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On b*ĝ - continuous functions and b*ĝ - open maps in Topological Spaces

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ABSTRACT

*In this paper, we define new class of functions namely b*ĝ-continuous functions and b*ĝ-open maps and we prove some of their basic properties. Also, we introduce a new class of b*ĝ-homeomorphisms and we prove some of their relationship among other homeomorphisms. Throughout this paper f: (X, τ) → (Y, σ) is a function from a topological space (X, τ) to a topological space (Y, σ).*

Keywords: *b*ĝ-continuous functions, b*ĝ-irresolute functions, b*ĝ-open maps, b*ĝ-closed maps.*

AMS Mathematics Subject Classification (2010): *54A05.*

1. INTRODUCTION

In 1996, D. Andrijevic[2] introduced b-open sets in topology and studied its properties. In 1970, N.Levine[9] introduced generalized closed sets and studied their basic properties. In 2003, M.K.R.S.Veerakumar[16] defined ĝ-closed sets in topological spaces and studied their properties. b*-closed sets have been introduced and investigated by Muthuvel[11] in 2012. In 2016, K.Bala Deepa Arasi and G.Subasini[4] introduced b*ĝ-closed sets and studied its properties. K.Balachandran et al introduced the concept of generalized continuous maps in Topological spaces.

These concepts motivate us to define a new version of maps b*ĝ-continuous, b*ĝ-irresolute and b*ĝ-open maps. Also, we prove some properties of these functions and establish the relationships between b*ĝ-continuous and other continuous functions.

2. PRELIMINARIES

Throughout this paper (X,τ) (or simply X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X,τ), Cl(A), Int(A) and A^c denote the closure of A, interior of A and the complement of A respectively. We are giving some basic definitions.

Definition: 2.1 A subset A of a topological space (X,τ) is called
1. a semi-open set[10] if A⊆Cl(Int (A)).
2. an α-open set[5] if A⊆Int(Cl(Int(A))).
3. a b-open set [2] if A⊆Cl(Int(A)) ∪ Int(Cl(A)).
4. a regular open set[14] if A=Int(Cl(A)).

The complement of semi-open (resp. α-open, regular open) set is called semi-closed (resp. α-closed, regular closed) set. The intersection of all semi-closed (resp. α-closed, regular closed) sets of X containing A is called the semi-closure (resp. α-closure, regular closure) of A and is denoted by sCl(A) (resp. αCl(A), rCl(A)). The family of all b*ĝ-open (resp. α-open, semi-open, b-open, regular open) subsets of a space X is denoted by b*ĝO(X) (resp. αO(X), sO(X), bO(X), rO(X)).

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Definition 2.2: A subset A of a topological space (X, τ) is called
1. a generalized closed set (briefly g-closed) [11] if Cl(A) ⊆U whenever A⊆U and U is open in X.
2. a gs-closed set[3] if sCl(A) ⊆ U whenever A⊆U and U is open in X.
3. a gb-closed set[1] if bCl(A) ⊆ U whenever A⊆U and U is open in X.
4. a ĝ-closed set[17] if Cl(A) ⊆ U whenever A⊆U and U is semi-open in X.
5. a bĝ-closed set[16] if bCl(A) ⊆ U whenever A⊆U and U is ĝ–open in X.
6. a gr*-closed set[9] if rCl(A) ⊆ U whenever A⊆U and U is g-open in X.
7. a g*s-closed set[15] if sCl(A) ⊆ U whenever A⊆U and U is gs-open in X.
8. a (gs)*-closed set[7] if Cl(A) ⊆ U whenever A⊆U and U is gs-open in X.
9. a b*ĝ-closed set[4] if b*Cl(A) ⊆ U whenever A⊆U and U is ĝ–open in X.

Definition 2.3: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a
1. continuous [18] if $f^{-1}(V)$ is closed in X for every closed set V in Y.
2. semi-continuous [8] if $f^{-1}(V)$ is semi-closed in X for every closed set V in Y.
3. α-continuous [5] if $f^{-1}(V)$ is α-closed in X for every closed set V in Y.
4. regular continuous[13] if $f^{-1}(V)$ is regular closed in X for every closed set V in Y.
5. gs-continuous [6] if $f^{-1}(V)$ is gs-closed in X for every closed set V in Y.
6. gb-continuous [19] if $f^{-1}(V)$ is gb-closed in X for every closed set V in Y.
7. bĝ-continuous [17] if $f^{-1}(V)$ is bĝ -closed in X for every closed set V in Y.
8. g*s-continuous[15] if $f^{-1}(V)$ is g*s-closed in X for every closed set V in Y.
9. gr*-continuous [9] if $f^{-1}(V)$ is gr*-closed in X for every closed set V in Y.
10. (gs)*-continuous [7] if $f^{-1}(V)$ (gs)*-closed in X for every closed set V in Y.

Definition 2.4: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a
1. open map[18] if $f(V)$ is open in Y for every open set V in X.
2. semi-open map[8] if $f(V)$ is semi-open in Y for every open set V in X.
3. α-open map[5] if $f(V)$ is α-open in Y for every open set V in X.
4. regular open map[13] if $f(V)$ is regular open in Y for every open set V in X.
5. gs-open map[6] if $f(V)$ is gs-open in Y for every open set V in X.
6. gb-open map[19] if $f(V)$ is gb-open in Y for every open set V in X.
7. bĝ-open map[17] if $f(V)$ is bĝ-open in Y for every open set V in X.
8. g*s-open map[15] if $f(V)$ is g*s-open in Y for every open set V in X.
9. gr*-open map[9] if $f(V)$ is gr*-open in Y for every open set V in X.
10. (gs)*-open map[7] if $f(V)$ is (gs)*-open in Y for every open set V in X.

Definition 2.5: A space (X, τ) is called a
1. T_b-space [3], if every gs-closed set in it is closed.
2. T_{gs}-space [1], if every gb-closed set in it is b-closed.
3. $T_{b\hat{g}}$-space [16], if every bĝ-closed set in it is b-closed.
4. $T^*_{b\hat{g}}$-space [16], if every bĝ-closed set in it is closed.
5. $T_{b*\hat{g}}$-space [4], if every b*ĝ-closed set in it is closed.

Remark 2.6: The family of all b*ĝ-closed (resp.α-closed, semi-closed, b-closed, regular closed) subsets of a space X is denoted by b*ĝC(X) (resp. αC(X), sC(X), bC(X), rC(X)).

3. b*ĝ-CONTINUOUS AND b*ĝ-IRRESOLUTE FUNCTIONS

We introduce the following definitions.

Definition 3.1: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be b*ĝ-continuous map if the inverse image of every closed set in (Y, σ) is b*ĝ-closed in (X, τ).

That is, $f^{-1}(V)$ is b*ĝ-closed of (X, τ) for every closed set V of (Y, σ).

Example 3.2: Let X = Y = {a,b,c} with topologies τ = {X,ϕ,{a},{b},{a, b}} and σ = {Y,ϕ,{a},{b, c}}. Define a map $f: (X, \tau) \rightarrow (Y, \sigma)$ by$f(a) = b$, $f(b) = a$, $f(c) = c$. Here, f is b*ĝ-continuous, since the inverse images of C(Y) {b, c} and {a} are {a,c} and {b} respectively which are b*ĝC(X).

Definition 3.3: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be b*ĝ-irresolute map if the inverse image of every b*ĝ -closed set in (Y, σ) is b*ĝ-closed in (X, τ).

That is, $f^{-1}(V)$ is b*ĝ-closed of (X, τ) for every b*ĝ-closed set V of (Y, σ).

Example 3.4: Let X = Y = {a, b, c} with topologies τ = {X, {b}, {c}, {b, c}} and σ = {Y, ϕ, {a, c}}. Define a map $f:(X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = c$, $f(b) = a$, $f(c) = b$. Here, f is b*ĝ-continuous, since the inverse images of b*ĝC(Y) {b, c}, {a, b} and {b} are {a, b},{a, c} and {a} respectively which are b*ĝC(X).

Proposition 3.5:

a) Every continuous map is b*ĝ-continuous.
b) Every α-continuous map is b*ĝ-continuous.
c) Every semi-continuous map is b*ĝ-continuous.
d) Every regular-continuous map is b*ĝ-continuous.
e) Every gr*-continuous map is b*ĝ-continuous.
f) Every g*s-continuous map is b*ĝ-continuous.
g) Every (gs)*-continuous map is b*ĝ-continuous.

Proof:

a) Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be continuous. Let V be a closed set in (Y,σ). Since f is continuous, $f^{-1}(V)$ is closed set in (X,τ). By proposition 3.4 in [4], $f^{-1}(V)$ is b*ĝ-closed in (X,τ). Hence f is b*ĝ-continuous.
b) Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be α-continuous. Let V be a closed set in (Y,σ). Since f is α-continuous, $f^{-1}(V)$ is α-closed set in (X,τ). By proposition 3.6 in [4], $f^{-1}(V)$ is b*ĝ-closed in (X,τ). Hence f is b*ĝ-continuous.
c) Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be semi-continuous. Let V be a closed set in (Y,σ). Since f is semi-continuous, $f^{-1}(V)$ is semi-closed set in(X,τ). By proposition 3.6 in [4], $f^{-1}(V)$ is b*ĝ-closed in (X,τ). Hence f is b*ĝ-continuous.
d) Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be regular-continuous. Let V be a closed set in (Y,σ). Since f is regular-continuous, $f^{-1}(V)$ is regular-closed set in (X,τ). By proposition 3.6 in [4], $f^{-1}(V)$ is b*ĝ-closed in (X,τ). Hence f is b*ĝ-continuous.
e) Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be gr*-continuous. Let V be a closed set in (Y,σ). Since f is gr*-continuous, $f^{-1}(V)$ is gr*-closed set in (X,τ). By proposition 3.16 in [4], $f^{-1}(V)$ is b*ĝ-closed in (X,τ). Hence f is b*ĝ-continuous.
f) Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be g*s-continuous. Let V be a closed set in (Y,σ). Since f is g*s-continuous, $f^{-1}(V)$ is g*s-closed set in (X,τ). By proposition 3.18 in [4], $f^{-1}(V)$ is b*ĝ-closed in (X,τ). Hence f is b*ĝ-continuous.
g) Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be (gs)*-continuous. Let V be a closed set in (Y,σ). Since f is (gs)*-continuous, $f^{-1}(V)$ is (gs)*-closed set in (X,τ). By proposition 3.20 in [4], $f^{-1}(V)$ is b*ĝ-closed in (X,τ). Hence f is b*ĝ-continuous.

The following examples show that the converse of the above proposition need not be true.

Example 3.6:

a) Let X = Y = {a, b, c} with topologies = {X, {a}} and σ = {Y, ϕ, {a}, {a, b},{a, c}}. Define a function $f:(X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = a, f(b) = c, f(c) = b$. Here, f is b*ĝ-continuous but not continuous, since the inverse image of C(Y) {b, c},{c} and {b} are {b, c},{b} and {c} which are b*ĝC(X) but not C(X).
b) Let X=Y={a, b, c} with topologies τ={X,ϕ,{a},{c},{a, c},{b, c}} and σ = {Y, ϕ,{a},{b, c}}. Define a function $f:(X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = b$, $f(b) = a, f(c) = b$. αC(X) = {X,ϕ, {a},{a, b},{b, c}}. Here, f is b*ĝ-continuous but not α-continuous, since the inverse image of C(Y) {b, c} and {a} are {a, b} and {b} which are b*ĝC(X) but not αC(X).
c) Let X=Y={a, b, c} with topologies τ = {X,ϕ,{a, c}} and σ = {Y,ϕ,{a}}. Define a function $f:(X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = c, f(b) = a, f(c) = b$. sC($X$)={$X$, ϕ,{b}}. Here, f is b*ĝ -continuous but not semi-continuous, since the inverse image of C(Y) {b,c} is {a,b} which is b*ĝC(X) but not sC(X).
d) Let X=Y={a, b, c} with topologies τ ={X, ϕ,{a},{b},{a, b}} and σ ={Y,ϕ, {a},{a, b},{a, c}}. Define a function $f:(X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = b, f(b) = c, f(c) = a$. rC($X$) = {$X$,$\phi$, {a, c},{b, c}}. Here, f is b*ĝ-continuous but not regular continuous, since the inverse image of C(Y) {b, c}, {c} and {b} are {a, c}, {a} and {c} which are b*ĝC(X) but not rC(X).
e) Let X = Y = {a, b, c} with topologies τ = {X, {b}} and σ = {Y, ϕ, {c}, {a, b}}. Define a function $f:(X,\tau) \rightarrow (Y,\sigma)$ by $(a) = c$, $f(b) = b, f(c) = a$. gr*C(X) = {X,ϕ,{a, c}}. Here, f is b*ĝ-continuous but not gr*-continuous, since the inverse image of C(Y) {a, b} and {c} are {b, c} and {a} which are b*ĝC(X) but not gr*C(X).
f) Let X = Y = {a, b, c} with topologies τ = {X, {b}} and σ = {Y, ϕ, {c}, {a, b}}. Define a function $f:(X,\tau) \rightarrow (Y,\sigma)$ by $(a) = c$, $f(b) = b, f(c) = a$. g*sC(X) = {X,ϕ,{a, c}}. Here, f is b*ĝ-continuous but not g*s-continuous, since the inverse image of C(Y) {a,b} and {c} are {b, c} and {a} which are b*ĝ C(X) but not g*sC(X).
g) Let X = Y = {a, b, c} with topologies = {X, {b}, {c}, {b, c}} and σ = {Y, ϕ, {a, c}}. Define a function $f:(X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = b, f(b) = c$, $f(c) = a$. (gs)*C(X)= {X,ϕ, {a},{a, b},{a, c}}. Here, f is b*ĝ-continuous but not (gs)*-continuous, since the inverse image of C(Y) {b} is {c} which is b*ĝC(X) but not (gs)*C(X).

Proposition: 3.7

a) Every b*ĝ-continuous is gb-continuous.
b) Every b*ĝ-continuous is gs-continuous.
c) Every b*ĝ-continuous is bĝ-continuous.

Proof:

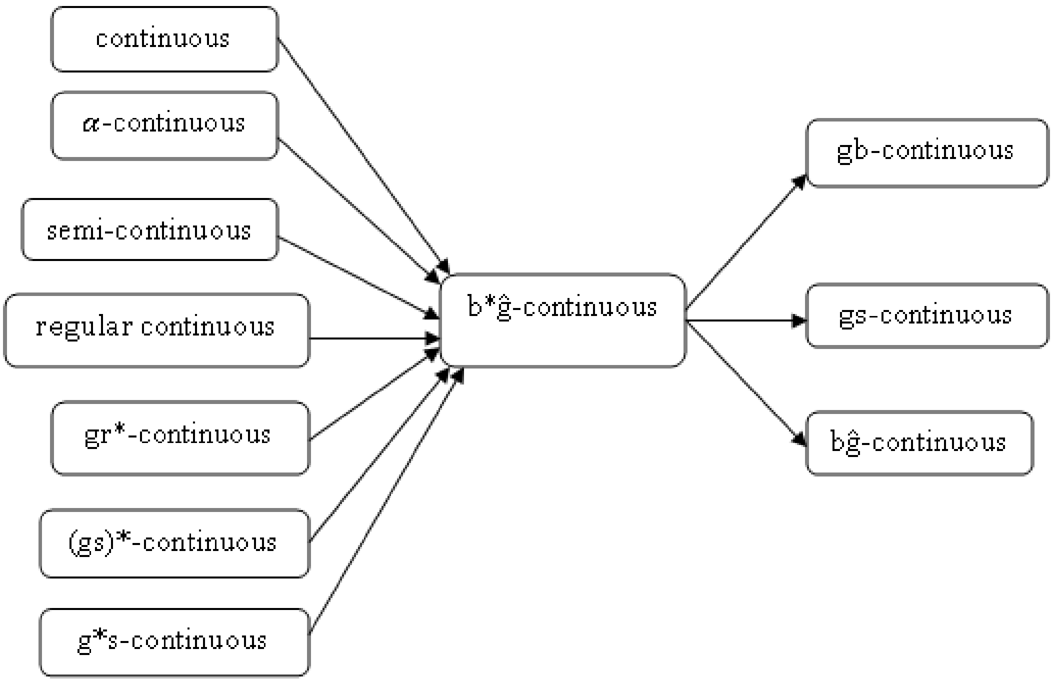
a) Let $f:(X,\tau)\to(Y,\sigma)$ be b*ĝ-continuous. Let V be a closed set in (Y,σ). Since f is b*ĝ-continuous,$f^{-1}(V)$ is b*ĝ-closed set in (X,τ). By proposition 3.12 in [4], $f^{-1}(V)$ is gb-closed in (X,τ). Hence f is gb-continuous.
b) Let $f:(X,\tau)\to(Y,\sigma)$ be b*ĝ-continuous. Let V be a closed set in (Y,σ). Since f is b*ĝ-continuous, $f^{-1}(V)$ is b*ĝ-closed set in(X,τ). By proposition 3.8 in [4], $f^{-1}(V)$ is gs-closed in (X,τ). Hence f is gs-continuous.
c) Let $f:(X,\tau)\to(Y,\sigma)$ be b*ĝ-continuous. Let V be a closed set in (Y,σ). Since f is b*ĝ-continuous,$f^{-1}(V)$ is b*ĝ-closed set in (X,τ). By proposition 3.10 in [4], $f^{-1}(V)$ is bĝ-closed in (X,τ). Hence f is bĝ-continuous.

The following examples show that the converse of the above proposition need not be true.

Example: 3.8

a) Let X = Y = {a, b, c} with topologies τ = {X,ϕ,{a, c}} and σ = {Y,ϕ,{a},{b}, {a, b},{a, c}}. Define a function $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=c, f(b)=a, f(c)=b$. b*ĝC(X)= {X,ϕ, {b},{a, b},{b, c}} and gbC(X)={X, ϕ,{a}, {b},{c },{a, b},{b, c}}. Here, f is gb-continuous but not b*ĝ -continuous, since the inverse image of C(Y) {b, c},{a, c},{c} and {b} are {a, b},{b, c},{b} and {a} which are gbC(X) but not b*ĝC(X).
b) Let X = Y = {a, b, c} with topologies τ = {X, {a}, {b, c}} and σ = {Y, ϕ, {a}}. Define a function $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=c, f(b)=a, f(c)=b$. b*ĝC(X)={X,ϕ,{a},{b, c}} and gsC(X)={X,ϕ,{a},{b}, {c},{a, b}, {b, c},{a, c} }. Here, f is gs-continuous but not b*ĝ -continuous, since the inverse image of C(Y) {b, c} is {a, b} which is gsC(X) but not b*ĝC(X).
c) Let X = Y = {a, b, c} with topologies τ = {X,,{c},{a, b}} and σ = {Y,ϕ,{a}}. Define a function $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b, f(b)=a, f(c)=c$. b*ĝC(X)={X,ϕ,{c},{a, b}} and bĝC(X)={X,ϕ,{a}, {b},{c}, {a, b},{b, c},{a, c} }. Then f is bĝ-continuous but not b*ĝ -continuous, since the inverse image of C(Y) {b, c} is {a, c} which is bĝC(X) but not b*ĝC(X).

Remark: 3.9 The following diagram shows the relationships of b*ĝ-continuous functions with other known existing functions. A→B represents A implies B but not conversely.



Proposition 3.10: Every b*ĝ-irresolute is b*ĝ-continuous.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be b*ĝ-irresolute. Let V be closed in (Y,σ). By proposition 3.4 in [4], V is b*ĝ-closed in (Y,σ). Since f is b*ĝ-irresolute, $f^{-1}(V)$ is a b*ĝ-closed set in (X,τ). Hence f is b*ĝ-continuous.

The following example shows that the converse of the above proposition need not be true.

Example 3.11: Let X = Y = {a, b, c} with topologies τ = {X, {a, c}} and σ = {Y, ϕ, {b}}. Define a function $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = b, f(b) = a, f(c) = c$. b*ĝC($X$)= {$X$,$\phi$, {b},{a, b},{b, c}} and b*ĝC(Y)={Y,ϕ,{a},{c},{a, b},{b, c},{a, c}}. Here, f is b*ĝ-continuous but not b*ĝ -irresolute, since the inverse image of C(Y) {a,c} is {b, c} which is b*ĝC(X) but the inverse image of b*ĝC(Y) {a},{c}, {a, b},{b, c} and {a, c} are {b},{c},{a, b},{a, c} and {b, c} which are not b*ĝC(X).

Proposition 3.12: Let $f:(X,\tau) \to (Y,\sigma)$ be a b*ĝ-continuous map. If (X,τ) is $T_{b*ĝ}$ -space then f is continuous.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be b*ĝ-continuous. Let V be a closed set in (Y,σ). Since f is b*ĝ-continuous, $f^{-1}(V)$ is b*ĝ-closed set in (X,τ). Since (X,τ) is $T_{b*ĝ}$ -space, $f^{-1}(V)$ is closed set in (X,τ). Hence f is continuous.

Proposition 3.13: Let $f:(X,\tau) \to (Y,\sigma)$ be a b*ĝ-continuous map. If (X,τ) is $T_{bĝ}$-space then f is b-continuous.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be b*ĝ-continuous. Let V be a closed set in (Y,σ). Since f is b*ĝ-continuous, $f^{-1}(V)$ is b*ĝ-closed set in (X,τ). By proposition 3.10 in [4], $f^{-1}(V)$ is bĝ-closed set in (X,τ). Since (X,τ) is $T_{bĝ}$-space, $f^{-1}(V)$ is b-closed set in (X,τ). Hence f is b-continuous.

Proposition 3.14: Let $f:(X,\tau) \to (Y,\sigma)$ be a b*ĝ-continuous map. If (X,τ) is T_{gs}-space then f is b-continuous.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be b*ĝ-continuous. Let V be a closed set in (Y,σ). Since f is b*ĝ-continuous, $f^{-1}(V)$ is b*ĝ-closed set in (X,τ). By proposition 3.12 in [4], $f^{-1}(V)$ is gb-closed set in (X,τ). Since (X,τ) is T_{gs}-space, $f^{-1}(V)$ is b-closed set in (X,τ). Hence f is b-continuous.

Proposition 3.15: Let $f:(X,\tau) \to (Y,\sigma)$ be a b*ĝ-continuous map. If (X,τ) is T_b-space then f is continuous.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be b*ĝ-continuous. Let V be a closed set in (Y,σ). Since f is b*ĝ-continuous, $f^{-1}(V)$ is b*ĝ-closed set in (X,τ). By proposition 3.8 in [4], $f^{-1}(V)$ is gs-closed set in (X,τ). Since (X,τ) is T_b-space, $f^{-1}(V)$ is closed set in (X,τ). Hence f is continuous.

Proposition 3.16: Let $f:(X,\tau) \to (Y,\sigma)$ be a b*ĝ-continuous map. If (X,τ) is $T^*_{bĝ}$ -space then f is continuous.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be b*ĝ-continuous. Let V be a closed set in (Y,σ). Since f is b*ĝ-continuous, $f^{-1}(V)$ is b*ĝ-closed set in (X,τ). By proposition 3.10 in [4], $f^{-1}(V)$ is bĝ-closed set in (X,τ). Since (X,τ) is $T^*_{bĝ}$-space, $f^{-1}(V)$ is closed set in (X,τ). Hence f is continuous.

4. b*ĝ-OPEN MAPS and b*ĝ-CLOSED MAPS

We introduce the following definitions.

Definition 4.1: Let X and Y be two topological spaces. A map $f:(X,\tau) \to (Y,\sigma)$ is called b*ĝ -open map if for each open set V of X, $f(V)$ is b*ĝ-open set in Y.

That is, image of every open set in (X,τ) is b*ĝ-open in (Y,σ).

Definition 4.2: Let X and Y be two topological spaces. A map $f:(X,\tau) \to (Y,\sigma)$ is called b*ĝ -closed map if for each closed set V of X, $f(V)$ is b*ĝ-closed set in Y.

That is, image of every closed set in (X,τ) is b*ĝ-closed in (Y,σ).

Example 4.3: Let X = Y = {a, b, c} with topologies τ = {X, {a},{b, c}} and σ ={Y, ϕ,{a}}. Define a function $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = b, f(b) = a, f(c) = c$. Then f is b*ĝ-open map, since the image of O(X) {a},{b, c} are {b},{a, c} which are b*ĝO(Y). Also, f is b*ĝ-closed map.

Proposition 4.4:

a) Every open map is b*ĝ-open map.
b) Every α-open map is b*ĝ-open map.
c) Every semi-open map is b*ĝ-open map.
d) Every regular open map is b*ĝ-open map.
e) Every gr*-open map is b*ĝ-open map.
f) Every g*s-open map is b*ĝ-open map.
g) Every (gs)*-open map is b*ĝ-open map.

Proof:

a) Let $f:(X,\tau)\to(Y,\sigma)$ be an open map and V be an open set in (X,τ). Since f is an open map, $f(V)$ is an open set in (Y,σ). By proposition 3.4 in [4], $f(V)$ is an b*ĝ-open set in (Y,σ). Hence f is b*ĝ-open map.
b) Let $f:(X,\tau)\to(Y,\sigma)$ be an α-open map and V be an open set in (X,τ). Since f is an α-open map, $f(V)$ is an α-open set in (Y,σ). By proposition 3.6 in [4], $f(V)$ is an b*ĝ-open set in (Y,σ). Hence f is b*ĝ-open map.
c) Let $f:(X,\tau)\to(Y,\sigma)$ be an semi-open map and V be an open set in (X,τ). Since f is an semi-open map, $f(V)$ is an semi-open set in (Y,σ). By proposition 3.6 in [4], $f(V)$ is an b*ĝ-open set in (Y,σ). Hence f is b*ĝ-open map.
d) Let $f:(X,\tau)\to(Y,\sigma)$ be an regular open map and V be an open set in (X,τ). Since f is an regular open map, $f(V)$ is an regular open set in(Y,σ). By proposition 3.6 in [4], $f(V)$ is an b*ĝ-open set in (Y,σ). Hence f is b*ĝ-open map.
e) Let $f:(X,\tau)\to(Y,\sigma)$ be an gr*-open map and V be an open set in (X,τ). Since f is an gr*-open map, $f(V)$ is an gr*-open set in (Y,σ). By proposition 3.16 in [4], $f(V)$ is an b*ĝ-open set in (Y,σ). Hence f is b*ĝ-open map.
f) Let $f:(X,\tau)\to(Y,\sigma)$ be an g*s-open map and V be an open set in (X,τ). Since f is an g*s-open map, $f(V)$ is an g*s-open set in (Y,σ). By proposition 3.18 in [4], $f(V)$ is an b*ĝ-open set in (Y,σ). Hence f is b*ĝ-open map.
g) Let $f:(X,\tau)\to(Y,\sigma)$ be an (gs)*-open map and V be an open set in (X,τ). Since f is an (gs)*-open map, $f(V)$ is an (gs)*-open set in (Y,σ). By proposition 3.20 in [4], $f(V)$ is an b*ĝ-open set in (Y,σ). Hence f is b*ĝ-open map.

The following example shows that the converse of the above proposition need not be true.

Example 4.5:

a) Let X = Y = {a, b, c} with topologies τ = {X, {a}, {b}, {a, b}, {a, c}} and = {Y, ϕ,{a}}. Define a function $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=a, f(b)=c, f(c)=b$. b*ĝO(Y)= {Y,ϕ, {a},{b},{c},{a, b},{a, c}}. Here, f is b*ĝ-open map but not open map, since the image of O(X) {a},{b},{a, b} and {a, c} are {a},{c},{a, c} and {a,b} which are b*ĝO(Y) but not O(Y).
b) Let X = Y = {a, b, c} with topologies τ = {X, ϕ,{a},{b},{a, b}} and σ = {Y,ϕ,{a, b}}. Define a function $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b, f(b)=a, f(c)=c$. b*ĝO(Y)= {Y,ϕ, {a},{b},{a, b}} and αO(Y) ={Y,ϕ,{a, b}}. Here, f is b*ĝ-open map but not α-open map, since the image of O(X) {a},{b} and {a, b} are {b},{a} and {a, b} which are b*ĝO(Y) but not αO(Y).
c) Let X = Y = {a, b, c} with topologies τ = {X, ϕ, {a},{b},{a, b}} and σ = {Y, ϕ,{a}}. Define a function $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=c, f(b)=a, f(c)=b$. b*ĝO(Y)= {Y,ϕ,{a}, {b},{c},{a, b},{a, c}} and sO(Y) = {Y,ϕ,{a},{a, b},{a, c}}. Here, f is b*ĝ-open map but not semi-open map, since the image of O(X) {a},{b} and {a, b} are {c},{a} and {a, c} which are b*ĝO(Y) but not sO(Y) .
d) Let X = Y = {a, b, c} with topologies τ = {X, {a},{b, c}} and σ = {Y,ϕ, {a},{b},{a, b}}. Define a function $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b, f(b)=a, f(c)=c$. b*ĝ O(Y)={Y,ϕ,{a},{b},{a, b},{b, c},{a, c}} and rO(Y)={Y, ϕ,{a },{b}}. Here, f is b*ĝ-open map but not regular open map, since the image of O(X) {a} and {b,c} are {b} and {a, c} which are b*ĝO(Y) but not rO(Y).
e) Let X=Y={a, b, c} with topologies τ={X,ϕ,{a},{b},{a, b},{a, c}} and σ={Y, ϕ,{a}, {b},{a, b}}. Define a function $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=a, f(b)=b, f(c)=c$. b*ĝO(Y)={Y,ϕ,{a},{b},{b, c},{a, b},{a, c}} and gr*O(Y)={Y, ϕ,{a},{b},{a, b}}. Here, f is b*ĝ-open map but not gr*-open map, since the image of O(X) {a},{b},{a, b} and {a, c} are {a},{b},{a, b} and {a, c} which are b*ĝO(Y) but not gr*O(Y).
f) Let X = Y = {a, b, c} with topologies τ = {X, ϕ,{a},{b},{a, b}} and σ = {Y, ϕ,{b}}. Define a function $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=c, f(b)=b, f(c)=a$. b*ĝO(Y)={Y,ϕ, {a}, {b},{c},{a, b},{b, c}} and g*sO(Y) ={Y,ϕ, {b},{a, b}, { b, c}}. Here, f is b*ĝ-open map but not g*s-open map, since the image of O(X) {a},{b} and {a, b} are {c},{b} and {b, c} which are b*ĝO(Y) but not g*sO(Y).
g) Let X = Y = {a, b, c} with topologies τ = {X, ϕ,{c},{a, b}} and σ = {Y,ϕ,{b },{c},{b, c}}. Define a function $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=a, f(b)=c, f(c)=b$. b*ĝO(Y)={Y,ϕ,{b},{c},{a, b},{b, c},{a, c}} and (gs)*O(Y)= {Y, ϕ,{b}, {c},{b, c}}. Here, f is b*ĝ-open map but not (gs)*-open map, since the image of O(X) {c} and {a, b} are {b} and {a,c} which are b*ĝO(Y) but not (gs)*O(Y).

Proposition 4.6:

a) Every b*ĝ-open map is gs-open map.
b) Every b*ĝ-open map is gb-open map.
c) Every b*ĝ-open map is bĝ-open map.

Proof:

a) Let $f:(X,\tau)\to(Y,\sigma)$ be a b*ĝ-open map and V be an open set in X. Since f is b*ĝ-open map, $f(V)$ is b*ĝ-open set in Y. By proposition 3.8 in [4], $f(V)$ is gs-open set in Y. Hence f is gs-open map.

b) Let $f:(X,\tau) \to (Y,\sigma)$ be a b*ĝ-open map and V be an open set in X. Since f is b*ĝ-open map, $f(V)$ is b*ĝ-open set in Y. By proposition 3.12 in [4], $f(V)$ is gb-open set in Y. Hence f is gb-open map.

c) Let $f:(X,\tau) \to (Y,\sigma)$ be a b*ĝ-open map and V be an open set in X. Since f is b*ĝ-open map, $f(V)$ is b*ĝ-open set in Y. By proposition 3.10 in [4], $f(V)$ is bĝ-open set in Y. Hence f is bĝ-open map.

The following example shows that the converse of the above proposition need not be true.

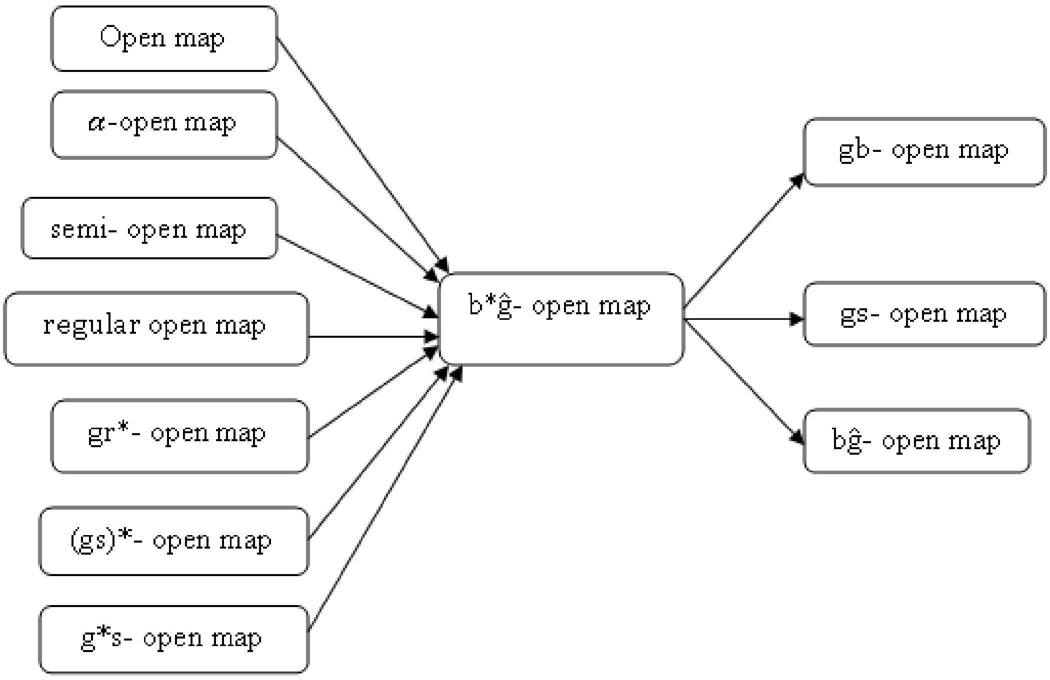
Example 4.7:

a) Let X = Y = {a, b, c} with topologies τ = {X, ϕ, {a}, {b},{a, b}} and σ = {Y,ϕ, {a},{b, c}}. Define a function $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = c, f(b) = a,\ f(c) = b$. b*ĝO($Y$)={$Y$,$\phi$, {a},{b,c}} and gsO($Y$)={$Y$,$\phi$,{a},{b},{c},{a, b},{b, c},{a, c}}. Here, f is gs-open map but not b*ĝ-open map, since the image of O(X) {a},{b} and {a, b} are {c},{a} and {a,c} which are gsO(Y) but not b*ĝO(Y).

b) Let X = Y = {a, b, c} with topologies τ = {X, ϕ,{b},{c},{b, c}} and σ = {Y,ϕ, {a},{b, c}}. Define a function $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = c, f(b) = a,\ f(c) = b$. b*ĝ O($Y$)={$Y$,$\phi$, {a},{b, c}} and gbO($Y$)={$Y$,$\phi$,{a},{b},{c},{a, b},{b, c},{a, c}}. Here, f is gb-open map but not b*ĝ -open map, since the image of O(X) {b},{c} and {b, c} are {a},{b} and {a, b} which are gbO(Y) but not b*ĝO(Y).

c) Let X = Y = {a, b, c} with topologies τ = {X, ϕ,{a},{b},{a, b}} and σ = {Y,ϕ, {c},{a, b}}. Define a function $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = b, f(b) = c,\ f(c) = a$. b*ĝ O($Y$)={$Y$,$\phi$, {a, b},{c}} and bĝO($Y$)={$Y$,$\phi$,{a},{b},{c},{a, b}, {b, c},{a, c}}. Here, f is bĝ-open map but not b*ĝ-open map, since the image of O(X) {a},{b} and {a, b} are {b},{c} and {b, c} which are bĝO(Y) but not b*ĝO(Y).

Remark 4.8: The following diagram shows the relationships of b*ĝ-open map with other known existing open maps. A→B represents A implies B but not conversely.



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