International Journal of Mathematical Archive-8(7), 2017, 197-204
Available online through www.ijma.info ISSN 2229 – 5046

ON SEMI α - REGULAR WEAKLY CLOSED SETS IN TOPOLOGICAL SPACES

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(Received On: 23-06-17; Revised & Accepted On: 12-07-17)

ABSTRACT

In topological space, a new class of closed sets is introduced and named as semi α-regular weekly closed sets (sαrw closed) is studied in this paper. Semi α-regular weekly closed sets (sαrw closed) that lies in-between the class of α closed sets and generalised semi closed sets whose basic properties are to be studied.

AMS Subject Classifications: *54A05, 54A10.*

Key words: *Topological space, semi closed sets, α closed sets, rw closed, αrw closed sets and gs closed sets and sαrw closed set*

1. INTRODUCTION

Levis S (19) was first to introduce generalized closed (g- closed) sets in1970. Later he defined the class of set in which he pursued his further investigations. Stone introduced another class termed regular open sets (31) in which he investigated further, whereas Bencheachi introduced and investigated rw-open sets (5).Few others also introduced new class of sets in which investigations were made. In recent times, R. S. Wali and Mendalgeri (37) introduced and studied α-regular weekly closed (αrw closed). A new class of closed set named **semi α- regular weakly closed (sαrw closed**) that lies between α closed & gs closed is introduced in this paper which is studied with its basic property.

2. PRELIMINARIES

In this research paper X or (X,τ) represents a non-empty topological space and A⊆X. For A⊆X in (X, τ), cl(A), int (A), scl (A), αcl(A), spcl (A) and gcl (A) represents the closure of A, the interior of A, the semi-closure of A, the α-closure of A, the semi pre closure of A and the g-closure of A in (X, τ) respectively.

The below stated are definitions of closed sets in topological space

Let (X,τ) be a topological space and A be a subset of X, and then A is defined as

(i) semi open (20) whenever A⊆ cl(int(A))and semi closed whenever int (cl (A)) ⊆ A
(ii) Pre-open (24) whenever A⊆ int(cl(A) and pre closed whenever cl(int(P)) ⊆A.
(iii) α-open set (16) whenever A⊆ int(cl(int(A))) and α closed whenever cl(int(cl(A))) ⊆ A
(iv) Semi pre-open (2) (β-open) whenever A⊆ cl int(cl(A)))) and semi-pre closed (β-closed) whenever int(cl(int(A)))⊆A.
(v) Regular open (31) whenever A= int(cl(A)) and regular closed whenever A = cl(int(A)).
(vi) Regular α-open (33) (rα-open) whenever there is a regular open U such that U ⊆ A ⊆ αcl(U).
(vii) Regular semi open (11) whenever there is a regular open set U such that U ⊆ A⊆ cl(U).
(viii) generalized closed (g closed) (19) whenever cl(A) ⊆ U and U be open in X.
(ix) Semi generalized closed (sg closed) (8) whenever scl(A)⊆U and U be semi-open in X.
(x) generalized semi closed (gs closed) (4) whenever scl(A) ⊆ U and U be open in X.

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(xi) α-generalized closed (αg closed)(21) whenever α -cl(A)⊆U and U be open in X.
(xii) regular generalized closed (rg closed)(28) whenever cl(A)⊆U and U be regular open in X.
(xiii) generalized pre regular closed (gpr closed)(14) whenever pcl(A)⊆ U and U be regular open in X.
(xiv) generalized semi pre closed (gsp closed)(13) whenever spcl(A) ⊆ U and U be open in X.
(xv) ω closed (32) whenever cl(A) ⊆ U and U be semi-open in X.
(xvi) strongly generalized closed (g*closed) (29) whenever cl (A) ⊆U and U be g- Open in X.
(xvii) Weakly generalized closed (wg closed) (27) whenever cl (Int (A)) ⊆ U and U be open in X.
(xviii) π generalized closed (πg closed)(12) whenever cl (A) ⊆ U and U be π-open in X.
(xix) Semi weakly generalized closed (swg closed) (27) whenever cl(Int(A) ⊆ U and U be semi open in X.
(xx) Regular weakly generalized closed (rwg closed) (27) whenever cl(Int(A))⊆ U and U be regular open in X
(xxi) Regular generalized α closed set (rgα closed)(33) whenever αcl (A) ⊆ U and U be regular α-open in X.
(xxii) g*pre closed (g*p closed) (34) whenever pcl(A)⊆ U and U be g-open in X
(xxiii) α generalized regular closed (αgr closed) (35) whenever αcl(A)⊆ U and U be regular-open in X
(xxiv) ωα closed (7) whenever α-cl(A) ⊆ U and U be ω-open in X
(xxv) generalized ωα closed (gωα closed) (6) whenever α-cl(A) ⊆ U and U be ωα -open in X
(xxvi) regular ω closed(rω closed) (5) whenever cl(A)⊆ U and U be regular semi- open inX
(xxvii) generalized regular closed (gr–closed) (9) whenever rcl(A)⊆ U and U be open in X
(xxviii) R*closed (R*closed) (15) whenever rcl (A) ⊆U and U be regular semi- open in X
(xxix) regular generalized weak (rgw closed) (26) whenever cl(int(A)) ⊆ U and U be regular semi open in X
(xxx) Weak generalized regular α closed (wgrα closed) (17) whenever cl(int(A)) ⊆ U and U be regular α-open in X
(xxxi) pre generalized pre regular closed (pgpr closed) (3) whenever pcl(A) ⊆ U and U be rg- open in X
(xxxii) regular pre semi closed (rps closed) (30) whenever it is regular and U be a rg- open in X
(xxxiii) generalized pre regular weakly closed (gprw closed)(18)whenever pcl(A)⊆U and U be regular semi-open in X
(xxxiv) generalized pre closed (gp closed) (23) whenever pcl(A)⊆U and U be open in X
(xxxv) α regular weakly closed(αrw closed) (37) whenever α-cl(A)⊆U and U be rω-open in X

Above is list of closed sets whose compliments are their open sets respectively.

3. PROPERTIES OF SEMI α -REGULAR WEAKLY CLOSED SETS IN TOPOLOGICAL SPACES

Definition 3.1: Let A be subset of topological space (X,τ), A defined semi α- regular weakly closed set (sαrw closed set) whenever scl(A)⊆U and A⊆U where U be αrw -open in X. The family of all semi α- regular weakly closed sets in (X,τ), is denoted by sαrw C(X).

The compliment of semi α-regular weakly closed sets in (X,τ), is semi α- regular weakly open sets in (X,τ), The family of all semi α-regular weakly open sets in (X,τ),is denoted by sαrw O(X).

Example 3.2: Let X = {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3}, {1, 2, 3}} be the topology then sαrw closed sets are X, ϕ, {1}, {4}, {1, 4}, {2, 3}, {2, 4}, {3, 4},{1, 2, 4}, {1, 3, 4}, {2, 3, 4} sαrw open sets are X, ϕ, {1}, {2}, {3}, {1, 4}, {1, 2}, {1, 3}, {1, 4},{2, 3}, {1, 2, 3}, {2, 3, 4}

First we have to be proving the collection of semi α regular weakly closed sets is lies between α-closed and generalised semi-closed.

Theorem 3.3: Each α closed is sαrw closed in X, its converse is untrue

Proof: Let A be α closed set in X. and U is any αrw-open in X such that A ⊆ U. We write scl(A) = A ⊆ U, that is, scl(A) ⊆ U. Therefore A is sαrw closed in X.

The example 3.4 proves each sαrw closed is not α closed is in X.

Example 3.4: Let X= {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3}, {1, 2, 3}} then the set A= {2, 4} is sαrw closed set but not α closed in X.

Corollary 3.5:
(i) Each closed set is sαrw closed in X, and its converse is untrue.
(ii) Each regular closed set is sαrw closed in X, and its converse is untrue.
(iii) Each π- closed set is sαrw closed in X, and its converse is untrue.
(iv) Each δ- closed set is sαrw closed in X, and its converse is untrue.

Proof:

(i) Following from Theorem 3.2. Each closed is α closed.
(ii) From Stone (31) & corollary 3.4 (i). Each regular closed set is closed.
(iii) Following Dontchev & Noiri (12), and corollary 3.4 (i). Each π- closed set is closed.
(iv) From Velicko. Each δ- closed set is closed (36) and corollary 3.4.

Example 3.6: Let X = {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3}, {1, 2, 3}} be the topology.

(i) Let A = {1} is sαrw closed set and is not closed in X.
(ii) Let A = {1, 2, 4} is sαrw closed set and is not regular closed in X.
(iii) Let A = {1, 2, 4} is sαrw closed set and is not π closed in X.
(iv) Let A = {2, 3} is sαrw closed set and is not δ closed in X.

Theorem 3.7: Each sαrw closed set is gs closed set in X and converse is untrue.

Proof: Assume A is sαrw closed set in X and U is open set in X such that A ⊆ U. We know that each open set is rw-open set (5), we write, scl(A)⊆U and since U is open in X. A is gs closed set in X.

The example 3.8 shows that the converse of the above theorem need not be true.

Example 3.8: Let X = {1, 2, 3, 4} and τ ={X, ϕ,{1},{2, 3} {1, 2, 3}} then the set A={ 2} is gs closed set but it is not sαrw closed set in X.

Theorem 3.9:

(i) Each sαrw closed set are gsp closed in X, and its converse is untrue.
(ii) Each sαrw closed set are rps closed in X, and its converse is untrue.
(iii) Each sαrw closed set are sg closed in X, and its converse is untrue.
(iv) Each sαrw closed set are gspr closed in X and its converse is untrue.

Proof:

(i) Assume A is sαrw closed set and an open set U in X, such that A ⊆ U. We know that each open set is regular ω- open set in X (5) and A is sαrw closed set in X it follows scl(A) ⊆ U, where spcl(A) ⊆ scl(A). Then spcl(A) ⊆ U , U is open in X. Thus A is a gsp closed set in X.
(ii) Assume A is sαrw closed set and a rg-open set U in X such that A ⊆ U. We know that each rg-open set is regular ω- open set in X (5) and A be sαrw closed set in X it follows scl(A) ⊆ U, where spcl(A) ⊆ scl(A). Then spcl(A)⊆U, U is rg-open in X. Thus A is a rps- closed set in X.
(iii) Assume A is sαrw closed set and semi open set U in X such that A ⊆ U. We know that each open set is regular ω- open set in X (5)and A be sαrw - closed set in X it follows that scl(A) ⊆ U and scl(A) ⊆ scl(A) is always, scl(A) ⊆ scl(A) ⊆ U, scl(A) ⊆ U, U is semi open in X. Thus A is a sg –closed set in X.
(iv) Assume A is sαrw closed set and regular open set U in X such that A ⊆ U. We know that each regular open set is regular ω- open set in X (5)and A be sαrw - closed set in M it follows scl(A) ⊆ U, where spcl(A) ⊆ scl(A). Then spcl(A)⊆ U ,U is regular open set. Thus A is a gspr closed set in X.

Example 3.10: Let X= {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3} {1, 2, 3}} be the topology

(i) Let A = {1, 2} is gsp closed which is not sαrw closed in X.
(ii) Let A = {1, 3} is rps closed which is not sαrw closed in X.
(iii) Let A = {2} is sg closed which is not sαrw closed in X.
(iv) Let A = {1, 2, 3} is gspr closed which is not sαrw closed in X.

Remark 3.11: In general, union of two sαrw closed subsets need not be sαrw closed set in X.

Example 3.12 Let X = {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3}, {1, 2, 3}} be the topology. Assume two sαrw closed subsets of X say, A = {1} and B = {2, 3}, then A⋃B = {1, 2, 3} which is not contained in sαrw closed set in X. Thus union of two sαrw closed sets isn't sαrw closed set in X.

Remark 3.13: In general, intersection of two sαrw closed sets is not sαrw closed set in X.

Example 3.14: Let X = {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3}, {1, 2, 3}} be the topology. Assume two sαrw closed subsets of X, say, A = {2, 3} and B = {2, 4}, then A∩B = {2} which is not contained in sαrw closed set in X. Therefore intersection of two sαrw closed sets isn't sαrw closed set in X.

Remark 3.15: Shown below are examples to prove sαrw closed sets are independent of g- closed, gprw- closed, rps - closed, R*- closed, gr- closed, wgrα- closed , rω -closed sets, swg- closed, rgw- closed, gpr- closed, g*p- closed sets, β- closed, semi closed, ω- closed, gωα- closed, pre closed, αrω -closed.

Example 3.16: Let X = {1, 2, 3, 4} and τ = {X, ϕ {1}, {2, 3}, {1, 2, 3}}. Then

(i) closed sets are X, ϕ, {4}, {1, 4}, {2, 3, 4}
(ii) sαrw closed sets are X, ϕ,{1}, {4},{1, 4},{2, 3},{2, 4},{3, 4},{1, 2, 4}, {1, 3, 4}, {2, 3, 4}
(iii) g closed sets are X, ϕ, {4}, {1, 4},{2, 4} {3, 4}, {1, 2 ,4}, {1, 3, 4},{2, 3, 4}
(iv) sg closed sets are X, ϕ,{1},{2},{3},{4},{1, 4},{2, 3},{2, 4},{3, 4}, {1, 2 ,4}, {1, 3, 4}, {2, 3,4}
(v) gs closed sets are X, ϕ,{1},{2},{3},{4},{1, 4},{2, 3},{2, 4},{3, 4 },{1, 2 ,4}, {1, 3, 4},{2, 3, 4}
(vi) gα closed sets are X, ϕ, {4}, {1, 4},{2, 4},{3, 4},{1, 2, 4}, {1, 3, 4}, {2, 3, 4}
(vii) αg closed sets are X, ϕ, {4}, {1, 4},{2, 4},{3, 4 },{1, 2, 4}, {1, 3, 4}, {2, 3, 4}
(viii) rw closed sets are X, ϕ, {4},{1, 2}{1, 3},{1, 4},{2, 4},{3, 4}, {1, 2, 3},{1, 2 ,4},{1, 3, 4},{ 2, 3, 4}
(ix) αrw closed sets are X, ϕ, {4}, {1, 4},{1, 2, 4},{1, 3, 4}, {2, 3, 4}
(x) gsp closed sets are X, ϕ,{1},{2},{3},{4},{1, 2},{1, 3},{1, 4},{2, 3},{2, 4},{3, 4}, {1, 2,4},{1, 3 ,4}, {2, 3, 4}
(xi) gp closed sets are X, ϕ, {2},{3}{4},{1,4},{2, 4},{3, 4},{1, 2, 4},{1, 3, 4},{2, 3, 4}
(xii) gpr closed sets are X, ϕ, {2},{3},{4},{1, 2}, {1, 3},{1, 4},{2, 4}, {3, 4}, {1, 2, 3},{1, 2, 4},{1, 3 ,4},{2, 3, 4}
(xiii) g* closed sets are X, ϕ, {4}, {1, 4},{2, 4},{3, 4},{1, 2, 4}, {1, 3, 4}, {2, 3, 4}
(xiv) πg closed sets are X, ϕ, {4}, {1, 4},{2, 4},{3, 4},{1, 2, 4}, {1, 3, 4}, {2, 3, 4}
(xv) w - closed sets are X, ϕ, {4}, {1, 4},{2, 4},{3, 4 },{1, 2, 4}, {1, 3, 4}, {2, 3, 4}
(xvi) swg closed sets are X, ϕ, {2},{3},{4}, {1, 4},{2, 4}, {3, 4},{1, 2, 4}, {1, 3,4}, {2, 3, 4}
(xvii) rg closed sets are X, ϕ,{4},{1, 2}{1, 3},{1, 4},{2, 4},{3, 4},{1, 2, 3},{1, 2 ,4}, {1, 3, 4}, {2, 3, 4}
(xviii) rwg closed sets are X, ϕ, {2},{3},{4},{1, 2}, {1, 3},{1, 4},{2, 4}, {3, 4},{1, 2, 3},{1, 2, 4},{1, 3 ,4}, {2, 3, 4}
(xix) gspr closed sets X, ϕ,{1},{2}{3}{4},{1, 2}{1, 3},{1, 4},{2, 3}{2, 4}{3, 4} {1,2,3}, {1,2,4}, {1,3,4}, {2,3, 4}
(xx) wg closed sets are X, ϕ,{2},{3} {4}, {1, 4},{2, 4},{3, 4 },{1, 2, 4}, {1, 3, 4},{2, 3, 4}
(xxi) αgr closed sets are X, ϕ, {4}, {1, 2}{1, 3},{1, 4},{2, 4},{ 3, 4}, {1, 2, 3},{1, 2, 4}, {1, 3, 4}, {2, 3, 4}
(xxii) wα- closed sets are X, ϕ, {4}, {1, 4},{2, 4},{3, 4},{1, 2, 4}, {1, 3, 4}, {2, 3, 4}
(xxiii) gprw closed sets are X, ϕ,{2},{3},{4},{1, 2}, {1, 3},{1, 4},{2, 4}, {3, 4 },{1, 2, 3}, {1,2,4}, {1,3,4},{2,3, 4}
(xxiv) rps closed sets are X, ϕ,{1},{2},{3},{4},{1, 2}, {1, 3},{1, 4}, {2, 3},{2, 4},{3, 4 },{1, 2,4},{1, 3 ,4},{2, 3, 4}
(xxv) R* closed sets are X, ϕ, {4},{1, 2},{1, 3},{1, 4},{2, 4} {3, 4}, {1, 2, 3},{1, 2, 4}, {1, 3, 4}
(xxvi) gr closed sets are X, ϕ, {4},{1, 4},{2, 4}, {3, 4},{1, 2, 4} {1, 3 ,4}, {2, 3, 4}
(xxvii) rgw closed sets are X, ϕ,{2},{3},{4},{1, 2}, {1, 3},{1, 4},{2, 4}, {3, 4},{1, 2, 3}, {1, 2, 4},{1, 3 ,4}
(xxviii) β closed sets are X, ϕ, {1},{2},{3},{4},{1 ,2},{1, 3} {1, 4}, {2, 3}, {2, 4},{3, 4}, {1, 2,4}, {1, 3, 4}, {2, 3, 4}
(xxix) semi closed sets are X, ϕ, {1},{4},{1, 4},{2, 3}, {2, 3, 4}
(xxx) pgpr closed sets are X, ϕ, {2},{3},{4},{1, 4}, {2, 4}, {3, 4 }, {1, 2, 4}, {1, 3,4}, {2, 3, 4}
(xxxi) g*p closed sets are X, ϕ, {2},{3},{4}, {1, 4},{2, 4}, {3, 4},{1, 2, 4}, {1, 3 ,4} {2, 3, 4}
(xxxii) gωα closed sets are X, ϕ, {4}, {1, 4},{2, 4}, {3, 4},{1, 2, 4}, {1, 3 ,4}, {2, 3, 4}
(xxxiii) pre closed sets are X, ϕ, {2},{3} ,{4},{1, 4}, {2, 4}, {3, 4},{1, 2, 4},{1, 3 ,4}, {2, 3, 4}
(xxxiv) π closed sets are X, ϕ, {4},{1, 4},{2, 3, 4}
(xxxv) δ closed sets are X, ϕ, {4},{1, 4},{2, 3, 4}

Remark 3.17: From the results discussed above and with the known facts, the relation between sαrw-closed sets and some existing closed sets in topological spaces is established in Figure 1.

A $\longrightarrow$ B denotes A implies B, and its converse is untrue.

A $\not\longleftrightarrow$ B denotes A and B are independent to each.

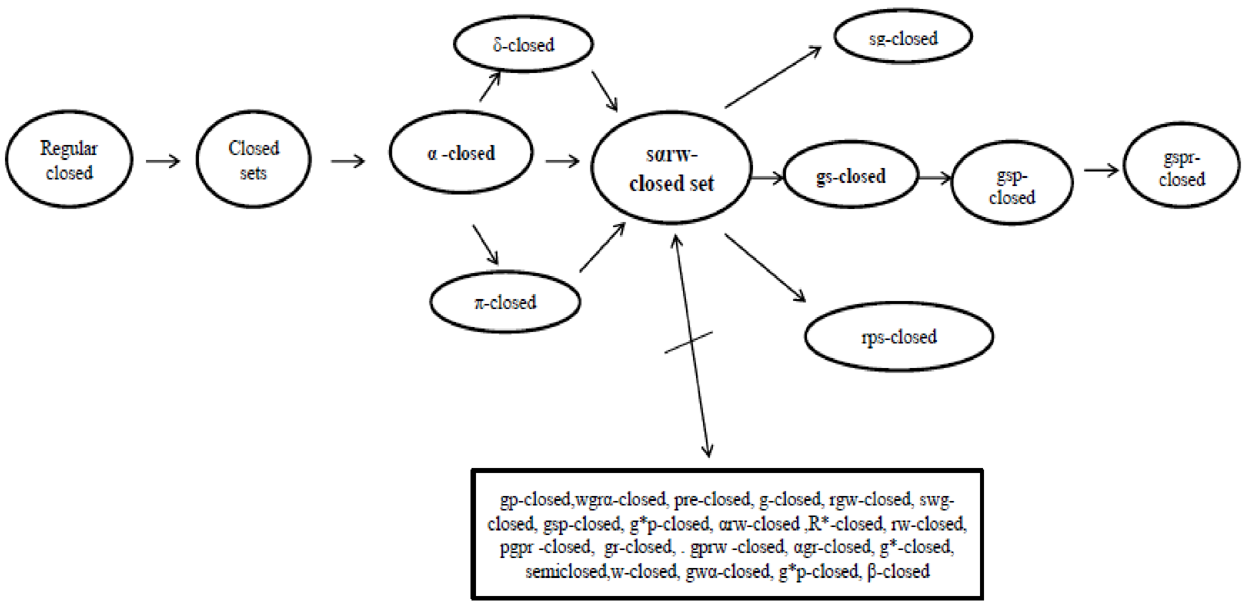

Figure-1: The relation between sαrw closed set and some existing closed sets in topological spaces

Theorem 3.18: A is sαrw closed set in X such that A⊆B⊆ scl(A) then B is sαrw closed set in X.

Proof: Given that A is sαrw closed in X then we need to prove B is also sαrw closed in X. A rω-open set U in X such that B ⊆ U. Given that A⊆B and A is sαrw closed set, scl(A) ⊆ U. Now B ⊆ scl(A) => scl (B) ⊆ scl(scl (A)) = scl(A) ⊆ U => scl(B) ⊆ U. Hence B is sαrw closed set in X. The converse is untrue which is proved through example below

Example 3.19: Let X= {1, 2, 3, 4} and τ = {X, ϕ,{1},{2,3},{1, 2, 3}} assume A={4}, B = {1, 4} where A&B are sαrw closed sets in X, such that A⊆ B which does not belong to scl(A),since scl(A)={4}

Theorem 3.20: Whenever A is regular open as well as A is αgr closed, and then A is sαrw closed set in X.

Proof: Given that A is regular open and αgr closed, rω-opens set U in X such that A⊆U. From definition we write, scl(A)⊆A ,where scl(A)⊆A⊆U. Thus A is sαrw closed set in X.

Remark 3.21: Whenever A is regular open as well as A is sαrw closed, A need not be αgr –closed. The example 3.22 proves the above remark

Example 3.22: Let X= {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3}, {1, 2, 3}} be the topology. Let A= {2, 3} is regular open and sαrw closed but not αgr closed in X.

Theorem 3.23: A is ω-open as well as ωα closed then A is sαrw closed set in X.

Proof: Given A is ω-open and ωα closed, any rω open set, say U in X such that A⊆U. Through definition we write, scl(A)⊆A therefore scl(A)⊆A⊆U, which proves A is sαrw closed set in X.

Remark 3.24 In general, when A is ω-open and sαrw closed, A need not be ωα closed. The example 3.25 proves the above remark

Example 3.25: Let X= {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3}, {1, 2, 3}} be the topology. Assume A= {1} is ω-open and sαrw closed and is not ωα closed in X.

Theorem 3.26: whenever A is open as well as αg closed, A is sαrw closed set in X.

Proof: Given that A be open and αg closed and U is any rω-open set in X A⊆ U. From definition we write, scl(A) ⊆A then scl(A) ⊆A⊆U, which proves A is sαrw closed set in X.

Remark 3.27 Example 3.28 proves that when A is open as well as sαrw closed, it need not be αg- closed

Example 3.28 Let X= {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3}, {1, 2, 3}} it's topology. Assume A= {2, 3} is open and sαrw closed but not αg closed in X.

Theorem 3.29: whenever A is regular open and A is rwg closed, A is sαrw closed set in X.

Proof: Let A be regular open and rwg closed where U is any rω-open set in X such that. A⊆U From definition we write, cl(int(A)) ⊆A we also know that cl(int(A)) ⊆ cl(int(cl(A))) ⊆A ⊆ U, then scl(A) ⊆U, which proves A is sαrw closed set in X.

Remark 3.30 The example 3.31 proves when A is both regular open and sαrw closed, A need not be rwg closed.

Example 3.31: Let X= {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3}, {1, 2, 3}} be the topology. Let A = {1} is regular open and sαrw closed and is not rwg closed in X

Corollary 3.32: When A is open and wg closed, A will be sαrw closed set in X.

Proof: It follows, each regular open is open set and each wg- closed is rwg closed set from theorem 3.29 (29)

Remark 3.33 In general, whenever A is open as well as sαrw closed, it need not be wg closed which is proven in an example 3.34.

Example 3.34: Let X= {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3}, {1, 2, 3}} be the topology. Assume A= {2, 3} is open and sαrw closed and is not wg closed in X.

Theorem 3.35: whenever A be regular open as well as gpr closed, A is s sαrw closed set in X.

Proof: Given A be regular open as well as gpr closed where U is any rω-open set in X such that A⊆U from definition we write, pcl(A) ⊆A. We know that pcl(A) ⊆ scl(A) ⊆A. Therefore scl(A) ⊆U, which proves A is sαrw closed set in X.

Remark 3.36: Example 3.37 proves when A is regular open as well as sαrw closed, A need not be gpr- closed

Example 3.37: Let X = {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3}, {1, 2, 3}} be the topology. Assume A= {1} is regular open and sαrw closed and is not gpr closed in X.

Corollary 3.38: whenever A be open and gp closed, A is sαrw closed set in X.

Proof: It follows each regular open is open and each gp closed set is gpr closed set from theorem 3.1 (15)

Remark 3.39: Example 3.40 proves whenever A is open and sαrw closed, A need not be gp closed

Example 3.40: Let X = {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3}, {1, 2, 3}} be the topology. Assume A= {2, 3} is open and sαrw closed and is not gp closed in X.

Theorem 3.41: A is subset of a topology X. Whenever A is regular open as well as sαrw closed then it is α closed.

Proof: Given A is regular open and sαrw closed, as each regular open is rω-open. Now A⊆A then definition of sαrw closed, scl(A) ⊆ A and also A⊆ scl(A) then scl(A)=A. This proves A is α closed.

Theorem 3.42: A is a subset of a topology X. Whenever A is regular semi open as well as rω closed then it is sαrw closed.

Proof: Given A is regular semi open as well as rω closed set such that A is subset of U, rω-open set U in X. Now A⊆ A by hypothesis cl (A) ⊆ A, we also know scl(A) ⊆ cl(A) ⊆ A⊆ U, which proves A is sαrw closed set in X.

Remark 3.43: whenever A is regular semi open as well as sαrw closed, it need not be rω closed

Example 3.44: Let X = {1, 2, 3, 4} and τ = {X, ϕ {1}, {2, 3}, {1, 2, 3}} be the topology. Assume A = {2, 3} is regular semi open and sαrw closed and not rω closed in X.

Theorem 3.45: A is subset of a topology X. Whenever A is semi open and ω closed then it is sαrw closed.

Proof: Given A is semi open and ω closed set such that A is subset of U, rω open set U in X. Now A ⊆ A by hypothesis cl(A) ⊆ A ,we also know scl(A) ⊆ cl(A) ⊆ A ⊆ U, which proves A is sαrw closed set in X.

Remark 3.46: Examples 3.47 prove when A is both semi open and sαrw closed; it need not be ω - closed.

Example 3.47: Let X = {1, 2, 3, 4} and τ = {X ϕ,{1},{2, 3}, {1, 2, 3}} be the topology. Assume A= {1} is semi open and sαrw closed and not ω closed in X.

Theorem 3.48: A is a subset of a topology X. Whenever A is both regular semi open and gprw closed then it is sαrw closed.

Proof: Given A is a regular semi open and gprw closed set such that A is subset of U, rω-open set U in X. Now A⊆ A by hypothesis pcl(A) ⊆A ,we also know that pcl(A) ⊆ scl(A) ⊆ A, therefore scl(A) ⊆ U, which proves A is sαrw closed set in X.

Remark 3.49: The example 3.50 shows when A is both regular semi open and sαrw closed, it need not be gprw closed.

Example 3.50: Let X = {1, 2, 3, 4} and τ = {X ϕ, {1}, {2, 3}, {1, 2, 3}} be the topology. Assume A= {2, 3} is regular semi open as well as sαrw closed but not gprw- closed in X.

Theorem 3.51: Let A be subset of a topology X, whenever A is both regular semi-open and rgw closed then it is sαrw closed.

Proof: Given A is a regular semi open and rgw closed set such that A subset of U, rω-open set U in X, now A⊆ A through hypothesis cl(int(A)) ⊆ A we also know that cl(int(A)) ⊆ cl(int(cl(A))) ⊆ A, therefore scl(A) ⊆ U. A is sαrw closed set in X.

Remark 3.52: Example 3.53 proves when A is regular semi open and sαrw closed it need not be rgw closed.

Example 3.53: Let X = {1, 2, 3, 4} and τ = {X, ϕ, {1}, {2, 3}, {1, 2, 3}} be the topology. Let A = {2, 3} is regular semi open as well as sαrw closed and not rgw closed in X.

REFERENCES

1. Abd El-Monsef M. E., El-Deeb S.N & R.A.Mahmoud(1983) β-open sets and β-continuous mappings Bull. Fac. Sci. Assiut Univ 12, 77-90.)
2. Andrijevic D., (1986), Semi-preopen sets, Mat. Vesnik., 38(1), 24-32.
3. Anitha M. & Thangavely P., (2005), On pre generalized pre regular closed sets (pgpr) Acta ciencia Indian , 31 M(4) , 1035-1040.
4. Arya S. P. and Nour T.M., (1990), chatcterizations of s-normal spaces, Indian J. Pure Appl, Math 21, 717-719.
5. BenchEachi S. S and Wali R.S., (2007), On rω- closed sets is Topological Spaces, Bull, Malays, Math, sci, soc30, 99-110.
6. BenchEachi S. S., Patil P. G. & Nalwad P.N, (2014), Generalized wαclosed sets is Topological Spaces, J. of new results in science, 7, 7-19.
7. BenchEachi S. S., Patil P. G. and Rayanagaudar T. D., (2009), ωαclosed sets is Topological Spaces, The Global. J. Appl. Math. and Math. Sci. 2, 53-63.
8. Bhattacharya P. and Lahiri B.K., (1987), Semi-generalized closed sets in topology, Indian J. Math., 29, 376-382.
9. Bhattacharya S., (2011) on generalized regular closed sets, Int J.Contemp .Math science Vol.6, 145-152.
10. Biswas N., (1970), On characterization of semi-continuous functions, Atti Accad. Naz. Lincei Rend,Cl. Sci. Fis. Mat. Natur. 48(8), 399–402.
11. Cameron D.E. (1978) Properties of sclosed spaces, prac Amer Math, soc 72,581-586.
12. Dontchev J. and Noiri T., (2000), Quasi-normal spaces and πgclosed sets, Acta Math. Hungar.89 (3), 211–219.
13. Dontchev J. (1995), On generalizing semi-preopen sets, Mem. Fac. Sci. Kochi Univ. Ser.A. Math., 16, 35- 48.
14. Gnanambal Y., (1997), on generalized preregular closed sets in topological spaces, Indian J. Pure. Appl. Math., 28(3), 351-360.
15. Janaki C. & Thomas R., (2012), on R*- closed sets in Topological Spaces, Int J of Math Archive 3(8), 3067- 3074.
16. Jastad O. N (1965) on some Classes of nearly open sets, Pacific J. Math15, 961- 970.
17. Jayalakshmi A. & Janaki C., (2012), On ωgrα closed sets in Topological Spaces,IntJ of maths 3(6), 2386-2392.
18. Joshi V., Gupta S., Bhardwaj N. & kumar R, (2012), On Generalised pre Regular weakly(gprω)closed set in sets in Topological Spaces,Int math foruro Vol(7), (40)1981-1992.
19. Levine N., (1970), Generalized closed sets in topology, Rend. Circ Mat. Palermo, 19(2), 89-96.
20. Levine N ., (1963), Semi-open sets and semi-continuity in topological spaces,70,36- 41.
21. Maki H., (1994), Devi and R. & Balachandran K., Associated topologies of generalized αclosed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser.A. Math., 15, 51-63.
22. Maki H., (1993), Devi and R. & Balachandran K., Generalized αclosed sets in topology, Bull. Fukuoka Univ. Ed. part- III 42, 13–21.
23. Maki H., Umehara J.and Noiri T., (1996), Each Topological space is pre T½ mem Fac sci, Kochi univ, Math,17,33-42.
24. Mashhour A.S., Abd El-Monsef M.E. and El-Deeb S.N., (1982), On pre-continuous and weak pre continuous mappings, Proc. Math. Phys. Soc. Egypt, 53, 47-53.
25. Mashhour A.S., Abd El-Monsef M.E. and El-Deeb S.N., α-open mappings Acta Nath Hungar 41, 1983, 213-218.
26. Mishra S., et, al, (2012), On regular generalized weakly (rgw) closed sets in topological spaces, Int. J. of Math Analysis Vol 6, No.(30), 1939-1952.
27. Nagaveni N., (1999), Studies on Generalizations of Homeomorphisms in Topological Spaces, Ph.D. Thesis, Bharathiar University, Coimbatore,.
28. Palaniappan N. and Rao K.C., (1993), Regular generalized closed sets, Kyungpook, Math. J., 33(2), 211-219.
29. Pushpalatha A.,(2000), Studies on generalizations of mapping in topological spaces, PhD Thesis, Bharathiar university, Coimbatore.
30. Shlya Isac Mary T. & Thangavelv P., (2010), On Regular pre-semi closed sets in topological spaces, KBM J. of Math Sc & comp Applications, (1), 9-17.
31. Stone M., (1937), Application of the theory of Boolean rings to general topology, Trans. Amer. Math.Soc.41, 374-481.
32. Sundaram P. and Sheik John M., (2000), On w closed sets in topology, Acta Ciencia Indica 4, 389–439.

33. Vadivel A.& Vairamamanickam K., (2009), rgα closed sets& rgα-open sets in Topological Spaces, Int J of math, Analysis Vol 3, 37,1803-1819.
34. Veera Kumar M. K. R. S(2002A),g*-pre closed sets, Acts Ciencia indica, 28(1), 51-60.
35. Veera Kumar M. K. R. S.,(2002B), On α-generalized regular closed sets, Indian J. of Math , 44(2), 165-181.
36. Velicko N. (1968), H closed topological spaces, Trans. Amer. Math. Soc78, 103-118.
37. Wali R. S. and Mendalgeri P. S., (2014), On α Regular w closed sets in topological spaces, Int. J. of Mathemaics Archive-5(10), 68-76.

Source of support: Nil, Conflict of interest: None Declared.