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EDGE NON-EDGE CROSSING NUMBER OF BIPARTITE GRAPH OF $\Gamma(Z_n)$

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ABSTRACT

Let R be a commutative ring and let Z (R) be its set of zero- divisors. We associate a graph $\Gamma(R)$ to R with vertices Z $(R)^*$= Z (R)-{0}, the set of non- zero zero divisors of R and for distinct u, v $\in$Z $(R)^*$, the vertices u and v are adjacent if and only if uv = 0 [1,2]. In this paper we introduce the edge non-edge crossing number of bipartite zero divisor graphs. We evaluate for any non-outer planar graph, the minimum number of crossings between an edge and a non-edge whose edges are simple arcs, by framing definition for the drawing D of ENE crossing number.

Subject Classification: *05C25, 05C10, 52C35.*

Keywords: *Crossing number, edge crossing number, Zero Divisor Graph, Rectilinear crossing number.*

1. INTRODUCTION

Planarity is of great significance from a theoretical point of view [3, 4]. In addition planarity and other related concepts are useful in many practical situations [4, 5]. For instance, in the design of a printed-circuit board, the electrical engineer must know if he can make the required connections without an extra layer of insulation [6]. In this paper we introduce the Edge - Non-edge crossing number of bipartite zero divisor graphs. We define the drawing D of Edge - Non-edge crossing number. Here we find for any non-outer planar graph, the minimum number of crossings between an edge and a non edge whose edges are simple arcs. So, we define a drawing G for such graphs, so that the minimum number of crossings are obtained. Let G be a simple non-outer planar graph with vertex set V(G) and edge set E(G).

An *Edge-Edge* crossing is the minimum number of crossing between two edges of G by a simple arc, but not a common vertex. An E*dge - non-Edge* crossing is the minimum number of crossings between an edge of G and the Jordan arc connecting two non-adjacent vertices of G and is denoted by ENE [cr(G)]. The Drawing D of ENE crossing number, of any connected graph G_n, for finding the minimum edge - non-edge crossings, is obtained by first drawing the largest possible star graph (subgraph $k_{1,n}$) from G in a circular manner. The rest of the vertices in G, can be placed either above or below the star graph, so that the maximum crossings is attained between the edges of G, but not with the edges of star graph. Strictly, the vertices must not be placed between the edges of star graph. Now the remaining vertices that are connected with the star graph are placed step by step, such that the minimum number of vertex sets are placed first which follows with the maximum one. Consequently, minimum number of ENE crossings can be attained between the edges and non- edges of G, for all outer planar graphs. Obviously, the edge - non-edge crossing number for an outer planar graph is zero [7].

Although the Drawing D is very tedious to obtain the minimum ENE crossing, we actually splitted the zero divisor graph into star graph connected with either complete graphs or complete bipartite graphs [7, 8, 9]. In the following section we prove the ENE crossing number of various zero divisor graphs [1, 2].

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2. THE ENE CROSSING NUMBER OF $\Gamma(Z_n)$

Theorem 2.1: For any graph $\Gamma(Z_{2p})$, where p is any prime, the ENE [cr((Z2p))] = 0.

Proof: Using the theorem in [7], $\Gamma(Z_{2p})$ is a planar star graph. That is $K_{1,p-1}$ is a star graph with centre as p. Clearly, the ENEcr $[\Gamma(Z_{2p})] = 0$.

Theorem 2.2: If p and q are distinct prime number with q > p, then
ENEcr $[\Gamma(Z_{pq})] = 2Q^2 + [Q^2 - 1] + [Q^2 - (1 + 2)] + \ldots + [Q^2 - (1 + 2 +\ldots+ Q)] + [R - (Q - 1)] + \ldots + [R- (Q - 1) - (Q - 2) - \ldots 2]$, where $Q = \frac{q-3}{2}$, $Q^2 = \left(\frac{q-3}{2}\right)^2$, $R = [Q^2 - (1 + 2 + \ldots + Q)]$

Proof: The vertex set of $\Gamma(Z_{pq})$ is $V[\Gamma(Z_{pq})] = \{p,2p, \ldots , p(q-1), q, 2q, \ldots, (p-1)q\}$ and $V|\Gamma(Z_{pq})| = p+q-2$. Let $V_1 = \{ p, 2p, \ldots , p(q-1)\}$, $V_2 = \{ q, 2q, \ldots, (p-1)q\}$, u = 2p and v=3q then uv = 6pq. Therefore, pq divides 6pq. But if u = p, v = 2p then uv = $2p^2$. That is pq does not divides $2p^2$ which means all the vertices in V_1 are non-adjacent. Similarly, all the vertices in V_2 are non-adjacent. But V_1 and V_2 are adjacent to each other. Therefore $K_{p-1,q-1}$ is a complete bipartite graph.

Now the drawing D of G involves in taking out the maximum star graph in $K_{p-1,q-1}$. Since p < q, we select any one vertex from among (p-1) vertices in V_2, say q. With q as centre point all the (q-1) vertices in V_1 are placed in a circular manner, such that a star graph $K_{p-1,q-1}$ is formed. The remaining (p-1) vertices say {2q, 3q,…, q(p 1)} vertices are placed far below the star graph $K_{p-1,q-1}$ in a straight line. Since $V_1 \cup V_2$ are adjacent, a bipartite graph is formed. Although we maximize the edge-edge crossings from outside, we aim to minimize the edge- non-edge crossings from inside. The edge - non-edge crossings will be minimum, only if the crossings pass among the edges in star graph $K_{p-1,q-1}$. As p < q and in the graph $\Gamma(Z_{pq})$, has q as the maximum number of vertices, always this (q - 1) vertices involves in drawing of the star graph $K_{1,q-1}$. Then clearly ENE cr $[(V_1, V_2)] = 0$.

Therefore the ENE crossings depend only on the choice of q. Even though we vary p, the ENE crossings depend only on q. Therefore the ENE crossings will be same for all pq, for particular q, and for any p. This will be clearer from the following cases.

Case-(i): Let p = 3,

Subcase-(i): Let q = 5, the vertex set of $\Gamma(Z_{15})$ is {3, 6, 9, 12, 5, 10}. Let V_1 = {3, 6, 9, 12} and V_2 = {5, 10}.

Therefore $K_{2,4}$ is a complete bipartite graph. With 5 as centre, a star graph $K_{1,4}$ is formed with 4 vertices in V_1. The edge - non-edge crossings of 4 vertices in $K_{1,4}$ is,

$$\begin{aligned}
\text{ENEcr } [\Gamma(Z_{15})] &= \text{ENEcr } [(V_1, V_1)] \\
&= \text{ENEcr } [(3, 6) + (3, 9) + (3, 12)] \\
&= 0 + 1 + 1 + 0 \\
&= 2 \\
&= 2 \times 1 \\
&= 2 \times \left(\frac{q-3}{2}\right)^2 + [1 - 1] \\
&= 2Q^2
\end{aligned}$$

where $Q = \frac{5-3}{2} = 1$, $Q^2 = 1$, $R = [1 - 1] = 0$

Subcase-(ii): Let q = 7, the vertex set of $\Gamma(Z_{21})$ is {3, 6, 9,…, 18, 7, 14}. Let V_1 = {3, 6, 9,…, 18} and V_2 = {7, 14}.

Therefore $K_{2,6}$ is a complete bipartite graph. With 7 as centre, a star graph $K_{1,6}$ is formed with 6 vertices in V_1. The edge - non-edge crossings of 6 vertices in $K_{1,6}$ is,

$$\begin{aligned}
\text{ENEcr } [\Gamma(Z_{21})] &= \text{ENEcr } [(V_1, V_1)] \\
&= \text{ENEcr}[(3,6) + \ldots + (3,18) +(6,9) + \ldots + (6,18) + (9,12) + (9,18)] \\
&= 4 + 4 + 3 + 1 = 12 = 2 \times 4 + (3 + 1) \\
&= 2 \times \left(\frac{7-3}{2}\right)^2 + [4 – 1] + [4 – (1 + 2)] \\
&= 2 \times \left(\frac{q-3}{2}\right)^2 + [Q^2 – 1] + [Q^2 – (1 + 2)]
\end{aligned}$$

where $Q = \frac{7-3}{2} = 2$, $Q^2 = 4$, $R = [4 - 1] = 3$

Case-(ii): Let p = 5

Subcase-(i): Let q = 7, The vertex set of $\Gamma(Z_{35})$ is {5, 10,…, 30, 7, 14,…,28}. Let V_1 = {5, 10,…, 30} and V_1 = {7, 14,…,28}. Therefore $K_{4,6}$ is a complete bipartite graph. With 7 as centre, a star graph $K_{1,6}$ is formed with 6 vertices in V_1. The edge - non-edge crossings of 6 vertices in $K_{1,6}$ is, ENEcr $[\Gamma(Z_{35})]$ = 12 (using above subcase (ii))

Subcase-(ii): Let q = 11,The vertex set of $\Gamma(Z_{55})$ is {5, 10, … , 50, 11, 22, … , 44} Here V_1 = {5, 10,…, 50} and V_2 = {11, 22, … 44}. Therefore $K_{4,10}$ is a complete bipartite graph. With 11 as centre, a star graph $K_{1,10}$ is formed with 11 vertices in V_1. The edge - non-edge crossings between 10 vertices in $K_{1,10}$ is,

$$\begin{aligned}
\text{ENEcr}[\Gamma(Z_{55})] &= \text{ENEcr}[(V_1, V_1)] \\
&= \text{ENEcr}[(5, 10) + \dots + (5, 50) + (10, 15) + \dots + (10, 50) + (15, 20) + \dots \\
&\quad + (15, 50) + \dots + (45, 50)] \\
&= 2 \text{ x } (1 + 2 + 3 + 4 + 3 + 2 + 1) + (1 + 2 + 3 + 4 + 3 + 2) + (1 + 2 + 3 + 4 + 3) \\
&\quad + (1 + 2 + 3 + 4) + (1 + 2 + 3) + (1 + 2) + (1) \\
&= 80 = 2 \text{ x } 16 + (15 + 13 + 10 + 6 + 3 + 1) \\
&= 2 \text{ x } \left(\frac{11-3}{2}\right)^2 + [16 - 1] + [16 - (1 + 2)] + [16 - (1 + 2 + 3) + (10 - 4) + (10 - 4 - 3) \\
&\quad + (10 - 4 - 3 - 2)] \\
&= 2\left(\frac{q-3}{2}\right)^2 + [Q^2 - 1] + [Q^2 - (1 + 2)] + \dots + [Q^2 - (1 + 2 + 3)] + R + [R - (Q - 1)] \\
&\quad + [R - (Q - 1) - (Q - 2)]
\end{aligned}$$

where $Q = \frac{11-3}{2} = 4$, $Q^2 = 16$, R = [16 - (1 + 2 + 3 + 4)] = 6.

Theorem 2.3: For any graph $\Gamma(Z_{pqr})$, p = 2, q = 3, r > 3, ENE cr $[\Gamma(Z_{6r})]$ = pq(r - 1) + 2p and if, p = 3, q = 5 and r > 5, ENE cr $[\Gamma(Z_{15r})] = q (p + q) (p + q - 1) + (q - 1)^2(r - 1) + p (p + q)^2 (r -1)$

Proof: The vertex set of $\Gamma(Z_{pqr})$ is,
V $[\Gamma(Z_{pqr})]$ = {p, 2p,…, p(qr - 1), q, 2q,…, q(pr - 1), r, 2r, … , (pq - 1)r} and V$|\Gamma(Z_{pqr})|$ = 2p(r - 1) + (pq - 1). Let V_1 = {qr, 2qr,…, (p - 1)qr}, V_2 = {pr, 2pr,…, (q - 1)pr}, V_3 = {r, 2r,…, (pq - 1)r}, V_4 = {pq, 2pq,…, (r - 1)pq}, V_5 = {q, 3q, 5q,…,(pr - 1)q}, V_6 = {p, 2p,…, (qr - 1)p}. Let u = qr, v_1 = pr, v_2 = pq, v_3 = p. Then $uv_1 = pqr^2$, $uv_2 = prq^2$, uv_3 = pqr divides pqr . This implies, the vertices in V_1 are adjacent to all the vertices in V_2, V_4, V_6. Therefore, $K_{p-1,q-1}$ and $K_{p-1,2(r-1)}$ are bipartite graphs.

Let x = pr, y_1 = pq, y_2 = q. Then $xy_1 = p^2qr$, xy_2 = pqr divides pqr . This implies, the vertices in V_2 are adjacent to all the vertices in V_4, V_5. Therefore, $K_{q-1,2(r-1)}$ is a complete bipartite graph. Similarly, let w = r and z = q. Then wz = rq divides pqr. This implies the vertices in V_3 are adjacent to the vertices in V_5, which is again a complete bipartite graph $K_{q-1,r-1}$. To find the Edge - non-edge crossings, the drawing of the graph $\Gamma(Z_{pqr})$ is obtained by arranging the vertices in the following manner. The vertices of V_6 are kept in an upper Semi-circular way followed by the vertices in V_1 which are kept in a straight line. The third row comprises the vertices of V_2 and V_3, where V_3 lies in the middle of V_2, such that half the vertices of V_2 are placed in one side and the other half on other side of V_3 horizontally. This again is followed by the vertices of V_4 and V_5, where the vertices of V_4 are placed in the middle of the vertices in V_5, in which the vertices of V_5 are divided equally in a lower semi-circular way.Therefore, the structure of the drawing is as follows. The complete bipartite graph $K_{p-1,2(r-1)}$ is followed by two bipartite graphs $K_{p-1,r-1}$ and $K_{p-1,2(r-1)}$ which in turn are followed by $K_{q-1,r-1}$ and $K_{q-1,2(r-1)}$, which are all connected. The proof is by induction. Case (i): Let p = 2, q = 3, r > 3

Subcase-(i): Let r = 5

The vertex set of $\Gamma(Z_{30})$ is {2, 4,…,28, 3, 6,…,24, 5, 10,…,25} and V$|\Gamma(Z_{30})|$=21, V_1 = {15}, V_4 = {6, 12, 18, 24}, V_2 = {10, 20}, V_5 = {3, 9, 21, 27}, V_3 = {5, 25}, V_6 = {2, 4,8,…, 28}. It is observed that,

ENE cr$[(V_1,V_2) + (V_1 ,V_3) + (V_1, V_4) + (V_1,V_5) + (V_1, V_6)] = 0$

That is ENE cr$\left[\sum_{j=2}^{6}(V_i, V_j) = 0\right]$, i= 1

ENE cr$[(V_2, V_4) + (V_2, V_5) + (V_2, V_6)] = 0$

That is ENE cr$\left[\sum_{j=4}^{6}(V_i, V_j) = 0\right]$, i= 2

ENE cr$[(V_4 ,V_5) + (V_4, V_6)] = 0$

That is ENE cr$\left[\sum_{j=5}^{6}(V_i, V_j) = 0\right]$, i= 4

ENE cr$[(V_5, V_6)] = 0$

Therefore, the ENE crossings contributes from the vertices of V_3. That is,

ENE cr[(V_3, V_2) + (V_3, V_5) + (V_3, V_6)] = ENEcr[(5, 20) + (5, 25) + (10, 25)] + ENEcr[(25, 3) + (25, 9) + (25, 21) + (25, 27)] + ENEcr[(5, 3) +… + (5, 27)] + ENEcr[(5, 2) +… + (5, 28)] + ENEcr[(25, 2) +…+(25, 28)]
= [(1 + 2 + 1) + (4 + 4) + (8 + 8)] = 4 + 8 + 16 = 2 x 2 + 2(5 - 1) + 4(5 - 1)
= 2 +2(r - 1) + 4(r - 1) = 2p + 6(r - 1) = 2p + pq(r - 1)

Subcase-(ii): Let r = 7
The vertex set of $\Gamma(Z_{42})$ is {2, 4,…, 40, 3, 6,…, 36, 7, 14,…,35}and V| $\Gamma(Z_{42})$| = 29, V_1 = {21}, V_4 = {6, 12,…,36}, V_2 = {14, 28}, V_5 = {3, 9,…,39}, V_3 = {7, 35}, V_6 = {2, 4, 8,…,40}. It is observed that,

ENE cr$\left[\sum_{j=2}^{6}(V_i, V_j) = 0\right]$, i= 1
ENE cr$\left[\sum_{j=4}^{6}(V_i, V_j) = 0\right]$, i= 2
ENE cr$\left[\sum_{j=5}^{6}(V_i, V_j) = 0\right]$, i= 4
ENE cr[(V_5, V_6)] = 0

Therefore, the ENE crossings contributes from the vertices of V_3. That is,

ENEcr[(V_3,V_2) + (V_3,V_5) + (V_3,V_6)]
= ENEcr[(7, 28) + (7, 35) + (14, 35)] + ENEcr[(35, 3) + (35, 9) +…+ (35, 39)] + ENEcr[(7, 3) +… + (7, 39)] + ENEcr[(7, 2) +… + (7, 40)] + ENEcr[(35, 2) +…+(35, 40)]
= [(1 + 2 + 1) + (6 + 6) + (12 + 12)] = 4 + 12 + 24
= 2 x 2 + 2(7 - 1) + 4(7 - 1) = 2 +2(r -1) + 4(r -1) = 2p + 6(r -1)
= 2p + pq(r -1).

Case-(ii): p=3, q=5, r>5.

Subcase-(i): Let r = 7
The vertex set of $\Gamma(Z_{105})$ is {3, 6, 9,…, 102, 5, 10,…, 100, 7, 14,…,35}and V| $\Gamma(Z_{42})$|= 50, V_1 = {35, 70}, V_4 = {15, 30,…,90}, V_2 = {21, 42, 63, 84}, V_5 = {5, 10, 20,…,100}, V_3 = {7, 14,…, 98}, V_6 = {3, 6,…, 102}, V'_2= {21, 42}, V''_2= {63, 84}, V'_3= {7, 14,…,49}, V''_3= {56, 77,.., 98}. It is observed that,

ENEcr$\left[\sum_{j=4}^{6}(V_i, V_j) = 0\right]$, i= 1

That is ENEcr$\left[\sum_{j=4}^{6}(V_i, V_j) = 0\right]$, i= 2
ENEcr[(V_4,V_5) + (V_4,V_6) + (V_5,V_6)] = 0

Therefore, the ENE crossings contributes from,
ENEcr[(V_1,V_3) + (V_2, V_3) + (V_2,V_6) + (V_3, V_5) + (V_3 ,V_6)]

Consider ENEcr[(V_1,V_3)] = ENEcr[(V_1, V'_3) + (V_1,V''_3)]
= ENEcr[(35, 7) + … + (35, 49) + (70, 7) + …+ (70, 49)]
+ENEcr[(35, 56) +…+ (35, 98) + (70, 56) +… + (70, 98)]
= [4(2) + 4(4) + 4(4) + 4(2)]
= 8 + 16 + 16 + 8 = 48

Consider ENEcr[(V_2,V_3)] = ENEcr[(V'_2,V'_3) + (V'_2,V''_3) + (V'_2,V''_2)]
+ ENEcr[(V'_3,V''_3) + (V'_3,V''_2) + (V''_3, V''_2)]
= ENEcr[(21, 7) +… + (21, 49) + (42, 7) +… + (42, 49)]
+ENEcr[(21, 56) +…+ (21, 98) + (42, 56) +…+ (42, 98)]
+ENEcr[(21, 84) + (21, 63) + (42, 63) + (42, 84)]
+ENEcr[(7, 56) +…+ (7, 98) + (14, 56) +…+ (14, 98)]
+ENEcr[(28, 56) +…+ (28, 98) + (49, 56) +…+ (49, 98)]
+ENEcr[(63, 7) +…+ (63, 49) + (84, 7) +…+ (84, 49)]
+ENEcr[(63, 56) +…+ (63, 98) + (84, 56) +…+ (84, 98)]
= [4(2) + 4(0) + 4(4) + 4(6)] + [1(0) + 1(2) + 1(2)1(4)]
+[4(8(4)) + 4(6) + 4(4) + 4(0) + 4(2)]
= 8 + 16 + 24 + 2 + 2 + 4 + 128 + 24 + 16 + 8 = 232

Consider ENEcr[(V_2,V_6)] = ENEcr[(21, 3) +…+ (21, 102) + (42, 3) +…+ (42, 102)]
+ENEcr[(63, 3) +…+ (63, 102) + (84, 3) +…+ (84, 102)]
= [24(0) + 24(2) + 24(2) + 24(0)] = 96

Consider ENEcr[(V_3,V_5)] = ENEcr[(7, 5) +…+ (7, 100) + (14, 5) +…+ (14, 100) +…]
+ENEcr[(98, 5) +…+ (98, 100)]
= [8(4(12))] = 8(48) = 384

Consider ENEcr[(V_3,V_6)] = ENEcr[(7, 3) +…+ (7, 102) + (14, 3) +…+ (14, 102) +…]
+ENEcr[(98, 3) +…+ (98, 102)]
= [8(4(24))] = 8(96) = 768

⇒ ENEcr[(V_1,V_3) + (V_2,V_3) + (V_2,V_6) + (V_3,V_5) + (V_3,V_6)]= 1528
= 48 + 232 + 96 + 384 + 768
= 280 + 96 + 384 + 768
= (5 x 8 x 7) + (16 x 6) + 1152
= (5 x 8 x 7) + (16 x 6) + 3 x $(8)^2$ x (6)
= (5 x 8 x 7) + (16 x 6) + 3 x $(2 + 3)^2$(7 - 1)
= q(p + q)(p + q - 1) + $(q - 1)^2$(r - 1) + p $(p + q)^2$(r - 1)

Subcase-(ii): Let r = 11 where p = 3, q = 5

The vertex set of $\Gamma(Z_{165})$ is {3, 6, 9,…, 162, 5, 10,…,160, 11, 22,…,154} and V| $\Gamma(Z_{165})$| = 74. Let V_1 = {55, 110}, V_4 = {15, 30,…,150},V_2 = {33, 66, 99, 132}, V_5 = {5, 10, 20,…,160},V_3 = {11, 22,…,154}, V_6 = {3, 6,…, 162}, V'_2= {33, 66}, V''_2= {99, 132},V'_3 = {11, 22, 44, 77}, V''_3= {88, 121, 143, 154}

It is observed that, ENEcr$\left[\sum_{j=4}^{6}(V_i, V_j) = 0\right]$, i= 1

That is ENEcr$\left[\sum_{j=4}^{6}(V_i, V_j) = 0\right]$, i= 2
ENEcr[(V_4,V_5) + (V_4,V_6) + (V_5,V_6)] = 0

Therefore, the ENE crossings contributes from,
ENEcr[(V_1,V_3) + (V_2, V_3) + (V_2,V_6) + (V_3, V_5) + (V_3 ,V_6)]

Consider ENEcr[(V_1,V_3)] = ENEcr[(V_1, V'_3) + (V_1,V''_3)]
= ENEcr[(55, 11) +…+ (55, 77) + (110, 88) +…+ (110, 154)]
= [4(2) + 4(4) + 4(4) + 4(2)] = 8 + 16 + 16 + 8 = 48

Consider ENEcr[(V_2,V_3)] = ENEcr[(V'_2,V'_3) + (V'_2,V''_3) + (V'_2,V''_2)]
+ ENEcr[(V'_3,V''_3) + (V'_3,V''_2) + (V''_3, V''_2)]
= ENEcr[(33,11) +…+ (33, 77) + (66, 11) +…+ (66, 77)]
+ ENEcr[(33, 88) +…+ (33, 154) + (66, 88) +…+ (66, 154)]
+ ENEcr[(33, 99) + (33, 132) + (66, 99) + (66, 132)]
+ ENEcr[(11, 88) +…+ (11, 154) + (22, 88) +…+ (22, 154)]
+ ENEcr[(44, 88) +…+ (44, 154) + (77, 88) +…+ (77, 154)]
+ ENEcr[(99, 11) +…+ (99, 77) + (132, 11) +…+ (132, 77)]
+ ENEcr[(99, 88) +…+ (99, 154) + (132, 88) +…+ (132, 154)]
= [4(2) + 4(0) + 4(4) + 4(6)] + [1(0) + 1(2) + 1(2) + 1(4)]
+[4(8(4)) + 4(6) + 4(4) + 4(0) + 4(2)]
= 8 + 16 + 24 + 2 + 2 + 4 + 128 + 24 + 16 + 8 = 232

Consider ENEcr[(V_2,V_6)] = ENEcr[(33, 3) +…+ (33, 162) + (66, 3) +…+ (66, 162)]
+ENEcr[(99, 3) +…+ (99, 162) + (132, 3) +…+ (132, 162)]
= [40(0) + 40(2) + 40(2) + 40(0)] = 160:

Consider ENEcr[(V_3, V_5)] = ENEcr[(11, 5) +…+ (11, 160) + (22, 5) +…+ (22, 160) +…]
+ ENEcr[(154, 5) +…+ (154, 160)]
= [8(4(20))] = 8(80) = 640

Consider ENEcr[(V_3, V_6)] = ENEcr[(11, 3) +…+ (11, 162) + (22, 3) +…+ (22, 162) +…]
+ENEcr[(154, 3) +…+ (154, 162)]
= [8(4(40))] = 8(160) = 1280

⇒ ENEcr[(V_1,V_3) + (V_2,V_3) + (V_2,V_6) + (V_3,V_5) + (V_3,V_6)] = 2360
= 48 + 232 + 160 + 640 + 1280|
= 280 + 160 + 1920
= (5 x 8 x 7) + (16 x 10) + 1920

$= (5 \times 8 \times 7) + (16 \times 10) + 3 \times (8)^2 \times (10)$
$= (5 \times 8 \times 7) + (16 \times 10) + 3 \times (2 + 3)^2(11- 1)$
$= q(p + q)(p + q - 1) + (q - 1)^2(r - 1) + p (p + q)^2(r - 1)$

CONCLUSION

In the above theorems we find the new approach of planarity by finding the crossing between the edge and non-edge. This can be extended for any composition of graphs. Note that that the edge non-edge crossing number is meant for all graphs except star graphs and complete graphs. Practically, this situation or set up may arise in future for launching network cables, electrical circuits. If we come across a situation in which case we need not care about the crossings of the cables (edges) which are connecting between the networks, but we need to minimize or avoid the crossings of the separators (non-edge) which will be of great use.

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