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ON THE GENERALIZED abc - BLOCK EDGE TRANSFORMATION GRAPHS

B. BASAVANAGOUD*, KEERTHI G. MIRAJKAR[1], POOJA B[2] AND SHREEKANT PATIL[3]

[*,3]Department of Mathematics,
Karnatak University Dharwad - 580 003, Karnataka, India.

[1,2]Department of Mathematics,
Karnatak University's Karnatak Arts College Dharwad - 580 001, Karnataka, India.

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ABSTRACT

Given a graph G with vertex set $V(G)$, edge set $E(G)$ and block set $U(G)$, let $\overline{G}$ be the complement, $L(G)$ the line graph and $B(G)$ the block graph of G. Let G^0 be the graph with $V(G^0) = V(G)$ and with no edges, G^1 the complete graph with $V(G^1) = V(G)$, $G^+ = G$, and $G^- = \overline{G}$. Let $bq(G)$ $(\overline{bq}(G))$ be the graph whose vertices can be put in one to one correspondence with the set of edges and blocks of G in such a way that two vertices of $bq(G)$ $(resp., \overline{bq}(G))$ are adjacent if and only if one corresponds to a block B of G and the other to an edge e of G and e is in (resp., is not in) B. Given $a, b, c \in \{0,1,+,-\}$, the abc - block edge transformation graph $Q^{abc}(G)$ of G is the graph with vertex set $V(Q^{abc}(G)) = E(G) \cup U(G)$ and the edge set $E(Q^{abc}(G)) = E((L(G))^a) \cup E((B(G))^b) \cup E(H)$ where $H = bq(G)$ if $c = +$, $H = \overline{bq}(G)$ if $c = -$, H is the graph with $V(H) = E(G) \cup U(G)$ and with no edges if $c = 0$, H is the complete bipartite graph with parts $E(G)$ and $U(G)$ if $c = 1$. In this paper, we investigate some basic properties such as order, size, vertex degree and connectedness of the these generalized abc - block edge transformation graphs $Q^{abc}(G)$.

2010 Mathematics Subject Classification: *05C12.*

Keywords: *line graph, block graph, abc - block edge transformation graphs.*

1. INTRODUCTION

All the graphs considered here are finite, undirected without loops or multiple edges. We refer to [3] or [4] for unexplained terminology and notation. A *block* of a graph is a maximal nonseparable subgraph. Let $G = (V, E)$ be a graph with block set $U(G)$={B_i; B_i is a block of G}, $|V(G)| = n$, $|E(G)| = m$ and $|U(G)| = r$. The *degree* of a vertex v_i in G is the number of edges incident to v_i and is denoted by $d_i = deg(v_i)$. As usual, K_n be the complete graph of order n, $K_{p,q}$ the complete bipartite graph, $S_{p,q}$ the double star and $d_G(v)$ the degree of a vertex v in G. If a block $B \in U(G)$ with the edge set $\{e_1, e_2, \ldots, e_s; s \geq 1\}$, then we say that the edge e_i and block B are incident with each other, where $1 \leq i \leq s$. If two distinct blocks are incident with a common cutvertex, then they are adjacent blocks. The *degree of a block* B in G, denoted by $d_G(B)$, is the number of blocks adjacent to B in G. We denote the number of edges incident with B in G by $D_G(B)$. The *line graph* $L(G)$ of a graph G is the graph with vertex set as the edge set of G and two vertices of $L(G)$ are adjacent whenever the corresponding edges in G have a vertex in common. The complement of line graph is a jump graph [2]. Let $bq(G)$ $(\overline{bq}(G))$ be the graph whose vertices can be put in one to one correspondence with the set of edges and blocks of G in such a way that two vertices of $bq(G)$ $(resp., \overline{bq}(G))$ are adjacent if and only if one corresponds to a block B of G and the other to an edge e of G and e is in (resp., is not in) B.

2. GENERALIZED abc - BLOCK EDGE TRANSFORMATION GRAPHS

Inspired by the definition of total transformation graphs [5] and block-transformation graphs [1], we introduce the graph valued functions namely generalized abc - block edge transformation graphs.

*Corresponding Author: B. Basavanagoud**
[,3]Department of Mathematics, Karnatak University Dharwad - 580 003, Karnataka, India.*

For a graph $G = (V, E)$, let G^0 be the graph with $V(G^0) = V(G)$ and with no edges, G^1 the complete graph with $V(G^1) = V(G)$, $G^+ = G$, and $G^- = \overline{G}$.

In this paper, we consider some graph valued functions $Q^{abc}: \mathcal{G} \to \mathcal{G}$ from set of graphs $\mathcal{G}$ into $\mathcal{G}$, depending on parameters $a, b, c \in \{0,1,+,-\}$ and call $Q^{abc}(G)$ the generalized abc - block edge transformation graph of G.

Definition: Given a graph G with edge set $E(G)$ and block set $U(G)$ and three variables $a, b, c \in \{0,1,+,-\}$, the generalized abc - block edge transformation graph $Q^{abc}(G)$ of G is the graph with vertex set $V(Q^{abc}(G)) = E(G) \cup U(G)$ and the edge set $E(Q^{abc}(G)) = E((L(G))^a) \cup E((B(G))^b) \cup E(H)$ where

(i) $H = bq(G)$ if $c = +$.
(ii) $H = \overline{bq}(G)$ if $c = -$.
(iii) H is the graph with $V(H) = E(G) \cup U(G)$ and with no edges if $c = 0$.
(iv) H is the complete bipartite graph with parts $E(G)$ and $U(G)$ if $c = 1$.

Thus we obtain 64 abc - block edge transformation graphs $Q^{abc}(G)$. Here note that $Q^{00+}(G) = bq(G)$ and $Q^{00-}(G) = \overline{bq}(G)$.

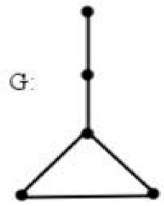

Figure-1: Graph G.

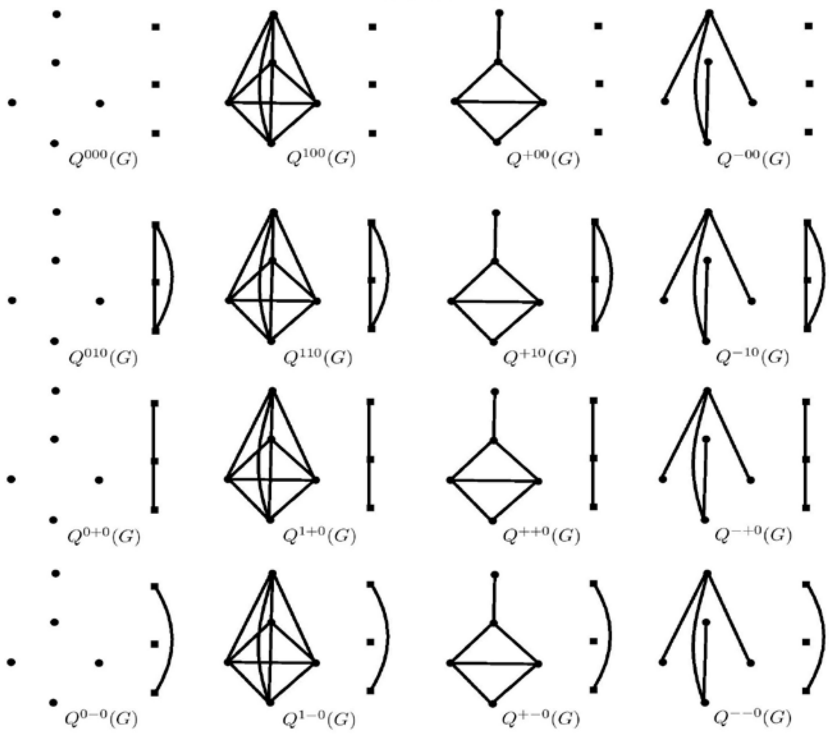

Figure-2: abc - block edge transformation graphs when $c = 0$.

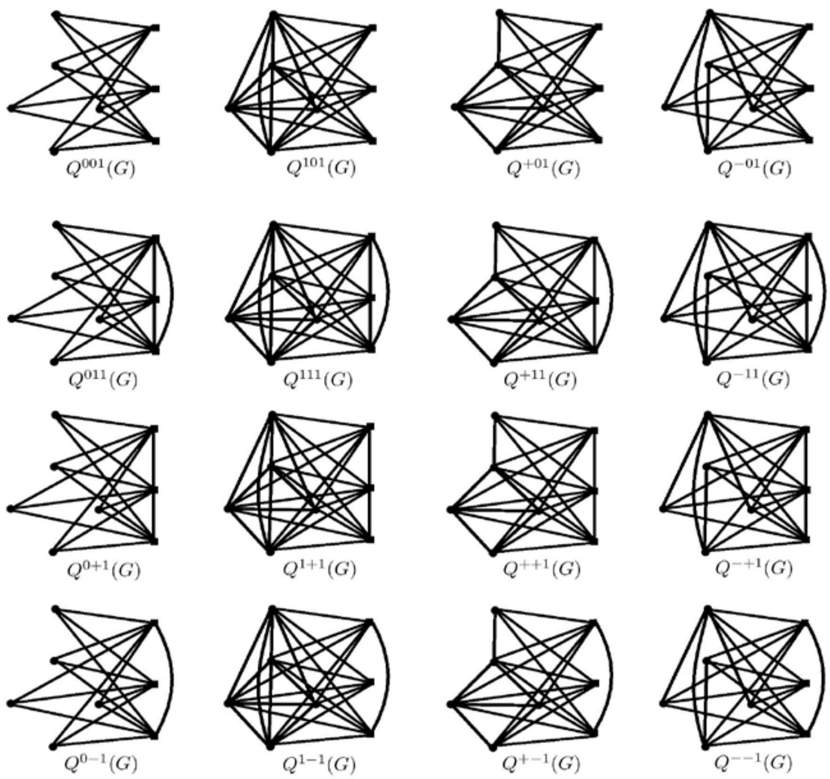

Figure-3: abc - block edge transformation graphs when $c = 1$.

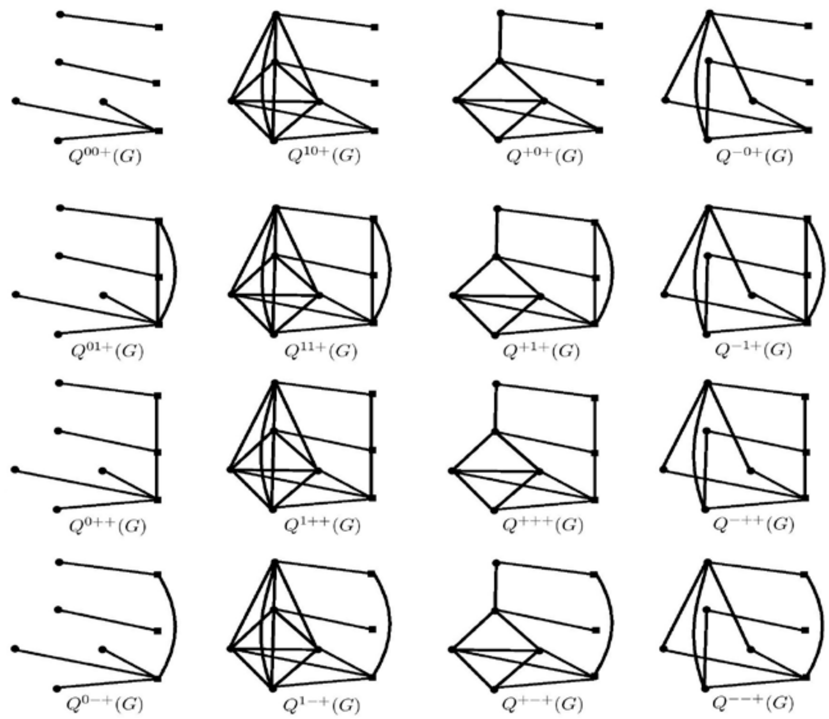

Figure-4: abc - block edge transformation graphs when $c = +$.

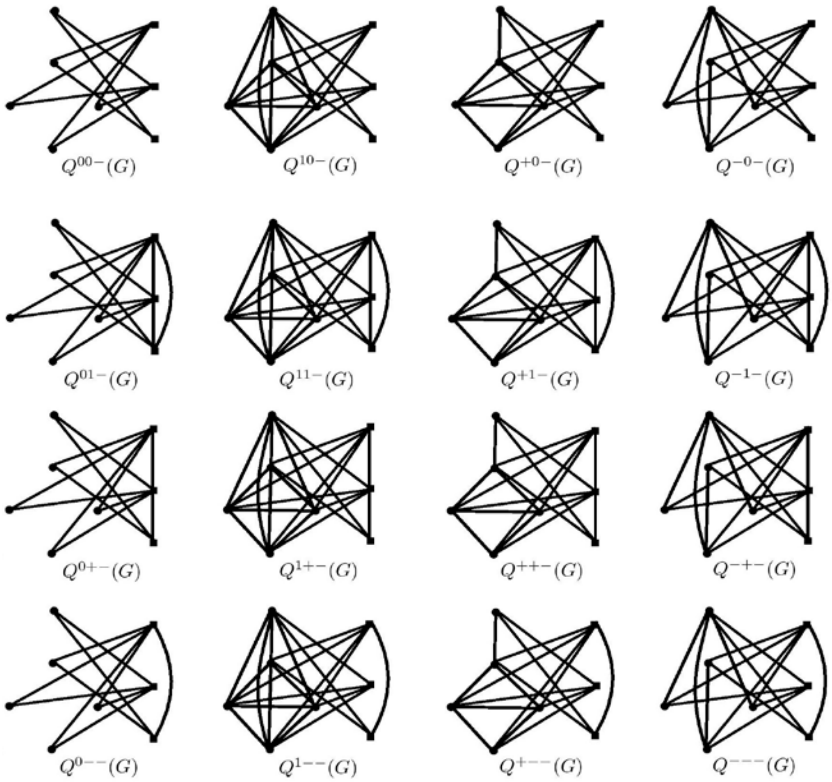

Figure-5: abc - block edge transformation graphs when $c = -$.

A graph G and all its 64 abc - block edge transformation graphs are shown in Figures 1-5. The vertex e_i' of $Q^{abc}(G)$ corresponding to edge e_i of G and is refereed as edge-vertex. The vertex B_i' of $Q^{abc}(G)$ corresponding to block B_i of G and is refereed as block-vertex. In Figure 2 to 5, the edge-vertices are denoted by circles and block-vertices are by squares.

The following remarks will be useful in the proof of our results.

Remark 2.1:
(i) $L(G)$ *is an induced subgraph of* $Q^{+bc}(G)$.
(ii) $\overline{L(G)}$ is an induced subgraph of $Q^{-bc}(G)$.
(iii) K_m is an induced subgraph of $Q^{1bc}(G)$.

Remark 2.2:
(i) $B(G)$ *is an induced subgraph of* $Q^{a+c}(G)$.
(ii) $\overline{B(G)}$ is an induced subgraph of $Q^{a-c}(G)$.
(iii) K_r is an induced subgraph of $Q^{a1c}(G)$.

Remark 2.3:
(i) $bq(G)$ *is a spanning subgraph of* $Q^{ab+}(G)$.
(ii) $\overline{bq}(G)$ is a spanning subgraph of $Q^{ab-}(G)$.
(iii) $K_{m,r}$ is a spanning subgraph of $Q^{ab1}(G)$.

Theorem 2.1 [6]: *Let* G *be a graph of size* $q \geq 1$. *Then* $\overline{L(G)}$ *is connected if and only if* G *contains no edge that is adjacent to every other edge of* G *unless* $G = K_4$ *or* C_4.

Since abc - block edge transformation graphs $Q^{abc}(G)$ are defined on the edge set and block set of a graph G. Isolated vertices of G (if G has) play no rule in $Q^{abc}(G)$, we assume that the graph G under consideration is nonempty and has no isolated vertices. In this paper, we investigate some basic properties such as order, size, vertex degree and connectedness of the these generalized abc - block edge transformation graphs $Q^{abc}(G)$.

3. ORDER, SIZE AND VERTEX DEGREE OF $Q^{abc}(G)$

It is shown in [3] that let $b_G(v)$ be the number of blocks to which vertex v belongs in a connected graph G. Then the number of blocks of G is given by $r = b(G) = 1 - n + \sum_{v \in V(G)} b_G(v)$.

Theorem 3.1: *Let G be an (n, m) connected graph with r blocks and let $b_G(v)$ be the number of blocks to which vertex v belongs in G. Then*

(i) The order of $Q^{abc}(G) = m + r$.

(ii) The size of $Q^{abc}(G)= \begin{cases} |E((L(G))^a)| + |E((B(G))^b)| & if\ \ c = 0. \\ |E((L(G))^a)| + |E((B(G))^b)| + mr & if\ \ c = 1. \\ |E((L(G))^a)| + |E((B(G))^b)| + m & if\ \ c = +. \\ |E((L(G))^a)| + |E((B(G))^b)| + mr - m & if\ \ c = -. \end{cases}$

where

$$E((L(G))^a)= \begin{cases} 0 & if\ \ a = 0. \\ \frac{m(m-1)}{2} & if\ \ a = 1. \\ -m + \frac{1}{2}\sum_{v \in V(G)} d_G^2(v) & if\ \ a = +. \\ \frac{m(m+1)}{2} - \frac{1}{2}\sum_{v \in V(G)} d_G^2(v) & if\ \ a = -. \end{cases}$$

and

$$E((B(G))^b)= \begin{cases} 0 & if\ \ b = 0. \\ \frac{r(r-1)}{2} & if\ \ b = 1. \\ \sum_{v \in V(G)} \frac{b_G(v)(b_G(v)-1)}{2} & if\ \ b = +. \\ \frac{r(r-1)}{2} - \sum_{v \in V(G)} \frac{b_G(v)(b_G(v)-1)}{2} & if\ \ b = -. \end{cases}$$

Theorem 3.2: *Let G be an (n, m)-graph with r blocks. Then the degree of edge-vertex e' $(e = uv$ in $G)$ and block-vertex B' in $Q^{abc}(G)$ when $c = 0$ are*

(i) $d_{Q^{ab0}(G)}(e') = \begin{cases} 0 & if\ \ a = 0\ \&\ b \in \{0,1,+,-\}. \\ m - 1 & if\ \ a = 1\ \&\ b \in \{0,1,+,-\}. \\ d_G(u) + d_G(v) - 2 & if\ \ a = +\ \&\ b \in \{0,1,+,-\}. \\ m + 1 - d_G(u) - d_G(v) & if\ \ a = -\ \&\ b \in \{0,1,+,-\}. \end{cases}$

(ii) $d_{Q^{ab0}(G)}(B') = \begin{cases} 0 & if\ \ b = 0\ \&\ a \in \{0,1,+,-\}. \\ r - 1 & if\ \ b = 1\ \&\ a \in \{0,1,+,-\}. \\ d_G(B) & if\ \ b = +\ \&\ a \in \{0,1,+,-\}. \\ r - 1 - d_G(B) & if\ \ b = -\ \&\ a \in \{0,1,+,-\}. \end{cases}$

Theorem 3.3: *Let G be an (n, m)-graph with r blocks. Then the degree of edge-vertex e' $(e = uv$ in $G)$ and block-vertex B' in $Q^{abc}(G)$ when $c = 1$ are*

(i) $d_{Q^{ab1}(G)}(e') = \begin{cases} r & if\ \ a = 0\ \&\ b \in \{0,1,+,-\}. \\ r + m - 1 & if\ \ a = 1\ \&\ b \in \{0,1,+,-\}. \\ r + d_G(u) + d_G(v) - 2 & if\ \ a = +\ \&\ b \in \{0,1,+,-\}. \\ r + m + 1 - d_G(u) - d_G(v) & if\ \ a = -\ \&\ b \in \{0,1,+,-\}. \end{cases}$

(ii) $d_{Q^{ab1}(G)}(B') = \begin{cases} m & if\ \ b = 0\ \&\ a \in \{0,1,+,-\}. \\ m + r - 1 & if\ \ b = 1\ \&\ a \in \{0,1,+,-\}. \\ m + d_G(B) & if\ \ b = +\ \&\ a \in \{0,1,+,-\}. \\ m + r - 1 - d_G(B) & if\ \ b = -\ \&\ a \in \{0,1,+,-\}. \end{cases}$

Theorem 3.4: *Let G be an (n, m)-graph with r blocks. Then the degree of edge-vertex e' $(e = uv$ in $G)$ and block-vertex B' in $Q^{abc}(G)$ when $c = +$ are*

(i) $d_{Q^{ab+}(G)}(e') = \begin{cases} 1 & if\ \ a = 0\ \&\ b \in \{0,1,+,-\}. \\ m & if\ \ a = 1\ \&\ b \in \{0,1,+,-\}. \\ d_G(u) + d_G(v) - 1 & if\ \ a = +\ \&\ b \in \{0,1,+,-\}. \\ m + 2 - d_G(u) - d_G(v) & if\ \ a = -\ \&\ b \in \{0,1,+,-\}. \end{cases}$

(ii) $d_{Q^{ab+}(G)}(B') = \begin{cases} D_G(B) & if\ \ b = 0\ \ \&\ \ a \in \{0,1,+,-\}. \\ D_G(B) + r - 1 & if\ \ b = 1\ \ \&\ \ a \in \{0,1,+,-\}. \\ D_G(B) + d_G(B) & if\ \ b = +\ \ \&\ \ a \in \{0,1,+,-\}. \\ D_G(B) + r - 1 - d_G(B) & if\ \ b = -\ \ \&\ \ a \in \{0,1,+,-\}. \end{cases}$

Theorem 3.5: *Let G be an (n,m)-graph with r blocks. Then the degree of edge-vertex e' $(e = uv$ in $G)$ and block-vertex B' in $Q^{abc}(G)$ when $c = -$ are*

(i) $d_{Q^{ab-}(G)}(e') = \begin{cases} r - 1 & if\ \ a = 0\ \ \&\ \ b \in \{0,1,+,-\}. \\ m + r - 2 & if\ \ a = 1\ \ \&\ \ b \in \{0,1,+,-\}. \\ d_G(u) + d_G(v) + r - 3 & if\ \ a = +\ \ \&\ \ b \in \{0,1,+,-\}. \\ m + r - d_G(u) - d_G(v) & if\ \ a = -\ \ \&\ \ b \in \{0,1,+,-\}. \end{cases}$

(ii) $d_{Q^{ab-}(G)}(B') = \begin{cases} m - D_G(B) & if\ \ b = 0\ \ \&\ \ a \in \{0,1,+,-\}. \\ m + r - D_G(B) - 1 & if\ \ b = 1\ \ \&\ \ a \in \{0,1,+,-\}. \\ d_G(B) + m - D_G(B) & if\ \ b = +\ \ \&\ \ a \in \{0,1,+,-\}. \\ m + r - 1 - D_G(B) - d_G(B) & if\ \ b = -\ \ \&\ \ a \in \{0,1,+,-\}. \end{cases}$

4. CONNECTEDNESS OF $Q^{abc}(G)$

The first theorem is well-known.

Theorem 4.1: *For a given graph G, $Q^{ab0}(G)$ is not connected.*

Theorem 4.2: *For a given graph G, $Q^{ab1}(G)$ is connected.*

Proof: The result follows from the fact of Remark 2.3 (iii) i. e., $K_{m,r}$ is a spanning subgraph of $Q^{ab1}(G)$ with parts $E(G)$ and $U(G)$. Therefore $Q^{ab1}(G)$ is connected
When $c = +$, we have the following theorems:

Theorem 4.3: *For a given graph G, $Q^{00+}(G)$ is connected if and only if G is a block.*

Proof: Suppose G is a block with m edges. Then $Q^{00+}(G) = K_{1,m}$ and which is connected.

Conversely, if G has at least two blocks, then $Q^{00+}(G)$ has at least two disjoint stars. Therefore $Q^{00+}(G)$ is disconnected, a contradiction.

Theorem 4.4: *For a given graph G, $Q^{1b+}(G)$ is connected.*

Proof: From Remark 2.1 (iii), we have K_m is an induced subgraph of $Q^{1b+}(G)$ with vertex set $E(G)$ and each block-vertex B' is adjacent to at least one edge-vertex e' where e is incident with a block B in G. Therefore $Q^{1b+}(G)$ is connected.

Theorem 4.5: *For a given graph G, $Q^{+0+}(G)$ is connected if and only if G is connected.*

Proof: Suppose G is connected. Then $L(G)$ is connected. By Remark 2.1 (i), $L(G)$ is a connected induced subgraph of $Q^{+0+}(G)$ and each block-vertex B' is adjacent to at least one edge-vertex e' where e is incident with a block B in G. Therefore $Q^{+0+}(G)$ is connected.

Conversely, suppose $Q^{+0+}(G)$ is connected. If G is a disconnected graph with at least two components G_1 and G_2, then $Q^{+0+}(G) = Q^{+0+}(G_1) \cup Q^{+0+}(G_2)$ is disconnected, a contradiction.

Theorem 4.6: *For a given graph G, $Q^{-0+}(G)$ is connected if and only if G contains no block K_2 that is adjacent to every other edge of G.*

Proof: Suppose a graph G contains no block K_2 that is adjacent to every other edge of G. If G is a block, then $Q^{-0+}(G) = \overline{L(G)} + K_1$ is connected. If G has more than one block, then we consider the following two cases:

Case-1: If G contains no edge that is adjacent to every other edge of G, then by Remark 2.1 and Theorem 2.1, $\overline{L(G)}$ is a connected subgraph of $Q^{-0+}(G)$, and in $Q^{-0+}(G)$, each block-vertex B_i' is adjacent to at least one edge-vertex e_j', where e_j is incident with B_i in G. Thus $Q^{-0+}(G)$ is connected.

Case-2: If G contains an edge e that is adjacent to every other edge of G, then clearly e is incident with a block B of size more than 2. And $Q^{-0+}(G-e)$ is a connected subgraph of $Q^{-0+}(G)$ and e', B', e_1' is a path in $Q^{-0+}(G)$, where e_1 is incident with B, and each block-vertex B_i' in $Q^{-0+}(G)$ is adjacent to at least one edge-vertex e_j', where e_j is incident with B_i in G. Hence $Q^{-0+}(G)$ is connected.

Conversely, suppose $Q^{-0+}(G)$ is connected. Assume G contains a block K_2, say e, that is adjacent to every other edge of G, then it is easy to see that $Q^{-0+}(G) = Q^{-0+}(G-e) \cup K_2$ is disconnected, a contradiction.

Theorem 4.7: *For a given graph G, $Q^{a1+}(G)$ is connected.*

Proof: From Remark 2.2 (iii), we have K_r is an induced subgraph of $Q^{a1+}(G)$ with vertex set $U(G)$ and each edge-vertex e' is adjacent to exactly one block-vertex B' where e is incident with a block B in G. Therefore $Q^{a1+}(G)$ is connected.

Theorem 4.8: *For a given graph G, $Q^{0++}(G)$ is connected if and only if G is connected..*

Proof: Suppose G is connected. Then $B(G)$ is connected. By Remark 2.2 (i), $B(G)$ is a connected induced subgraph of $Q^{0++}(G)$ and each edge-vertex e' is adjacent to exactly one block-vertex B' where e is incident with a block B in G. Therefore $Q^{0++}(G)$ is connected.

Conversely, suppose $Q^{0++}(G)$ is connected. If G is disconnected graph with at least two component G_1 and G_2, then $Q^{0++}(G) = Q^{0++}(G_1) \cup Q^{0++}(G_2)$ is disconnected, a contradiction.

Theorem 4.9: *For a given graph G, $Q^{+++}(G)$ is connected if and only if G is connected.*

Proof: Suppose G is connected. Then by Theorem 4.8, $Q^{0++}(G)$ is connected, and we have $Q^{+++}(G)$ is spanning subgraph of $Q^{+++}(G)$. Therefore, $Q^{+++}(G)$ is connected.

Conversely, suppose $Q^{+++}(G)$ is connected. If G is disconnected graph with at least two component G_1 and G_2, then $Q^{+++}(G) = Q^{+++}(G_1) \cup Q^{+++}(G_2)$ is disconnected, a contradiction.

Theorem 4.10: *$Q^{-++}(G)$ is connected for any graph G.*

Proof: If G is connected, then by Remark 2.2 (i), $B(G)$ is a connected induced subgraph of $Q^{-++}(G)$, and each edge-vertex e_i' in $Q^{-++}(G)$ is adjacent to exactly one block-vertex B_x', where B_x is incident with e_i in G. Thus $Q^{-++}(G)$ is connected.

If G is disconnected, then by Remark 2.1 (ii) and Theorem 2.1, $\overline{L(G)}$ is a connected induced subgraph of $Q^{-++}(G)$, and each block-vertex B_x' in $Q^{-++}(G)$ is adjacent to at least one edge-vertex e_i', where e_i is incident with B_x in G. Thus $Q^{-++}(G)$ is connected.

Theorem 4.11: *For a given graph G, $Q^{0-+}(G)$ is connected if and only if G contains no block that is adjacent to every other block of G.*

Proof: Suppose a graph G contains no block that is adjacent to every other block of G. Then $\overline{B(G)}$ is a connected induced subgraph of $Q^{0-+}(G)$ with vertex set $U(G)$, and each edge-vertex e' is adjacent to exactly one block-vertex B' where e is incident with a block B in G. Therefore $Q^{0-+}(G)$ is connected.

Conversely, suppose $Q^{0-+}(G)$ is connected. Assume G contains a block B that is adjacent to every other blocks of G, and B is incident with $e_1, e_2, \ldots, e_s$ edges. Then it is easy to see that $Q^{0-+}(G) = Q^{0-+}(G - \{e_1, e_2, \ldots, e_s\}) \cup K_{1,s}$ is disconnected, a contradiction.

Theorem 4.12: *$Q^{+-+}(G)$ is connected for any graph G.*

Proof: If G is connected, then by Remark 2.1 (i), $L(G)$ is a connected induced subgraph of $Q^{+-+}(G)$, and each block-vertex B_x' in $Q^{+-+}(G)$ is adjacent to at least one edge-vertex e_i', where e_i is incident with B_x in G. Thus $Q^{+-+}(G)$ is connected.

If G is disconnected, then by Remark 2.2 (ii), $\overline{B(G)}$ is a connected induced subgraph of $Q^{+-+}(G)$, and each edge-vertex e_i' in G^{+-+} is adjacent to exactly one block-vertex B_x', where B_x is incident with e_i in G. Thus $Q^{+-+}(G)$ is connected.

Theorem 4.13: *For a given graph G, $Q^{--+}(G)$ is connected if and only if G contains no block K_2 that is adjacent to every other edge of G.*

Proof: Suppose a graph G contains no block K_2 that is adjacent to every other edge of G. Then by Theorem 4.6, $Q^{-0+}(G)$ is connected, and we have $Q^{-0+}(G)$ is a spanning subgraph of $Q^{--+}(G)$. Therefore, $Q^{--+}(G)$ is connected. Conversely, suppose $Q^{--+}(G)$ is connected. Assume G contains a block K_2, say e, that is adjacent to every other edge of G. Then it is easy to see that $Q^{--+}(G) = Q^{--+}(G-e) \cup K_2$ is disconnected, a contradiction.

When $c = -$, we have the following theorems:

Theorem 4.14: *For a given graph G, $Q^{00-}(G)$ is connected if and only if G has at least three blocks.*

Proof: Suppose G contains at least three blocks. Then each edge-vertex is adjacent to at least two block-vertex in $Q^{00-}(G)$. Therefore it is sufficient to prove every pair of edge-vertices are connected. Let e'_1 and e'_2 be the edge-vertices of $Q^{00-}(G)$. Then there exist a block B' which is not incident with e_1 and e_2 in G such that e'_1 and e'_2 are connected through B' in $Q^{00-}(G)$. Therefore, every pair of vertices in $Q^{00-}(G)$ are connected. Hence $Q^{00-}(G)$ is connected.

Conversely, suppose $Q^{00-}(G)$ is connected. Assume G is a block. Then $Q^{00-}(G) = (m+1)K_1$ is disconnected, a contradiction. Assume G has two blocks B_1 and B_2 with x and y number of incident edges respectively. Then $Q^{00-}(G) = K_{1,x} \cup K_{1,y}$ is disconnected, a contradiction.

Theorem 4.15: *For a given graph G, $Q^{1b-}(G)$ is connected if and only if G is not a block.*

Proof: Suppose G is not a block. By Remark 2.1 (iii), we have K_m is an induced subgraph of $Q^{1b-}(G)$ with vertex set $E(G)$, and each block-vertex B' is adjacent with at least one edge-vertex e' where e is not incident with block B in G. Therefore $Q^{1b-}(G)$ is connected.

Conversely, suppose $Q^{1b-}(G)$ is connected. Assume G is a block. Then $Q^{1b-}(G) = K_m \cup K_1$ is disconnected, a contradiction.

Theorem 4.16: *For a given graph G, $Q^{+0-}(G)$ is connected if and only if G is neither a block nor a union of two blocks.*

Proof: Suppose G is neither a block nor a union of two blocks. Then we consider the following cases:

Case-1: Suppose G is connected. Then it has at least two blocks. Hence by Remark 2.1 (i), $L(G)$ is a connected subgraph of $Q^{+0-}(G)$, and also each block-vertex B'_i in $Q^{+0-}(G)$ is adjacent to at least one edge-vertex e'_j, where e_j is not incident with B_i in G. Thus $Q^{+0-}(G)$ is connected.

Case-2: Suppose G is disconnected. Then it has at least three blocks. We see that in $Q^{+0-}(G)$, each block-vertex B'_i is adjacent at least two edge-vertices e'_j, where e_j is not incident with B_i in G, and each edge-vertex e'_j is adjacent to edge-vertex e'_k and at least two block-vertices B'_i in $Q^{+0-}(G)$, where e_k is adjacent to e_j, and B_i is not incident with e_j in G.

Since in such a case, there is a path between any two vertices of $Q^{+0-}(G)$. Hence $Q^{+0-}(G)$ is connected.

Conversely, suppose $Q^{+0-}(G)$ is connected. If G is a block, then $Q^{+0-}(G) = L(G) \cup K_1$ is disconnected, a contradiction. If $G = B_1 \cup B_2$ is a union of two blocks, then $Q^{+0-}(G)$ is a disconnected graph having two components namely $L(B_1) + K_1$ and $L(B_1) + K_1$, a contradiction.

Theorem 4.17: *For a given graph G, $Q^{-0-}(G)$ is connected if and only if $G \neq P_3$ is not a block.*

Proof: Suppose $G \neq P_3$ is not a block. We consider the following two cases:

Case-1: Suppose G contains no edge that is adjacent to every other edge of G. Then by Remark 2.1 (ii) and Theorem 2.1, $\overline{L(G)}$ is a connected subgraph of $Q^{-0-}(G)$, and each block-vertex B'_i is adjacent to at least one edge-vertex e'_j in $Q^{-0-}(G)$, where e_j is not incident with B_i in G. Thus $Q^{-0-}(G)$ is connected.

Case-2: Suppose G contains an edge e that is adjacent to all other edge of G. Then by definition of $Q^{-0-}(G)$, each edge-vertex e_i' is adjacent to edge-vertex e_k' and at least one block-vertex B_j', where B_j is not incident with e_i, and e_k is not adjacent to e_i in G. And also each block-vertex B_j' is adjacent to at least one edge-vertex e_i', where e_i is not incident with B_j in G. Hence there is a path between any two vertices of $Q^{-0-}(G)$. Therefore $Q^{-0-}(G)$ is connected.

Conversely, suppose $Q^{-0-}(G)$ is connected. If G is a block, then $Q^{-0-}(G) = \overline{L(G)} \cup K_1$ is disconnected, a contradiction. If $G = P_3$, then $Q^{-0-}(G) = 2K_2$ is disconnected, a contradiction.

Theorem 4.18: *For a given graph G, $Q^{a1-}(G)$ is connected if and only if G is not a block.*

Proof: Suppose G is not a block. By Remark 2.2 (iii), we have K_r is an induced subgraph of $Q^{a1-}(G)$ with vertex set $U(G)$, and each edge-vertex e' is adjacent to at least one block-vertex B' where e is not incident with block B in G. Therefore, $Q^{a1-}(G)$ is connected.

Conversely, suppose $Q^{a1-}(G)$ is connected. Assume G is a block. Then $Q^{a1-}(G) = (L(G))^a \cup K_1$ is disconnected, a contradiction.

Theorem 4.19: *For a given graph G, $Q^{0+-}(G)$ is connected if and only if G is neither a block nor a union of two blocks.*

Proof: Suppose G is neither a block nor a union of two blocks. Then we consider the following cases:

Case-1: Suppose G is connected. Then it has at least two blocks. By Remark 2.2 (i), $B(G)$ is a connected induced subgraph of $Q^{0+-}(G)$ with vertex set $U(G)$, and also each edge-vertex e_i' in $Q^{0+-}(G)$ is adjacent to at least one block-vertex B_j', where e_i is not incident with B_j in G. Therefore $Q^{0+-}(G)$ is connected.

Case-2: Suppose G is disconnected. Then it has at least three blocks and we have $Q^{00-}(G)$ is a spanning subgraph of $Q^{0+-}(G)$. Therefore by Theorem 4.14, $Q^{0+-}(G)$ is connected.

Conversely, suppose $Q^{0+-}(G)$ is connected. If G is a block, then $Q^{0+-}(G) = (m+1)K_1$ is disconnected, a contradiction. If $G = B_1 \cup B_2$ is not a union of blocks, where B_1 and B_2 are blocks incident with x and y number of edges respectively, then $Q^{0+-}(G) = K_{1,x} \cup K_{1,y}$ is disconnected, a contradiction.

Theorem 4.20: *For a given graph G, G^{++-} is connected if and only if G is neither a block nor a union of two blocks.*

Proof: Suppose G is neither a block nor a union of two blocks. Then by Theorem 4.19, $Q^{0+-}(G)$ is connected, and we have $Q^{0+-}(G)$ is spanning subgraph of $Q^{++-}(G)$. Therefore, $Q^{++-}(G)$ is connected.

Conversely, suppose $Q^{++-}(G)$ is connected. If G is a block, then $Q^{++-}(G) = L(G) \cup K_1$ is disconnected, a contradiction. If $G = B_1 \cup B_2$ is not a union of blocks, then $Q^{++-}(G) = (L(B_1) + K_1) \cup (L(B_2) + K_1)$ is disconnected, a contradiction.

Theorem 4.21: *For a given graph G, $Q^{-+-}(G)$ is connected if and only if G is not a block.*

Proof: Suppose $G \neq P_3$ is not a block. Then by Theorem 4.17, $Q^{-0-}(G)$ is connected, and we have $Q^{-0-}(G)$ is spanning subgraph of $Q^{-+-}(G)$. Therefore, $Q^{-+-}(G)$ is connected. If $G = P_3$, then $Q^{-+-}(G) = P_4$ is connected.
Conversely, if G is a block, then $G^{-+-} = \overline{L(G)} \cup K_1$ is disconnected, a contradiction.

Theorem 4.22: *For a given graph G, $Q^{0--}(G)$ is connected if and only if G is not a connected graph with one or two blocks.*

Proof: Suppose G is not a connected graph with one or two blocks. We consider the following cases:

Case-1: If G is a connected graph. Then G contains at least three blocks and we have $Q^{00-}(G)$ is a spanning subgraph of $Q^{0--}(G)$. Hence by Theorem 4.14, $Q^{0--}(G)$ is connected.

Case-2: If G is not a connected graph, then G contains at least two blocks. Suppose G contains at least three blocks. Then result is oblivious by Theorem 4.14. If $G = B_i \cup B_j$ where B_i and B_j are blocks with x and y number of incident edges respectively, then $Q^{0--}(G) = S_{x,y}$ is connected.

Conversely, suppose $Q^{0--}(G)$ is connected. Assume G is a block. Then $Q^{0--}(G) = \overline{B(G)} \cup K_1$ is disconnected, a contradiction. Assume G is a connected graph with two blocks B_1 and B_2 having x and y number of incident edges respectively, then $Q^{0--}(G) = K_{1,x} \cup K_{1,y}$ is disconnected, a contradiction.

Theorem 4.23: *For a given graph* G, $Q^{+--}(G)$ *is connected if and only if* G *is not a block.*

Proof: Suppose G is not a block. Then by Theorem 4.22, $Q^{0--}(G)$ is connected, and we have $Q^{0--}(G)$ is spanning subgraph of $Q^{+--}(G)$. Therefore, $Q^{+--}(G)$ is connected. If G is connected graph with two blocks, then $Q^{+--}(G)$ is connected.

Converse is obvious.

Theorem 4.24: *For a given graph* G, $Q^{---}(G)$ *is connected if and only if* $G \neq P_3$ *is not a block.*

Proof: Suppose $G \neq P_3$ is not a block. Then by Theorem 4.17, $Q^{-0-}(G)$ is connected, and we have $Q^{0--}(G)$ is spanning subgraph of $Q^{---}(G)$. Therefore, $Q^{---}(G)$ is connected.

Conversely, suppose $Q^{---}(G)$ is connected. If G is a block, then $Q^{---}(G) = \overline{L(G)} \cup K_1$ is disconnected, a contradiction. If $G = P_3$, then $Q^{---}(G) = 2K_2$ is disconnected, a contradiction.

5. CONCLUSION

In this paper, we have introduced 64 abc - block edge transformation graphs and studied their order, size, vertex degree and connectedness of these 64 abc-block edge transformation graphs. The study of diameter, traversability, planarity, chromatic number, domination number, spectra, energy and topological indices of these new graphs can be interesting. Characterization of these 64 abc - block edge transformation graphs can be quite challenging, (i.e., to prove that: A graph G is a generalized abc - block edge transformation graph if and only if it is isomorphic to the generalized abc - block edge transformation graph $Q^{abc}(H)$ of some graph H).

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