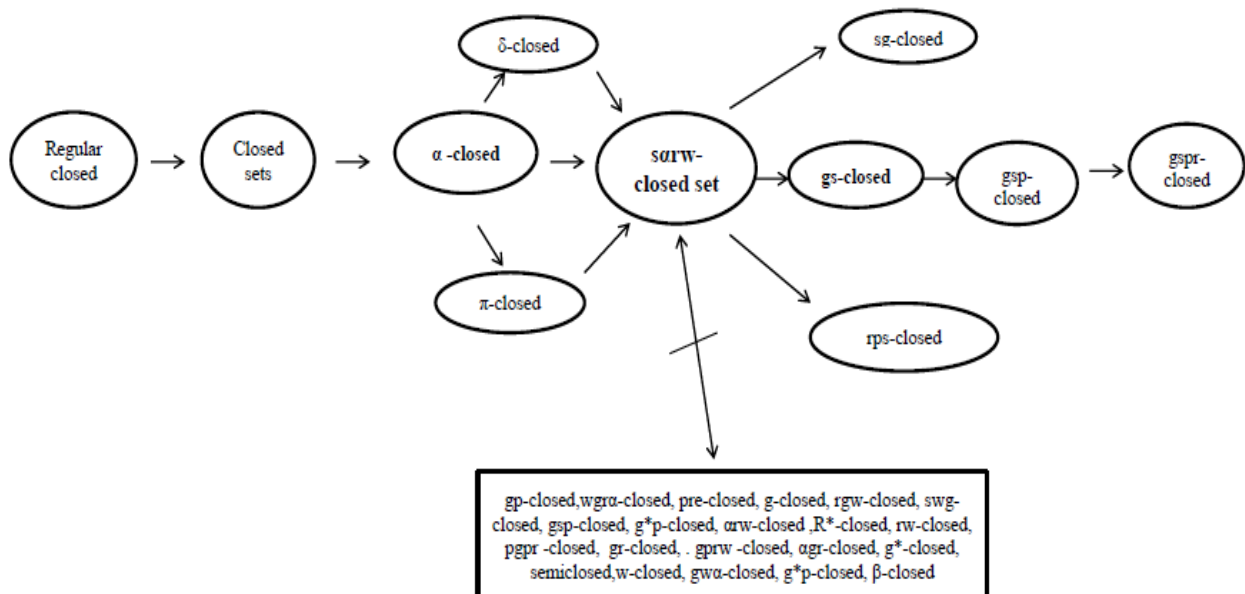


Figure-1: The relation between sarw closed set and some existing closed sets in topological spaces

Theorem 3.18: A is $s\alpha$ rw closed set in X such that $A \subseteq B \subseteq \text{scl}(A)$ then B is $s\alpha$ rw closed set in X.

Proof: Given that A is $s\alpha$ rw closed in X then we need to prove B is also $s\alpha$ rw closed in X. A $r\omega$ -open set U in X such that $B \subseteq U$. Given that $A \subseteq B$ and A is $s\alpha$ rw closed set, $\text{scl}(A) \subseteq U$. Now $B \subseteq \text{scl}(A) \Rightarrow \text{scl}(B) \subseteq \text{scl}(\text{scl}(A)) = \text{scl}(A) \subseteq U \Rightarrow \text{scl}(B) \subseteq U$. Hence B is $s\alpha$ rw closed set in X. The converse is untrue which is proved through example below

Example 3.19: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2,3\}, \{1, 2, 3\}\}$ assume $A = \{4\}$, $B = \{1, 4\}$ where A & B are $s\alpha$ rw closed sets in X, such that $A \subseteq B$ which does not belong to $\text{scl}(A)$, since $\text{scl}(A) = \{4\}$

Theorem 3.20: Whenever A is regular open as well as A is α gr closed, and then A is $s\alpha$ rw closed set in X.

Proof: Given that A is regular open and α gr closed, $r\omega$ -opens set U in X such that $A \subseteq U$. From definition we write, $\text{scl}(A) \subseteq A$, where $\text{scl}(A) \subseteq A \subseteq U$. Thus A is $s\alpha$ rw closed set in X.

Remark 3.21: Whenever A is regular open as well as A is $s\alpha$ rw closed, A need not be α gr -closed. The example 3.22 proves the above remark

Example 3.22: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology. Let $A = \{2, 3\}$ is regular open and $s\alpha$ rw closed but not α gr closed in X.

Theorem 3.23: A is ω -open as well as $\omega\alpha$ closed then A is $s\alpha$ rw closed set in X.

Proof: Given A is ω -open and $\omega\alpha$ closed, any $r\omega$ open set, say U in X such that $A \subseteq U$. Through definition we write, $\text{scl}(A) \subseteq A$ therefore $\text{scl}(A) \subseteq A \subseteq U$, which proves A is $s\alpha$ rw closed set in X.

Remark 3.24 In general, when A is ω -open and $s\alpha$ rw closed, A need not be $\omega\alpha$ closed. The example 3.25 proves the above remark

Example 3.25: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology. Assume $A = \{1\}$ is ω -open and $s\alpha$ rw closed and is not $\omega\alpha$ closed in X.

Theorem 3.26: whenever A is open as well as α g closed, A is $s\alpha$ rw closed set in X.

Proof: Given that A be open and α g closed and U is any $r\omega$ -open set in X $A \subseteq U$. From definition we write, $\text{scl}(A) \subseteq A$ then $\text{scl}(A) \subseteq A \subseteq U$, which proves A is $s\alpha$ rw closed set in X.

Remark 3.27 Example 3.28 proves that when A is open as well as $s\alpha$ rw closed, it need not be α g- closed

Example 3.28 Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ it's topology. Assume $A = \{2, 3\}$ is open and $s\alpha$ rw closed but not α g closed in X.

Theorem 3.29: whenever A is regular open and A is $r\omega$ g closed, A is $s\alpha$ rw closed set in X.

Proof: Let A be regular open and $r\omega$ g closed where U is any $r\omega$ -open set in X such that $A \subseteq U$ From definition we write, $\text{cl}(\text{int}(A)) \subseteq A$ we also know that $\text{cl}(\text{int}(A)) \subseteq \text{cl}(\text{int}(\text{cl}(A))) \subseteq A \subseteq U$, then $\text{scl}(A) \subseteq U$, which proves A is $s\alpha$ rw closed set in X.

Remark 3.30 The example 3.31 proves when A is both regular open and $s\alpha$ rw closed, A need not be $r\omega$ g closed.

Example 3.31: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology. Let $A = \{1\}$ is regular open and $s\alpha$ rw closed and is not $r\omega$ g closed in X

Corollary 3.32: When A is open and w g closed, A will be $s\alpha$ rw closed set in X.

Proof: It follows, each regular open is open set and each w g- closed is $r\omega$ g closed set from theorem 3.29 (29)

Remark 3.33 In general, whenever A is open as well as $s\alpha$ rw closed, it need not be w g closed which is proven in an example 3.34.

Example 3.34: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology. Assume $A = \{2, 3\}$ is open and $s\alpha$ rw closed and is not w g closed in X.

Theorem 3.35: whenever A be regular open as well as gpr closed, A is s sarw closed set in X.

Proof: Given A be regular open as well as gpr closed where U is any $r\omega$ -open set in X such that $A \subseteq U$ from definition we write, $pcl(A) \subseteq A$. We know that $pcl(A) \subseteq scl(A) \subseteq A$. Therefore $scl(A) \subseteq U$, which proves A is sarw closed set in X.

Remark 3.36: Example 3.37 proves when A is regular open as well as sarw closed, A need not be gpr- closed

Example 3.37: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology. Assume $A = \{1\}$ is regular open and sarw closed and is not gpr closed in X.

Corollary 3.38: whenever A be open and gp closed, A is sarw closed set in X.

Proof: It follows each regular open is open and each gp closed set is gpr closed set from theorem 3.1 (15)

Remark 3.39: Example 3.40 proves whenever A is open and sarw closed, A need not be gp closed

Example 3.40: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology. Assume $A = \{2, 3\}$ is open and sarw closed and is not gp closed in X.

Theorem 3.41: A is subset of a topology X. Whenever A is regular open as well as sarw closed then it is α closed.

Proof: Given A is regular open and sarw closed, as each regular open is $r\omega$ -open. Now $A \subseteq A$ then definition of sarw closed, $scl(A) \subseteq A$ and also $A \subseteq scl(A)$ then $scl(A) = A$. This proves A is α closed.

Theorem 3.42: A is a subset of a topology X. Whenever A is regular semi open as well as $r\omega$ closed then it is sarw closed.

Proof: Given A is regular semi open as well as $r\omega$ closed set such that A is subset of U, $r\omega$ -open set U in X. Now $A \subseteq A$ by hypothesis $cl(A) \subseteq A$, we also know $scl(A) \subseteq cl(A) \subseteq A \subseteq U$, which proves A is sarw closed set in X.

Remark 3.43: whenever A is regular semi open as well as sarw closed, it need not be $r\omega$ closed

Example 3.44: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology. Assume $A = \{2, 3\}$ is regular semi open and sarw closed and not $r\omega$ closed in X.

Theorem 3.45: A is subset of a topology X. Whenever A is semi open and ω closed then it is sarw closed.

Proof: Given A is semi open and ω closed set such that A is subset of U, $r\omega$ open set U in X. Now $A \subseteq A$ by hypothesis $cl(A) \subseteq A$, we also know $scl(A) \subseteq cl(A) \subseteq A \subseteq U$, which proves A is sarw closed set in X.

Remark 3.46: Examples 3.47 prove when A is both semi open and sarw closed; it need not be ω - closed.

Example 3.47: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology. Assume $A = \{1\}$ is semi open and sarw closed and not ω closed in X.

Theorem 3.48: A is a subset of a topology X. Whenever A is both regular semi open and gprw closed then it is sarw closed.

Proof: Given A is a regular semi open and gprw closed set such that A is subset of U, $r\omega$ -open set U in X. Now $A \subseteq A$ by hypothesis $pcl(A) \subseteq A$, we also know that $pcl(A) \subseteq scl(A) \subseteq A$, therefore $scl(A) \subseteq U$, which proves A is sarw closed set in X.

Remark 3.49: The example 3.50 shows when A is both regular semi open and sarw closed, it need not be gprw closed.

Example 3.50: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology. Assume $A = \{2, 3\}$ is regular semi open as well as sarw closed but not gprw- closed in X.

Theorem 3.51: Let A be subset of a topology X, whenever A is both regular semi-open and rgw closed then it is sarw closed.

Proof: Given A is a regular semi open and rgw closed set such that A subset of U , ω -open set U in X , now $A \subseteq U$ through hypothesis $\text{cl}(\text{int}(A)) \subseteq A$ we also know that $\text{cl}(\text{int}(A)) \subseteq \text{cl}(\text{int}(\text{cl}(A))) \subseteq A$, therefore $\text{scl}(A) \subseteq U$. A is sarw closed set in X .

Remark 3.52: Example 3.53 proves when A is regular semi open and sarw closed it need not be rgw closed.

Example 3.53: Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1\}, \{2, 3\}, \{1, 2, 3\}\}$ be the topology. Let $A = \{2, 3\}$ is regular semi open as well as sarw closed and not rgw closed in X .

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